Numerical Modeling for Information Diffusion on Social Media: a Review of Recent Works

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Abstract. Social media play a central role in information diffusion, enabling users to rapidly generate and share contents, often without adequate verification. This makes such platforms particularly susceptible to the spread of fake or partially verified information. In this work, we will present some deterministic models for the analysis of information propagation, inspired by epidemiology and formulated as systems of ordinary differential equations. Firstly, we will review the Ignorant–Spreader–Recovered (ISR) model, and some of its possible extensions that incorporate additional behavioral and structural features, including the Ignorant–Exposed–Spreader–Skeptic (IESZ) and the Ignorant–Spreader–Counter spreader–Recovered (ISCR) models. Furthermore, we will consider an age–structured version of the ISR model to better capture heterogeneity in users' activities and interactions. Firstly, we will focus on the qualitative features of these models, particularly highlighting the role of stiffness and its connection to the characteristic time scales of information spread. Secondly, since some properties of the considered models are a–priori known, as positivity, we will review the construction of adapted numerical methods, with an emphasis on the so–called Nonstandard Finite Difference (NSFD) schemes. Finally, we will show an application of the presented models and numerical approaches for the analysis of the spread of true and fake news recently shared on X.

Keywords: Information diffusion · Epidemiological models · ISR model · IESR model · IESZ model · ISCR model · age-group model · NSFD scheme · positivity preservation · stiffness analysis

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1 Introduction

The rapid growth of digital technologies has reshaped the way information is produced, distributed, and consumed. In today's hyperconnected world, social media platforms have become primary channels for news dissemination, gradually surpassing traditional media such as newspapers, since they are often freely accessible and easy to use. As a result, users increasingly rely on personalized feeds, streaming content, and online communities to stay informed, contributing to a complex and dynamic information ecosystem that also includes blogs, forums, and other digital platforms.

However, alongside the benefits of accessibility and speed, this transformation has introduced significant challenges concerning the quality and reliability of information. Social media facilitate not only the spread of accurate news but also the rapid diffusion of misleading or false content. Although the dissemination of false information is not a new phenomenon, its impact has been amplified in the digital era. To better understand and address this phenomenon, recent research has focused on detection methods for false information using machine learning and AI techniques, ranging from probabilistic methods such as Bayesian classifiers, to neural network - based models that consider textual features and patterns of user interactions, as well as approaches leveraging text pre-processing and representation, including tokenization, stemming, and N-grams [18,19]. This review paper aims to present, in a structured and self-contained manner, selected recent contributions to the modeling of information diffusion and their numerical treatment, with a focus on ordinary differential equation (ODE) models for describing and predicting news dissemination, whether the information is true or false, adapted numerical methods for their solution, and strategies for data acquisition and parameter estimation, highlighting their scope of application and main limitations.

Mathematical Models

A key role in this field is played by epidemiological models, i.e. the mathematical models used to describe epidemics [27]. These models are generally of compartmental type, that is they divide the population affected by the virus in classes, as susceptibles, infected, recovered and so on, each of which ruled by a differential equation, treating the news as a virus that spreads in a population of living beings. This formulation can be done through ODEs, as e.g. in [5,7,15–17,26,32], Partial Differential Equations (PDEs) [37], as well as a stochastic formulation that incorporate probabilistic aspects of user behavior (see [32] and references therein).

Among the classical models of this type, the so–called Ignorant–Spreader–Recovered ODE model is very well known. It takes into account three groups of individuals, as the ones who can see a news and share it, called Ignorants (I), the ones who share a news called Spreaders (S), and the group of individuals who do not believe anymore in the news, called Recovered (R). There is no unique formulation of the ISR model, as several models of these kind can be found in literature. As example, in [32,33] the authors propose several ISR models in order to better catch the dynamics of information diffusion depending on whether

a news item was spread on a particular website or social media, proposing at the same time also a possible ISR model with a decay, in order to consider that a possible loss of interest towards the news may occur as time passes. Many models of interest that extend the ISR class are also mentioned in [16,32,33] and reference therein, where the authors propose an Ignorant–Exposed–Spreader–Recovered (IESR) model that takes into account the possibility that an ignorant can see a news and believe it, but without sharing anything related to it. In this case, this individual is considered as a member of the exposed class (E). This class is the counter part of the exposed class of the epidemiological models, i.e. the class of all the individuals that have been in contact with an infectious but have not develop the symptoms of the disease yet. Worth of mention are also the classes of methods of Ignorant - Spreader - Counter Spreader - Recovered (ISCR) [12,32,33] and of Ignorant - Exposed - Spreader - Skeptic (IESZ) [12,13,26]. Both of them are built in such a way that it is possible to capture more complex dynamics of the information diffusion process. In fact, the former takes into account two classes of spreaders, that is the spreaders that share a piece of news and the counter spreaders that share news with an opposite content. The latter, on the contrary, is suitable for the description of the initial phases of the information spread. It takes into account, apart from ignorants and spreaders, the classes of exposed (E), defined before, and skeptics (Z), which is the class of individuals that have seen a news but remained indifferent to it. All these models can be further refined by considering their age-structured counterparts, as done for a specific ISR model in [4,5]. In this framework, the system is formulated by accounting for the presence of individuals belonging to different age groups. Consequently, the spreading dynamics of a given piece of information may vary across age groups, reflecting different levels of interest. Each compartment of individuals is further subdivided into a set of subclasses, each corresponding to a specific age group. These subclasses retain the same epidemiological meaning and characteristics as the original compartment, while being distinguished by their associated age group index.

Furthermore, in order to more accurately characterize the dynamics of information diffusion, it is essential to investigate the fundamental properties of this typology of mathematical models. In particular, it has been rigorously established in [15,16] that the propagation speed of information is strongly associated with specific features of the underlying problem, such as stiffness. More precisely, these studies show that an increase in the stiffness of the problem described by an ISR model corresponds to a higher likelihood of information diffusion.

