Preliminary Insights on the Modelling of a Wine Region Landscape with Human Factor and Willingness to Pay

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Abstract. This contribution describes preliminary insights on a research work that extends a three-dimensional model formulated as a system of Ordinary Differential Equations (ODEs). The model includes the Willingness To Pay (WTP) as an economic parameter to explore the potential dynamics within an environmental system decomposed into landscape units (LUs) in which the human factor influences the environment. The case study is the UNESCO wine landscape in Piedmont, and the results here presented refer to two representative areas. The purpose of this ongoing work is to test the sensitivity of the mathematical model with the WTP parameter conceived as the payment of a certain monetary amount in view of supporting the preservation of the landscape. Therefore, the authors recognise this model as a potentially useful decision-aiding tool for UNESCO landscape planning and management.

Keywords: Mathematical modelling · Human-environment system · WTP

1 Introduction

Landscapes continuously evolve according to the relationships between ecological processes, cross-spatial levels and human behavior [1]. Landscape planning requires multidisciplinary analysis and assessment capable of taking into account the complexity of landscape value [2]. In the case of cultural landscapes, the above mentioned interlinkages are crucial to preserve and manage this complex value through human generations. Among the methods and tools commonly applied in landscape planning, three-dimensional mathematical models falling

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into the family of Ordinary Differential Equations (ODEs) are carrying interesting insights to this research domain.

This paper describes a mathematical model for landscape ecology including economic and social features in the dynamics. This paper is the result of the experience and previous research works of the authors on mathematical modelling for landscape ecology and on landscape economics to support decision-aiding processes. Starting from the pioneer work in paper [3] until to the recent review in [4], this class of mathematical models shows a great potential in predicting the time evolution of landscape ecology characteristics in relation to social and/or human behavior. Particularly, this work provides a novel extension through the integration of an economic variable corresponding to the Willingness to Pay (WTP) expressed by society who live/visit a region. This direct integration represents a step forward at the interface between landscape ecology and landscape economics domains. Economic variables of pure public goods (e.g. cultural landscape), monetary and non-monetary, can be determined through decision-aiding evaluation techniques for the economic appraisal of project profitability [5,6], as well as for policy design [7]. From a methodological point of view, these tend to be employed individually or matched to the numerical simulations [8,9]. The authors searched on Scopus the following keywords to investigate whether these are directly integrated within the dynamical model: "ordinary differential equations", "willingness to pay" "mathematical model", "integration", "landscape" did not give back results in this resear. There are no valid results in this field, but rather in product development and epidemiology, so that they have been excluded [12,13]. Therefore it is possible to state that this work is carrying an original and novel contribution to the current state of academic literature.

The WTP represents the community's willing to support economically both landscape preservation and management solutions and it is obtained from a survey which belongs to a past study [10]. This is here normalised and integrated into the dynamical model together with ecological and social parameters to investigate the way of this economic attitude relates with ecological and human features in determining future states within a landscape. The dynamical model is applied to a wine region landscape in Southern Piedmont (Italy), namely "Vineyard Landscape of Langhe-Roero and Monferrato", which has been included in the UNESCO World Heritage List since 2014 [11]. This territorial context depicts an interesting case study to prove the reliability of the model, since the social dimension continues to shape the identity of this living cultural landscape over the centuries, whereas the wine-making process is the main driving force of the local economic development. The expected outcome of this research work is twofold. On the one hand, the paper extends a class of mathematical models for landscape ecology by directly incorporating an economic variable as one of the model's parameters. On the other hand, the results are finalised to support the definition of landscape preservation and management policies by enhancing the interdependencies between society and cultural identity, ecological state, and local economic development. After this introduction, the paper is organised in the following sections: Sect. 2 presents the model and describes the methodology

used to estimate the model's parameters based on ecological and economic data; Sect. 3 illustrates the application to the case study focused on two core zones of the UNESCO site; Sect. 4 presents the numerical solutions to explore how ecological conditions, social behaviour and the normalised WTP jointly influence the temporal evolution of the system; Sect. 5 closes with conclusions and future perspectives.

2 Methodology

This section presents the methodology used to develop the mathematical model and its ecological stability.