Numerical Methods

Regardless the model being considered, that depends on the problem we are describing, note that in all cases the solution of the ODEs system is positive. Moreover, equilibrium points could be present. Thus, these aspects should be taken into account while solving the chosen model. For this reason, positivity preserving, elementary stable and conservative numerical methods are required. Several schemes of this kind are available in literature, as e.g. modified Patankar methods [3] or numerical methods based on the use of Nonstandard Finite Dif-

ferences (NSFD) [8,9,12,28–30]. The latter class is gaining more and more attention in recent years, since NSFD methods are easy to formulate, stable and not expensive in terms of computational cost. However, NSFD schemes have order of consistency equal to one. For this reason, in recent works, as [14], also higher order NSFD schemes for general ODE problems have been formulated. In the following, we will examine these typologies of schemes for the solution of the presented models. As we will see, NSFD methods, due to their good properties, can be advantageous for a phase of parameter estimation.

Data Acquisition and Parameter Estimation

Unlike the epidemiological scenario, information diffusion models do not benefit from a data-rich framework for parameter identification. In epidemiology, parameterization is typically supported by well-structured datasets, and model parameters have a clear empirical interpretation, being directly linked to measurable biological processes. For example, clinical evidence can be used to quantify characteristic time scales of the disease, which can then be systematically translated into model rates. By contrast, such a direct correspondence between data and parameters is largely missing in information diffusion. Available datasets are often limited and unstructured and, more importantly, the parameters themselves lack a consistent interpretation across contexts, since diffusion dynamics depend strongly on the specific content and the environment in which it spreads. Several approaches have been proposed in the literature to address this goal, ranging from classical optimization-based procedures to data-driven techniques based on Machine Learning approaches. Among the latter category, a possible strategy consists in the use of Physics Informed Neural Networks (PINNs), as for example done in [31], where PINNs where used to estimate the parameters of an epidemiological model employed to describe the COVID-19 epidemic. In this work, we focus on an optimization-based approach, following [7,12,13,26,34], as it proves to be effective even when only limited data are available, such as those corresponding to the early stages of the diffusion process. In this context, after an initial data acquisition phase, a subset of the dataset is employed for parameter calibration. Starting from an initial guess of the parameters vector, the estimation procedure is formulated as a nonlinear least-squares problem. At each iteration, the ODE system is solved and the error between the model output and the observed data, restricted to the initial phase of the diffusion, is evaluated in terms of the mean squared error. This quantity is then minimized using the *lsqnonlin* MATLAB built-in routine. As we will see, although the optimization procedure requires repeated numerical solutions of the ODE system, the overall computational cost remains limited thanks to the efficiency of the NSFD schemes. This makes the approach particularly suitable for practical applications, where reliable predictions of the overall evolution of the information spread can be obtained even from a relatively small amount of early-time data.

The paper is organized as follows. Section 2 is devoted to the presentation of epidemiological models for information diffusion of ISR type, considering both the classical and age-structured formulation, while Sect. 3 is focused on more

advanced models recently proposed in literature. In Sect. 4 an in depth analysis of some key features of the above mentioned models is carried out, with a particular focus on the relation between stiffness and the velocity of news propagation. Section 5 describes the data acquisition process and parameter estimation, both fundamentals for model validation. Section 6 is dedicated to the presentation and description of NSFD schemes, while Sect. 7 present some numerical experiments related to the diffusion of true or fake news shared on X (formerly Twitter) in recent years.

2 Epidemiological Models for Information Diffusion

Epidemiological-like models have recently gained a central role in the process of information diffusion, since they allow to catch several dynamics of the considered phenomenon. The idea is to treat the information shared online as a virus that spread in a population of living beings. In particular, epidemiological models are generally formulated as compartmental models, that is they divide the population under consideration into several classes, each of which including all the individuals with the same features. Thus, according to the epidemics being described, several classes can be observed. Among the foundational models in this class, the Susceptible–Infectious–Recovered (SIR) framework proposed by Kermack and McKendrick in 1927 remains one of the most widely adopted [27,34]. It provides a division of the population in the three classes: Susceptibles, i.e. the healthy individuals that can contract a virus; Infectious, i.e. the individuals who are ill and can spread the virus; Recovered, i.e. the individuals who have recovered from the disease and have gained immunity towards it. Similarly, a model for information diffusion can be formulated. That is, the individuals who surf the net or use a social network can be divided in classes according to their status during the information spreading process. In particular, this section will be devoted to the counter part of the aforementioned SIR model for epidemics. Specifically, the Ignorant–Spreader–Recovered (ISR) model and its variants will be introduced.

2.1 ISR Models

The ISR model provides a division of the population in the following classes:

- the ignorant class $I(t)$, that reunites those who are unaware of the news under consideration, and that is the counter part of the susceptible class of the SIR model;
- the spreader class $S(t)$, that is composed of all the individuals that, believing the news in question, decide to reshare it. It is the counter part of the infectious class of the SIR model;
- the recovered class $R(t)$, composed of those who have lost interest in the previously shared news and that, as a consequence, stop sharing it. It is the counter part of the recovered class of the SIR model.

Several approaches can be adopted to formulate this model. Of particular interest is the ISR model introduced in [15]:

$$\begin{cases} I'(t) = -\bar{\beta} I(t) S(t), \\ S'(t) = \bar{\beta} I(t) S(t) - \bar{\alpha} S(t), \\ R'(t) = \bar{\alpha} S(t). \end{cases} \tag{2.1}$$

Let us consider $N(t) = S(t) + I(t) + R(t)$, i.e. the total population size at time t. Note that $N'(t) = 0$, that is, the total population size is constant, since no entries or exits from the population are taken into account. The involved parameters $\bar{\beta}$ and $\bar{\alpha}$, respectively, define the contact rate between ignorants and spreaders that rules the transitions from the ignorant class to the spreader class and the rate of transition from the spreader class to the recovered class. Specifically, these parameters are defined as:

$$\bar{\beta} = \frac{\text{INT}}{10} \qquad \text{and} \qquad \bar{\alpha} = \frac{\text{HDI}}{100},$$

where INT stands for *Internet Penetration Index* and is a parameter representing the ease of access to the Internet in a country, while HDI stands for *Human Development Index* and indicates the social development rate of the country where the news is spreading. These indexes, in particular, are recorded per country, as reported in [23,24].

The dynamics among different classes can be also described through a graph. For the ISR model, the graph is in the following Fig. 1.

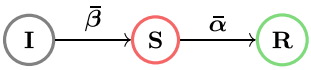

Fig. 1. Graph of the ISR model (2.1).

The aforementioned ISR model has been addressed by the authors for the analysis of fake news on social media. However, the ISR model can be formulated also for the process of information dissemination, regardless the truth of the news being shared. Interesting formulation is the one presented in [32,33] and analyzed in depth in [7]. It is an ISR model specific for social media, formulated taking into account the following parameters:

- κ, the average number of connections among users of a social network;
- β, the per capita news spreading rate per unit of time, that rules transition from the ignorant class to the spreader class;
- γ, the recovery rate, that rules the transition from the spreader class to the recovered class.

Note that the role of the parameter κ is fundamental. In fact, contrary to the epidemics, on social media encounters among individuals happen more frequently

since an encounter happens whenever people get in touch trough a reaction or a comment to a given post. This makes the process of transition between the spreader class to the recovered class much more faster than the corresponding transition from susceptibles to infectious in the epidemic scenario. The ISR model for social media reads as follows:

$$\begin{cases} I'(t) = -\beta\kappa I(t)S(t), \\ S'(t) = \beta\kappa I(t)S(t) - \gamma\kappa S(t)S(t), \\ R'(t) = \gamma\kappa S(t)S(t). \end{cases} \tag{2.2}$$

Observe that, unlike the ISR model (2.1), in this case transitions to the recovered class are governed by interactions between two spreaders. When one spreader encounters the news shared by another, it may realize that the information is false or, more generally, may lose interest in it. In both cases, this leads to a transition from the spreader class to the recovered class. This transition is mathematically expressed by the term $\gamma\kappa S(t)S(t)$.

2.2 Age-Structured ISR Models

As aforementioned, a possible approach to improve a model for information diffusion consists in considering an age-structured formulation. Such model has been proposed in [4,5] and summarized below.

Let assume that the population is divided into the n age groups. For each $i-$th group, we distinguish corresponding number of spreaders S_i, ignorants I_i and recovered R_i. The ISR model which describes the diffusion of information, proposed in [4,5], is:

$$\begin{cases} I_i'(t) = -\beta \displaystyle\sum_{j=1}^{n} M_{ij} \frac{S_j(t)}{N_j} I_i(t), \\ S_i'(t) = \beta \displaystyle\sum_{j=1}^{n} M_{ij} \frac{S_j(t)}{N_j} I_i(t) - \gamma S_i(t), \qquad t \in [0,T], \\ R_i'(t) = \gamma S_i(t), \end{cases} \tag{2.3}$$

with $i = 1, \ldots, n$. The contact rate $\beta > 0$ and the recovery rate $\gamma > 0$ are assumed to be uniform in the population groups; N_i is the (constant) number of individuals belonging to the age group g_i, $i = 1, \ldots, n$. The contacts among the spreaders and the ignorant of the different groups are modelled by the contact matrix M, which is nonnegative. In particular, in the diffusion of information in social media, M_{ij}/N_j stands for the probability of contact between an individual of the age-group i and another one of the age-group j. A practical and easy way to compute these values consist of setting $M_{ij}/N_i = p_j$, where p_j is the fraction of individuals of the $j-$th group.

In [5] it has been proved that the solution is nonnegative, for nonegative initial data, the total population is preserved, the reproduction number was

computed and is equal to $R_0 = \beta\rho(M)/\gamma$, the final size of the epidemic was derived. Moreover, a suitable NSFD method was formulated, and a Modified Patankar–Runge–Kutta scheme was adopted, both ensuring the preservation of positivity in the numerical solution.

3 Advanced Models for Information Diffusion

In the previous section, the ISR model class has been introduced, with a focus also on a possible age-structured formulation. However, in some cases, the ISR model cannot catch all the details of the scenario being described. For example, it may happen, in particular in the initial phases of a news spread, that not all the ignorant individuals that read a news are convinced of its content. Some people can be interested in the news, but need their time before sharing it. Others, on the contrary, can remain totally indifferent. Moreover, in the particular case of fake news dissemination, not all the spreaders share fake content. It can happen that a group of individuals, knowing that the shared news is fake, tries to restore the truth by sharing a news with an opposite content. Thus, to better capture these specific situations, more elaborate models extending the ISR framework can be introduced: the IESR model described in [16], the IESZ model described in [13,26] and the ISCR model proposed in [12].