2.1 The Mathematical Model

We consider an environmental system consisting of N landscape units (LUs), representing distinct regions separated by natural or anthropogenic barriers, such as rivers, roads, motorways, railways, buildings, and industrial infrastructures.

Our framework builds upon the model introduced in [14], where human action is incorporated through the fraction of environmentalists within the population. This model is mathematically consistent and exhibits a rich dynamical behavior, in particular inducing periodic oscillations of environmental and human variables, associated with resilient territories.

In this work, we extend that model by incorporating an economic indicator representing the Willingness To Pay (WTP) of a community in a given region. This quantity captures the propensity of the population to allocate resources either to benefit from a natural asset or to support the restoration of green areas. Namely, we consider three state variables that describe the ecological state of each LU and the impact of the human behaviour on the ecological quality of the environment.

These indicators are represented by the normalized state variables $v(t)$, $b(t)$ and $x(t)$ at each time t, where v indicates the green area of the LU with high ecological quality, b measures the biological energy produced by the vegetation inside the LU, and x is the proportion of environmentalists that preserve and protect the green areas. The human-landscape model proposed in paper [14] to describe the ecological state of each LU is given by

$$\dot{v} = dv(1-v) - hUv - s(1-x)v, \tag{1a}$$

$$\dot{x} = \kappa x(1-x)[c - v + \delta(2x-1)], \tag{1b}$$

$$\dot{b} = ab(1-b) - l(1-v)b. \tag{1c}$$

Parameters d, h, U, a, and l describe structural and environmental characteristics of the LU, whereas s, κ, c, and δ govern the social dynamics. In particular, the parameter s links the social behaviour to the ecological dynamics. The specific meaning of the parameters is described as follows:

- d depends on solar exposure, relative humidity and ecotonal length, with this last quantity being the length of the borders among LUs; d is therefore an indicator of the ecological quality of the LU; it measures the intrinsic ecological quality and regenerative capacity of the LU;
- h is the ratio between the total perimeter of built-up sectors and the total perimeter of the LU; it captures fragmentation and edge effects generated by urbanization;
- $U \in (0, 1]$ is the ratio between the total built-up area and the total area of the LU, and it quantifies the level of urbanization within the LU;
- s is the human-caused environmental degradation rate, which measures the negative impact of the non-environmentalists on green areas and acts as a coupling parameter translating social behavior into ecological damage;
- κ represents the imitation speed, which determines how fast individuals modify their behavior through social interaction.
- c represents the perceived conservation value of green areas and the social benefit attributed to environmental quality and promotes pro-environmental behavior;
- δ measures the uniform pressure to remain in the same group due to injunctive social norms; it models conformity and social inertia and may reinforce majority behaviour;
- $a \in [0, 1[$ is the weighted average of the area of the LU exposed to SE-W-NE and measures the solar exposure of the LU;
- $l \in (0, 1]$ is the ratio between the surface area of the impermeable barriers included in the LU and the total area of the LU; it represents structural ecological constraints limiting vegetation growth.

In the present work, we introduce a variant of model (1) defined by incorporating an additional economic variable representing the Willingness To Pay (WTP) of a community in a given region. This quantity captures the propensity of the population to allocate resources in order to benefit from a natural asset. The new human-landscape model is described by

$$\dot{v} = dv(1 - v) - hUv - s(1 - x)v + w_1 v(1 - v), \tag{2a}$$

$$\dot{x} = \kappa x(1 - x)[c - v + \delta(2x - 1)], \tag{2b}$$

$$\dot{b} = ab(1 - b) - l(1 - v)b, \tag{2c}$$

where the parameter w_1 denotes normalized monetary quantities representing the WTP for the preservation of high-quality green areas within the region.

Remark 1. As in paper [14], we assume that the behavior of the non-environmentalists has a direct impact on the degradation of v, an effect captured by the loss term $-s(1-x)v$, with s denoting the rate of human-induced environmental degradation.

Remark 2. The new term $w_1 v(1 - v)$ in Eq. (2a) is a logistic type term representing the contribution of individuals willing to pay a maximum amount for the preservation of high quality green areas.

2.2 Equilibria and Stability

The temporal dynamics of model (2) can be studied in two steps, since Eq. (2c) decouples from the system (2a–2b), and therefore the dynamics of v and x are independent of that of b.