3.1 IESR Model

The Ignorant - Exposed - Spreader - Recovered (IESR) model is one of the possible direct extension of the ISR model. Following the corresponding model for epidemics (see for e.g. [26]), it introduces the class of Exposed individuals E, i.e., those who have seen a piece of news and believed it but have not yet shared anything related to it yet. In particular, following [16], we present a more general IESR model, with a non constant population, in order to take into account that new individuals can enter the population, for example because influenced by oral confrontation or by news on the same topic spread through other means of communication. The new add rate of individuals to the ignorant population is ruled by the parameter Λ. Note that according to this model, when ignorant individuals get in touch with a news shared by a spreader, they believe in the news transiting into the expoded class, at rate $\tilde{\beta}$, and remaining there until the moment when they share something related to the news or when they definitely lose interest towards it. In the first case a transition into the spreader class, ruled by the parameter $\tilde{\gamma}$ can happen, while in the second case a transition into recovered class, at rate $\tilde{\delta}$, is possible. Note that also a transition from spreader to recovered is possible, ruled by the parameter $\tilde{\alpha}$ and determined by the time during which a piece of news remains trending among the most widespread news

(Fig. 2). Thus, the IESR model reads as follows:

$$\begin{cases} I'(t) = \Lambda - \tilde{\beta}\, I(t)S(t), \\ E'(t) = \tilde{\beta}\, I(t)S(t) - (\tilde{\gamma} + \tilde{\delta})E(t), \\ S'(t) = \tilde{\gamma}\, E(t) - \tilde{\alpha} S(t), \\ R'(t) = \tilde{\alpha}\, S(t) + \tilde{\delta} E(t). \end{cases} \tag{3.4}$$

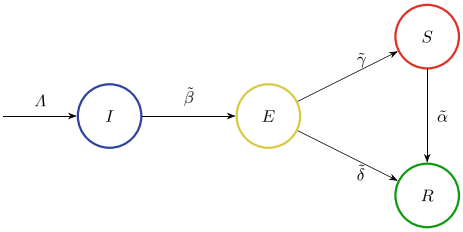

Fig. 2. Graph of the IESR model (3.4).

3.2 IESZ Model

The Ignorant–Exposed–Spreader–Skeptic model (IESZ), introduced in [13,26], slightly modifies the ISR model by considering, apart from ignorants and spreaders, the classes of Exposed E and Skeptics Z. It provides a distinction between the individuals who have seen a news, believed it but that have not shared anything related to it yet that transit into the exposed class, and the individuals who remain indifferent to the news. The latter ones are the individuals who will transit into the skeptic class. Note that, contrary to the ISR or IESR model, this one does not provide an exit from the spreader class, as clearly visible from the graph in Fig. 3. Once becoming spreader, an individual will remain in this status forever. Indeed, this model is particularly suitable to describe the initial phases of the epidemic, when we are interested in knowing how fast the number of spreaders will increase, without being interested in possible losses of interest towards the news.

The IESZ model reads as follows:

$$\begin{cases} I'(t) = -\hat{\beta} I(t) \frac{S(t)}{N} - bI(t) \frac{Z(t)}{N}, \\ E'(t) = (1-p)\hat{\beta} I(t) \frac{S(t)}{N} + (1-l)bI(t) \frac{Z(t)}{N} - \rho E(t) \frac{S(t)}{N} - \epsilon E(t), \\ S'(t) = p\beta I(t) \frac{S(t)}{N} + \rho E(t) \frac{S(t)}{N} + \epsilon E(t), \\ Z'(t) = lbI(t) \frac{Z(t)}{N}. \end{cases} \tag{3.5}$$

The model parameters are defined as follows: $\hat{\beta}$ denotes the rate at which ignorant individuals come into contact with the news and transition either to the

spreader class with probability p or to the exposed class with probability $1-p$; b denotes the rate at which ignorant individuals transition either to the skeptic class with probability l or to the exposed class with probability $1-l$. The parameter ρ represents the contact rate governing transitions from the exposed to the spreader class due to interactions with spreaders, while ϵ denotes the transition rate from exposed to spreader driven by an internal decision process, independent of further contacts. In particular, the relations among the different classes are described by the graph in Fig. 3.

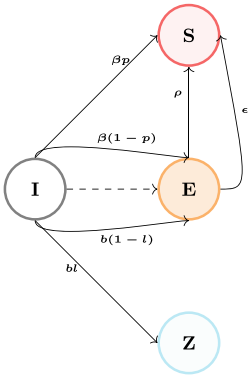

Fig. 3. Graph of the ISEZ model in (3.5).

3.3 ISCR Model

The Ignorant - Spreader - Counter spreader - Recovered (ISCR) model, firstly introduced in [32,33], is another direct extension of the ISR type models, and in particular of the ISR for social media described in the previous Sect. 2. This model is suitable to describe situations in which there are two groups of spreaders: the Spreaders S that share a news and the Counter spreaders C that try to stop the news spread by sharing content with opposite topic. Particularly interesting is the new ISCR model proposed in [12]. By considering, as done for the ISR model (2.2), κ the average number of connections of individuals, β the transition rate from the ignorant class to the spreader class and γ the transition rate from the spreader class to the recovered class, there the authors propose an ISCR model of the following form, whose corresponding graph is given by Fig. 4:

$$\begin{cases} I'(t) = \Lambda - \beta\kappa I(t)S(t) - \alpha\kappa I(t)C(t) - \delta I(t), \\ S'(t) = \beta\kappa I(t)S(t) - (\omega_1 - \omega_2)\kappa S(t)C(t) - \gamma\kappa S(t)[S(t) + C(t)] - \delta S(t), \\ C'(t) = \alpha\kappa I(t)C(t) + (\omega_1 - \omega_2)\kappa S(t)C(t) - \mu\kappa C(t)[S(t) + C(t)] - \delta C(t), \\ R'(t) = \gamma\kappa S(t)[S(t) + C(t)] + \mu\kappa C(t)[S(t) + C(t)] - \delta R(t). \end{cases} \tag{3.6}$$

The parameters α and μ are the counterparts of β and γ referred to counter spreaders, while ω_1 and ω_2 take into account the relation between spreaders and counter spreaders and rule the transitions of individuals between these classes. Furthermore, two additional parameters are considered: Λ, that as for the IESR model (3.4), is the the new addition rate to the ignorant class, and δ that has a similar meaning of the death rate for epidemics. In this context, this parameter has been introduced to take into account that during the process of information spreading some individuals might have problems accessing their accounts because reported as fake profiles or due to technical issues. Thus, the resulting model provides a total population size that evolve according to the simplified logistic equation $N'(t) = \Lambda - \delta N(t)$, where $N(t)$ indicates the total population size.