The analysis in this section will focus on the determination of equilibria of the reduced system (2a–2b), for the sake of brevity. Accordingly, we consider the reduced model

$$\dot{v} = dv(1-v) - hUv - s(1-x)v + w_1 v(1-v), \tag{3a}$$
$$\dot{x} = \kappa x(1-x)[c - v + \delta(2x-1)]. \tag{3b}$$

The above system retains the same mathematical structure as the one studied in [14]. Therefore, the analysis of the equilibria, including their existence and stability conditions, follows the same arguments as in [14].

The equilibrium states of model (3) are as follows

$$E_0 = (0,0), \quad E_1 = \left(1 - \frac{hU+s}{d+w_1}, 0\right), \quad E_2 = (0,1),$$
$$E_3 = \left(1 - \frac{hU}{d+w_1}, 1\right), \quad E_4 = \left(0, \frac{\delta - c}{2\delta}\right), \tag{4}$$
$$E_5 = \left(1 + \frac{s(-1+c+\delta) + 2hU\delta}{s - 2(d+w_1)\delta}, 1 + \frac{(d+w_1)(1+c-\delta) + hU}{s - 2(d+w_1)\delta}\right).$$

The conditions ensuring the existence of the equilibrium states (4), as well as their stability properties, are analogous to those obtained in [14] for the equilibria of the corresponding model, the details are therefore omitted.

3 Application

This section describes the case study and the estimation of the model's parameters, based on ecological and economic data, together with suitable assumptions for the social parameters.

3.1 Case Study: the Wine Region Landscape of Langhe-Roero and Monferrato

The case study falls in the UNESCO World Heritage Site "Vineyard landscape of Piedmont: Langhe-Roero and Monferrato" and consists of six core zones representing the Outstanding Universal Value (OUV) and two buffer zones aimed at preserving the serial site's value [11]. In this work, the term core zone is operationally labelled with the term landscape units (LUs), that is used in the literature on landscape ecology to refer to spatial portions composed of sets of

biotopes with a certain degree of homogeneity and all contribute to the configuration of a comprehensive ecomosaic of the Environment [15,16]. Indeed, the Landscape Regional Plan of Piedmont designated the perimeter of this UNESCO site following both the Landscape Ambits and Units, since they result in recognisable interactions between morphological, natural and cultural components.

This work focuses on two core zones of this UNESCO site as reference landscape units (Fig. 1). The core zone 1 "Langa del Barolo" corresponding here to the Landscape Unit 1 "Diano d'Alba', where Nebbiolo's vines form outstanding terraces and alternated by historical testimonies (e.g. castles, or boroughs, among others). The core zone 4 "Nizza Monferrato and the Barbera" is labelled here as "Landscape Unit 4" 'Nizza Monferrato' representing the beating core of Barbera's wine-making production of Monferrato.

The core zones were not chosen a priori by the authors, but were selected after performing numerical simulations of the mathematical model over all six core zones.

For each landscape unit, we investigated the system response under different degradation levels and in the presence or absence of the economic contribution, with the aim of assessing the combined role of ecological conditions, social behaviour, and Willingness To Pay mechanisms in shaping the system dynamics. The results revealed that these two landscape units, which have distinct configurations and dynamics, provide the most meaningful outputs.

3.2 Economic Parameter Estimation

The economic component of the model is based on the Willingness To Pay (WTP), estimated from the results of the paper [10] that developed a Multiple Regression Analysis based on a Choice Experiment (see Table 1) by surveying both residents and tourists, by asking their willingness to pay for three alternative scenarios of this landscape by varying its attributes (i.e.naturalness, vineyards extension, landscape quality, intangible heritage, cultural accessibility, and cost). That survey was conducted in those years immediately before the inclusion of this wine landscape into the UNESCO World Heritage List [10]. Those years were crucial in terms of social awareness on the value of these lands, as a result of a process begun in the early 2000s [11]. The choice of focusing on the results of a past study is motivated by the fact that the attention of the authors is currently on proving the reliability of this mathematical modelling extension and reflecting outputs' expectations. The Choice Experiment regressors are taken into account to estimate the individual WTP parameter (€) for both residents and tourists, and for each alternative scenario presented by that survey. Due to the limited number of pages, the authors decided to focus on the first alternative scenario of that study (Alternative 1) [10], which represented the state of the art, characterized by:

- Naturalness and protected species: medium/low
- Vineyards: medium-sized vineyard areas
- Landscape quality: excellent and high landscape quality

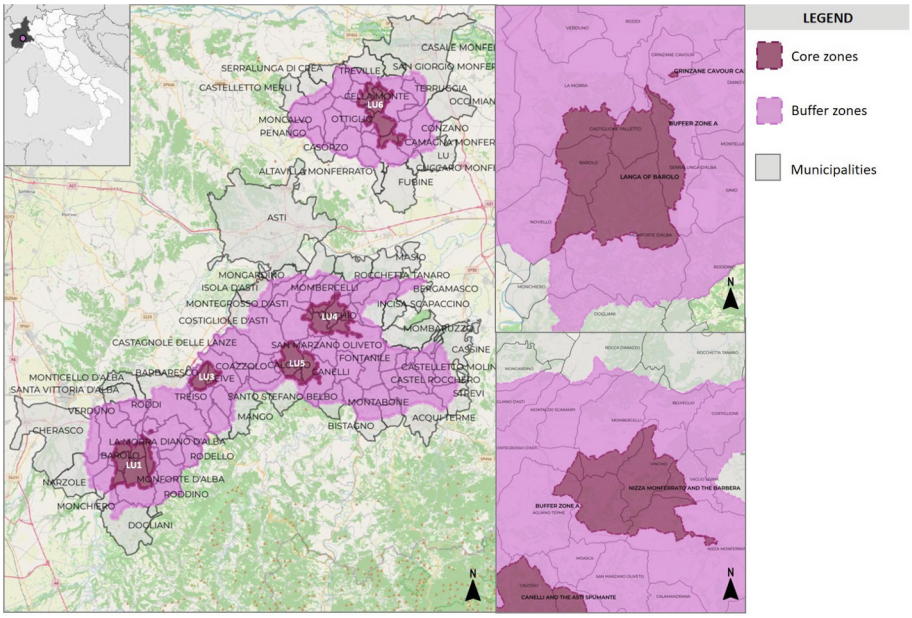

Fig. 1. Territorial localisation of Langhe-Roero Monferrato (left), selection of the core zones 1 (Diano d'Alba) and 2 (Nizza Monferrato) as landscape units for numerical simulations.

- Intangible heritage: high
- Cultural accessibility: extremely difficult
- Cost: 100 €

To estimate the WTP, we applied the conditional mean formula reported in paper [17]:

$$WTP = \frac{\ln(1 + e^a)}{-b}, \quad WTP \geq 0 \tag{5}$$

where e refers to the regressor value, the apex a is the weight of the regressor, and b is the regression coefficient related to the monetary attribute (Table 2).

At a certain point, the two individual WTP of the respondents (i.e. residents and tourists) are merged with an arithmetic mean for each LU

$$WTP_{\text{mean}} = \frac{WTP_{ri} + WTP_{ti}}{2} \tag{6}$$

where WTP_{ri} refers to the WTP of residents in the i-th LU, whereas WTP_{ti} concerns the WTP of tourists in the same LU.

The WTP parameter is firstly normalized on the maximum WTP value of the considered LUs. The highest WTP value equal to 1 does not represent really the general maximum WTP value may be expected, so that we retained useful to calculate a ratio between the WTP mean and the maximum value being expected

Table 1. Estimated coefficients of the Multiple Regression Analysis for the Choice Experiment, as reported in [10].

Attributes	Tourists		Residents	
	B	Sign.	B	Sign.
Vineyards surface	0.509	0.000	−0.297	0.043
Landscape quality	1.081	0.000	0.71	0.000
Intangible heritage	0.496	0.000	−0.281	0.075
Cultural accessibility	0.376	0.000	0.198	0.1
Price	−0.017	0.000	−0.036	0.000
Constant	−5.17	0.000	−0.999	0.329

Table 2. Individual WTP values for tourists and residents in Alternative 1.