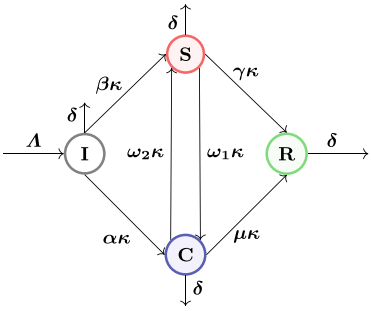

Fig. 4. Graph of the ISCR model (3.6).

4 Stiffness Analysis for the Prediction of Information Diffusion

By analyzing the structural properties of the ODE system underlying the given mathematical model, further insights into the mechanisms governing news diffusion can be obtained. In particular, as stressed in [15–17], there exist a strict link between the stiffness ratio of the system and the velocity of news dissemination, particularly in the case of fake news spread. The stiffness ratio is defined as the ratio in modulus between the largest and smallest nonzero eigenvalues of the Jacobian matrix associated with the linearized system corresponding to the ODE_S system of the model [25].

A large stiffness ratio indicates that fast and slow dynamics coexist, making the system *stiff*. In the particular case of the ISR model as explained in [15], a higher stiffness ratio corresponds to a faster transition of fake news across the population. This behavior is explained by the presence of a dominant fast time scale, which drives a rapid increase and subsequent decline in the number of spreaders. The same mechanism also leads to a faster re-establishment of the

truth, suggesting that stiffness captures not only the intensity of diffusion but also the system's responsiveness.

For example, we show below how to compute the stiffness ratio related to the ISR model (2.1). Suppose to have the following vector of initial conditions: $[I_0, S_0, R_0]^T = [I(0), S(0), R(0)]^T$. By computing the linearized system, the following Jacobian matrix is obtained:

$$J_{\alpha,\beta}(I_0, S_0) = \begin{bmatrix} -\bar{\beta}S_0 & \bar{\beta}I_0 & 0 \\ \bar{\beta}S_0 & \bar{\beta}I_0 - \bar{\alpha} & 0 \\ 0 & \bar{\alpha} & 0 \end{bmatrix}.$$

By computing the eigenvalues of this matrix, it is possible to obtain the stiffness ratio: the larger this value, the more the news will spread, affecting a high number of users. In this sense, the stiffness ratio provides a quantitative link between the mathematical structure of compartmental models and the qualitative behavior of information diffusion. It captures not only the speed of propagation, but also the efficiency of the mechanisms leading to the re-establishment of the truth, thus offering a valuable tool for the analysis of misinformation dynamics.

5 Data Acquisition and Parameter Estimation

The previous sections have introduced several mathematical models for the description of information diffusion, and also a link between mathematical properties of the model and the evolution of the phenomenon. However, in order to practically analyze this type of phenomena, it is fundamental to first construct a proper dataset to work with. In fact, there exist some tools to extract data from social networks, as for example the Python data scraper *Tweepy* or the X (formerly Twitter) portal *Twitter Developer Option*. However, the extracted data come in a qualitative format and are referred to a chosed "hashtag". In particular, the extracted information is related to user ID of the user who has shared the tweet, the date and time of the tweet and the written message. Thus, after the phase of data collecting, it is fundamental to organize the information in a quantitative way in order to construct a numerical dataset. Data are organized according to the following Algorithm 1, proposed in [7]. The algorithm assigns each user to the ignorant class until time t_0, when the first tweet is posted. At that point, the user moves into the spreader class and remains there until time $t_1 + t_{\text{exit}}$, that is, a duration t_{exit} after the moment of the last tweet t_1. The time t_{exit} needed to transition between compartments depends on the specific news being tracked. Therefore, when constructing the dataset, it is necessary to account for different transition times from the spreader to the recovered state.

Once constructed the dataset, other fundamental critical phase to be carried out for the model validation process is represented by the computation of the model parameters. Indeed, even if for particular models as the ISR given by (2.1) and presented in [15], parameters are computed starting from given tabulated indexes, generally the computation of these parameters is not trivial. On the

Algorithm 1. Constructing I, S, R vectors from Twitter data

```
I ← Ignorant Population
S ← Spreader Population
R ← Recovered Population
t ← First Data of dataset
t_end ← Last Data of dataset
t_exit ← Time to pass from S to R
for each user do
    t_0 ← first time this user tweeted
    t_1 ← last time this user tweeted
end for
loop
if t ≤ t_0 then
    I ← I + 1
    t ← t + t_exit
    go to loop
else
    if t_0 ≤ t ≤ t_1 + t_exit then
        S ← S + 1
        t ← t + t_exit
        go to loop
    else
        if t_1 + t_exit ≤ t ≤ t_end then
            R ← R + 1
            t ← t + t_exit
            go to loop
        else
            close loop
        end if
    end if
end if
```

contrary of the epidemiological scenarios in which for example parameters can be estimated by knowing some properties of the virus as its incubation period or the response of infected to medical treatments, with news dissemination this is not possible. Thus, in order to estimate the model parameters, a parameter estimation strategy is necessary. Following what has been proposed in [7,12,13], it is possible to determine a set of optimized parameters for the chosen model by minimizing a given error function. A possible choice for the latter consists in considering the mean squared error between real data at each time step t_j and the corresponding real values. This error, in particular, can be computed by considering the error on the actual number of spreaders and the number of the recovered at a given time step, in order to take into account the individuals who are spreading the considered news and the individuals who will not be involved in the news dissemination anymore. In particular, the error function reads as

follows:

$$f(param) = \sum_{j=1}^{n} (S_j - S(t_j))^2 + \sum_{j=1}^{n} (R_j - R(t_j))^2. \tag{5.7}$$

Here, n is the total number of data taken in consideration and *param* is the set of parameters to be optimized. From a computational point of view, several tools are available for the process of parameter optimization. In particular, in order to minimize the mean square error described by (5.7), it is possible to make use of the MATLAB built in function *lsqnonlin*. This routine allows the user to select the optimization algorithm used in the estimation process. Among the available options there is the trust-region reflective method, which supports parameter bounds and thus prevents solutions from taking on non-physical values, such as negative ones.

6 NSFD Schemes

In the previous section, we introduced a strategy for data acquisition and parameter estimation. As will be clear in the numerical experiments section, the approach we adopt consists of optimizing the model parameters by minimizing the error over a given set of real data. However, this procedure requires solving the system repeatedly, which makes the use of computationally expensive numerical methods impractical. Therefore, we need numerical schemes that are not only stable and accurate, but also efficient and computationally affordable. Moreover, we are dealing with models whose solutions must, e.g., remain positive, since they represent the number of individuals in each compartment of the population. For this reason, it is essential to employ numerical methods that preserve, e.g., positivity.

Among the available approaches, Nonstandard Finite Difference (NSFD) schemes, originally introduced by Mickens [28–30], provide a suitable framework, especially in the context of epidemiological-type models. These methods have been widely studied in the literature due to their ability to preserve essential qualitative properties of the continuous problem, such as positivity and stability of the equilibrium points, when appropriate construction principles are followed (see, e.g., [1,2,8–10,30]). To briefly recall these principles, consider an ODE problem of the form

$$\begin{cases} y'(t) = f(t, y), \\ y(t_0) = y_0, \end{cases} \qquad t \in [t_0, t_{end}],$$

where $f : [t_0, t_{end}] \times \mathbb{R}^m \to \mathbb{R}^m$ and $y : [t_0, t_{end}] \to \mathbb{R}^m$. Let $\{t_n = t_0 + nh,\ n = 0, \ldots, N\}$ be a uniform discretization of the time interval, with $t_N = t_{end}$. Starting from a standard finite difference (SFD) scheme, an NSFD scheme can be constructed by modifying two key components. First, the first order derivative of the unknown function is approximated through the generalized discrete form

$$y'(t_n) \approx \frac{y_{n+1} - \Psi(h)\, y_n}{\Phi(h)},$$

where $y_n \approx y(t_n)$, and $\Psi(h)$ and $\Phi(h)$ are nonnegative functions, referred to as the numerator and denominator functions. They, respectively, satisfy:

$$\Psi(h) = 1 + \mathcal{O}(h), \quad \Phi(h) = h + \mathcal{O}(h^2), \quad \text{as } h \to 0.$$

Note that the classical finite difference approximation is recovered when $\Psi(h) = 1$ and $\Phi(h) = h$. Second, nonlinear terms in $f(t, y)$ are approximated in a nonlocal manner, i.e. by approximating terms as $y^2(t_n)$ as $y_n y_{n+1}$.
By appropriately applying these modifications to a given standard explicit method, it is possible to construct schemes that remain explicit while better capturing the qualitative behavior of the continuous model, such as positivity of the solution. Moreover, since the modifications to the corresponding standard method are obtained through a perturbation only of order $\mathcal{O}(h)$, the consistency is automatically ensured (see, e.g., [2]).
In the following, we illustrate the construction of an NSFD scheme by considering the age-structured ISR model (2.3) and the ISCR model (3.6) as a representative examples.

6.1 NSFD Scheme for the Age-Structured ISR Model

In [5], these nonlocal representation of the right-hand side and denominator function have been adopted

$$\beta I_j S_i \to \beta I_j^{(k)} S_i^{(k+1)}, \gamma I_i \to \gamma I_i^{(k+1)}, \qquad \phi(h, \gamma) = \frac{1 - e^{-\gamma h}}{\gamma},$$

to derive the following NSFD method for the age-group ISR model (2.3):

$$\begin{cases} \dfrac{S_i^{(k+1)} - S_i^{(k)}}{\phi(h,\gamma)} = -\beta \displaystyle\sum_{j=1}^{n} M_{ij} \frac{I_j^{(k)}}{N_j} S_i^{(k+1)}, \\ \dfrac{I_i^{(k+1)} - I_i^{(k)}}{\phi(h,\gamma)} = \beta \displaystyle\sum_{j=1}^{n} M_{ij} \frac{I_j^{(k)}}{N_j} S_i^{(k+1)} - \gamma I_i^{(k+1)}, \\ \dfrac{R_i^{(k+1)} - R_i^{(k)}}{\phi(h,\gamma)} = \gamma I_i^{(k+1)}, \end{cases} \quad i = 1, \ldots, n, \quad k = 0, \ldots, K.$$

The proposed method preserves the total population and the positivity of the solution, for any choice of the stepsize $h > 0$. To overcome the accuracy limitations of first-order convergence of this method, Modified Patankar-Runge-Kutta methods provide a viable alternative for higher-order positivity preservation. Their formulation, however, requires a specific mathematical formalism that is beyond the scope of this work; the interested reader is referred to [5].