Tourists		Residents		Alternative 1	
e^a	b	e^a	b	WTP_{it}^1	WTP_{ir}^1
2.599	−0.017	0.712	−0.036	75.33 €	14.94 €

(100 €). This numerical adaptation of the WTP parameter does not compromise its structural meaning but is an adaptation necessary to be included into the mathematical model together the other dimensionless parameters.

$$WTP_i^G = \frac{WTP_{\text{mean},i}}{WTP_i^{\max}} \tag{7}$$

The analysis focuses on two representative landscape units, namely Diano d'Alba (Core zone 1) and Nizza Monferrato (Core zone 4), denoted by LU1 and LU4, respectively. The corresponding economic values, expressed in terms of WTP mean and its normalized form, are reported in Table 3.

Table 3. Mean Willingness To Pay (WTP) and corresponding normalized values for the selected landscape units.

Landscape Units	mean WTP	mean WTP/100 €
LU1	65.22 €	0.652
LU4	33.74 €	0.337

3.3 Ecological Parameters

The ecological parameters used in this family of mathematical models are mainly based on european and regional spatial geodatabases (e.g. EU Project Copernicus, or Geoportale Piemonte). In the latest years, researches have benefitted from

the Copernicus Project that has enriched its mapping services and data elaboration. Since the results of that survey are referred to 2014, the closest spatial data useful to calculate the ecological parameters identified are: (i) Corine Land Cover Map 2012, and (ii) Digital Territorial Model (DTM, LoD = 10 meters) made available by Geoportale Piemonte geocatalogue. Both the vector and raster files were imported within the QGIS Software (version 3.4) and calculated surfaces and perimeters as basis for the calculation of the ecological parameters. To know the equation formulas, please see the paper [8] and publications therein. Table 4 below lists the ecological parameters for the selected LUs, namely (i) Diano d'Alba and (ii) Nizza Monferrato:

Table 4. Ecological parameters for each landscape unit LU1 and LU4.

Landscape Units	U_i	h_i	a_i	d_i	l_i
LU1 - Diano D'Alba	0.001	0.016	0.502	0.394	0.001
LU4 - Nizza Monferrato	0.008	0.052	0.598	0.423	0.008

3.4 Social Parameters

The selection of the social parameters is based on a previous work by some of the authors [14]. Their definition was guided by the available literature and, as discussed therein, was shown to be suitable for the study of the coupled social-ecological dynamics. As already mentioned before, the focus of this paper is to prove the reliability of the mathematical model, even if the authors recognise a research gap reflecting on the opportunity to calculate these indicators through a more sociological approach. Due to the lack of suitable information in the 2014 survey to estimate these social parameters, and in the absence of indirect data, we refer instead to plausible social behaviours at that time in relation to the potential evolution of the wine landscape as a UNESCO site. Thus, the values shown in the Table 5 below are the numerical translation.

Table 5. Social parameters for each landscape unit LU1 and LU4.

Landscape Units	s_i	k_i	c_i	δ_i
LU1 - Diano D'Alba	0.6	1	0.1	0.18
	0.9	1	0.1	0.18
LU4 - Nizza Monferrato	0.3	1	0.1	0.18
	0.7	1	0.1	0.18

4 Results

In this section, we present some numerical simulations aimed at illustrating the potential of the proposed model as a quantitative support tool for the analysis of coupled ecological and social dynamics in wine landscape areas. More precisely, the simulations are used to explore how ecological conditions, social behaviour, and the normalized Willingness To Pay jointly influence the temporal evolution of the system.

4.1 Initial Data and Input Data

The simulations are performed using model (2), with initial conditions given in Table 6. The corresponding ecological and social values are chosen consistently with the procedure and the assumptions explained in Subsects. 3.3 and 3.4 for parameters of the same nature.

Table 6. Initial conditions for the landscape units LU1 and LU4.

Landscape Units	v_0	x_0	b_0
LU1 - Diano D'Alba	0.073	0.50	0.328
LU4 - Nizza Monferrato	0.117	0.50	0.339

Moreover, the ecological parameters of system (2) are those from Table 4 and the social parameters are from Table 5. The monetary component is incorporated through the normalized WTP reported in Table 3.