6.2 NSFD Scheme for the ISCR Model

By applying the previously introduced NSFD construction rules, it is possible to obtain a NSFD scheme for the ISCR model (3.6):

$$\begin{cases} \frac{I_{n+1}-I_n}{\Phi(h)} &= \Lambda - \beta\kappa I_{n+1}S_n - \alpha\kappa I_{n+1}C_n - \delta I_n, \\ \frac{S_{n+1}-S_n}{\Phi(h)} &= \beta\kappa I_{n+1}S_n - (\omega_1-\omega_2)\kappa S_{n+1}C_n - \gamma\kappa S_{n+1}(S_n+C_n) - \delta S_n, \\ \frac{C_{n+1}-C_n}{\Phi(h)} &= \alpha\kappa I_{n+1}C_n + (\omega_1-\omega_2)\kappa S_{n+1}C_n - \mu\kappa C_{n+1}(S_n+C_n) - \delta C_n, \\ \frac{R_{n+1}-R_n}{\Phi(h)} &= \gamma\kappa S_{n+1}(S_n+C_n) + \mu\kappa C_{n+1}(S_n+C_n) - \delta R_n. \end{cases}$$

For implementation, it is convenient to rewrite the NSFD scheme in explicit recursive form.

$$\begin{cases} I_{n+1} &= \dfrac{(1-\delta\Phi(h))I_n + \Lambda\Phi(h)}{1+\beta\kappa\Phi(h)S_n + \alpha\kappa\Phi(h)C_n}, \\ S_{n+1} &= \dfrac{(1-\delta\Phi(h)+\beta\kappa\Phi(h)I_{n+1})S_n}{1+(\omega_1-\omega_2)\kappa\Phi(h)C_n + \gamma\kappa\Phi(h)(S_n+C_n)}, \\ C_{n+1} &= \dfrac{(1-\delta\Phi(h)+\alpha\kappa\Phi(h)I_{n+1}+(\omega_1-\omega_2)\kappa\Phi(h)S_{n+1})C_n}{1+\mu\kappa\Phi(h)(S_n+C_n)}, \\ R_{n+1} &= (1-\delta\Phi(h))R_n + \Phi(h)\,\kappa(\gamma S_{n+1}+\mu C_{n+1})(S_n+C_n). \end{cases} \tag{6.8}$$

In particular, we indicate with N_n the discrete total population, computed as $N_n = I_n + S_n + C_n + R_n$. By construction, the obtained scheme (6.8) has first-order accuracy. Moreover, it is positivity preserving if the following denominator function is chosen:

$$\Phi(h) = \frac{1-e^{-\delta h}}{\delta}.$$

Further qualitative properties of the scheme, including the preservation of equilibrium points and their stability (elementary stability), have been proved in [12].

7 Numerical Experiments

In the previous sections, we have seen some mathematical models for the analysis of information diffusion on social media, as well as a possible strategy to estimate model parameters. Furthermore, Section 6 introduces a possible approach to develop a positivity preserving numerical scheme based on NSFD schemes. As pointed out in [15,16], in the first phases of information spread, to analyze news trend, it is possible to rely on the ISR model (2.1), since the parameters α and β are tabulated. This allows us to gain a general understanding of how a piece of news spreads within a given country. However, since our goal is to present a more accurate analysis of information spread on social media, in the following, we will analyze two specific pieces of news, rather than focusing on the general trend of news diffusion per country. By making advantage of the data extraction and parameter estimation strategies previously introduced, we analyze the evolution of the spread of the following news:

- the fake news spread with hashtag #DCBlackout [7,12,13,26], related to eventual issues with communications in the city of Washington D.C. at the beginning of June 2020, due to the Black Lives Matter movement;
- the true news spread with hashtag #MashaAmini [12], referring to the death of the Kurdish woman Mahsa Amini, who was detained by Iranian police in September 2022 after being accused of improperly wearing the hijab, in violation of Iranian law.

For further case studies, it is possible to refer to the GitHub repository [20], or to the numerical experiments sections of [7,11,12]. In this context, when a piece of news is disseminated, the focus is often not on the evolution itself, but on the identification of its peak diffusion. This corresponds to the time at which the spreading process reaches its maximum intensity, both in terms of the number of spreaders and the temporal location of this maximum. Such information is particularly crucial in the case of misinformation, as it enables the timely deployment of countermeasures to mitigate or even prevent further propagation. On the other hand, for verified or reliable news, identifying the peak can provide significant practical benefits. For example, when the information is related to a specific product, companies can leverage this insight to detect periods of highest public interest and accordingly optimize the timing of targeted marketing strategies. For these reasons, by employing the data acquisition and parameter estimation strategy as described in Section 5, particularly minimizing the error function (5.7), we compare the performances of the ISR model for social media (2.2) and of the ISCR model (3.6). In particular, we consider just a part of the dataset for the optimization strategy, that is the one with green $*$, while the remaining part of the dataset (in red $\bullet$) is used just to check how good predictions with optimmized parameters are. Note that, in the estimation procedure, all model parameters are optimized, with the exception of the add rate Λ and the mortality rate δ of the ISCR model. These two parameters are fixed consistently with the literature, specifically $\Lambda = 0.00023$ and $\delta = 0.001$, as suggested in [12,16]. Moreover, for both the considered models we assume $\kappa = \frac{\kappa_0}{N_0}$, in order to account for the dependence of the interaction rate on the population size. From a numerical perspective, the new ISCR model is repeatedly solved during the parameter estimation process by means of the NSFD scheme (6.8) introduced in Section 6. This approach presents a significant advantage, as it allows the use of relatively large time steps while still ensuring accurate results. This is particularly important since the ODE model provides only an approximation of real data: even when increasing the numerical accuracy, the error with respect to observed data does not decrease below a certain threshold. Therefore, the use of larger step sizes improves computational efficiency without compromising the overall quality of the estimation. For the ISR model, instead, we employ the MATLAB routine *ode45*. Results are displayed by Fig. 5, while optimized parameters are reported in Table 1 for the ISCR model and Table 2 for the ISR model. Moreover, Table 3 shows the errors on the total number of spreaders and on the time of the news spread peak, as well as CPU time for parameter estimation, when the ISCR model is used.

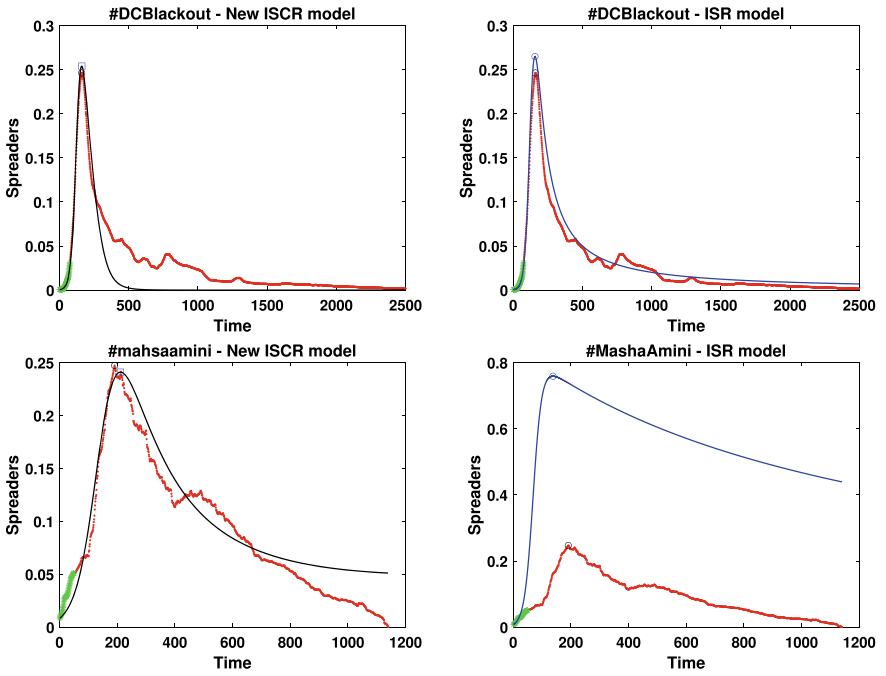

Fig. 5. News spread predictions using the ISCR model (3.6) and the ISR model (2.2), for #DCblackout (top) and #mashaamini (below). *:data for the fitting phase. •: data for prediction. **-**: solution of the model.

As it can be seen, the ISCR model produces better results than the ISR model on both the two case studies, since it better describes both the news evolution through time and the moment of the maximum spread of the news. This behavior is even more evident in the case of the news spread with hashtag #mashaamini, when the ISR model completely fails to catch the evolution dynamics. This outcome is not surprising, due to the main features of these two models. In fact, the ISR model provides a general description of news diffusion, whereas in the case of fake or higly debated news the ISCR model is expected to perform better, since it has been specifically designed to capture the underlying dynamics of such phenomena. Furthermore, Table 3 highlights a clear advantage of the NSFD scheme over MATLAB's ode45 solver. Indeed, even when ode45 is used with a relative tolerance of 10^{-3} to achieve errors of the same order of magnitude as the NSFD scheme (employed here with a discretization step $h = 2$), the NSFD method requires less computational time for solving the ISCR model during the parameter estimation phase. This demonstrates that NSFD achieves comparable accuracy while being more efficient. In [12], the authors also showed that the NSFD method (6.8) outperformes both the explicit and implicit Euler method, preserving the stability properties of the equilibrium points of the continuous model. Figure 5 shows also a peculiarity of the ISCR model. Although originally

Table 1. Parameter values of the ISCR model (3.6) for #DCBlackout and #mashaamini news.

News	β	γ	α	μ	k_0	ω_1	ω_2
#DCBlackout	0.30	0.03	0.30	0.04	0.20	1.66	1.05
#mashaamini	0.10	0.10	0.10	0.30	0.30	0.50	0.40

Table 2. Parameter values of the ISR model (2.2) for #DCBlackout and #mashaamini news.

News	β	γ	k_0
#DCBlackout	0.30	0.30	0.20
#mashaamini	2.02	0.03	0.03

Table 3. Comparison between NSFD and ode45 methods in terms of ErrTp, ErrSp, and CPUtime for the parameter estimation process, for the #DCBlackout and #mashaamini news.

News	**Method**	**ISCR**			**ISR**		
		ErrTp	**ErrSp**	**CPUtime**	**ErrTp**	**ErrSp**	**CPUtime**
#DCBlackout	NSFD ($h = 2$)	0.046	0.050	2.2188	0.046	0.050	2.2188
	ode45	0.045	0.050	3.5781	0.045	0.050	3.5781
#mashaamini	NSFD ($h = 2$)	0.082	0.015	0.71875	0.082	0.015	0.71875
	ode45	0.082	0.015	0.85938	0.082	0.015	0.85938

developed to study the propagation of fake news, where spreaders disseminate misinformation while counter spreaders attempt to restore the truth, the model is also suitable for the description of true news spread, as e.g. for the case of the news spread with the hashtag #mashaamini. This happens in the case of true but highly debated news, where distinct groups of individuals actively share and discuss content related to the topic.

8 Conclusion

In this work, several epidemiological models for analyzing information diffusion on social networks have been reviewed, ranging from basic frameworks such as the ISR class to more advanced formulations like the IESZ and ISCR models. In addition, a data acquisition and parameter estimation strategy has been discussed, highlighting its crucial role in the model validation phase. The importance of developing numerical methods that are both problem-oriented and computationally efficient has also been emphasized. Finally, numerical experiments have been presented, in which the proposed approach is applied to analyze the spread of news recently shared on X.

Several directions for future research are possible. First, the dataset construction process could be improved. Currently, without analyzing the content of individual tweets, it is only possible to construct the classes of ignorants, spreaders, and recovered users, while other classes, such as counter spreaders, cannot yet be defined. Furthermore, the number of data points used for parameter optimization (the green data) is currently selected empirically based on the news dynamics. Future work could aim to systematically link the number of points used for optimization phase to the total number of available data. Second, regarding the numerical method, it would be interesting to explore the application of positivity-preserving NSFD schemes of high order, following recent developments in the literature [14], to improve both accuracy and efficiency.

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