The numerical results highlight the role of the economic contribution w_1 in shaping the evolution of the system. In the considered scenarios, we analyse baseline configurations corresponding to different values of the human-induced environmental degradation rate s, and compare them with configurations including the normalized WTP. This setting enables a direct assessment of the impact of the economic contribution on the system dynamics. Overall, the introduction of the WTP term leads to a more favourable evolution of the green area variable v in both landscape units. This behaviour is consistently observed across all the considered scenarios and provides clear evidence of the role of the economic component in supporting ecological preservation.

4.2 Results of LU1 - Diano D'Alba

Figure 2 illustrates the results for LU1 (Diano d'Alba). In the first row, we analyse the scenarios associated with $s = 0.6$, which represents a degradation rate comparable in magnitude to the WTP contribution w_1.

In the baseline case ($w_1 = 0$), on the left plot, the system is strongly affected by environmental degradation. Despite the initial presence of a community of

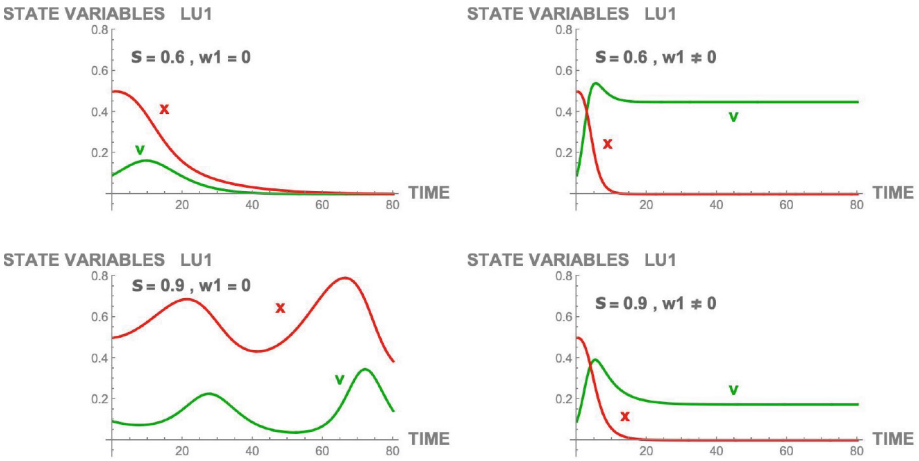

Fig. 2. LU1 – Diano d'Alba. Asymptotic behaviour of the green area v and the proportion of environmentalists x. The first row corresponds to $s = 0.6$, while the second row corresponds to $s = 0.9$. The first column shows the case without economic contribution ($w_1 = 0$), whereas the second column includes the normalized WTP ($w_1 = 0.652$). The comparison highlights the combined effect of social pressure and economic incentives on the evolution of the system.

environmentalists, the social response is not sufficient to counterbalance the degradation pressure. After a short transient phase, the green area of high ecological quality declines rapidly, eventually vanishing. At the same time, the proportion of environmentalists decreases towards zero, indicating a progressive loss of social engagement in environmental protection. The asymptotic trend clearly points to a fully degraded state, characterised by the disappearance of high-quality green areas and the absence of active environmentalists. This scenario highlights that, in the absence of effective mitigation measures, the long-term evolution of the system is dominated by degradation, raising critical concerns in terms of sustainability.

In contrast, on the right plot of the first row in Fig. 2, the corresponding scenario with economic contribution ($w_1 = 0.652$) shows a significant qualitative change in the system dynamics. The introduction of the willingness-to-pay term effectively counteracts the degradation process, leading to a stable configuration where the green area of high ecological quality is preserved at a significant level (approximately 50%). Interestingly, in this case, the proportion of environmentalists still tends to zero asymptotically. This behaviour can be interpreted as follows: once the ecological state of the system improves and stabilises at a sufficiently high level, the perceived need for active environmental engagement decreases. As a consequence, the social pressure sustaining environmentalist behaviour weakens, and the population gradually shifts towards a passive equilibrium in which preservation is maintained primarily by the economic contribution rather than by collective social action.

We now turn to a more extreme configuration, in the second row of Fig. 2, corresponding to $s = 0.9$, which can be interpreted as a limit scenario used to test the robustness of the system under high degradation pressure. In this case, the dynamics exhibit a pronounced oscillatory behaviour over a relatively long time interval. In particular, the proportion of environmentalists fluctuates approximately between 40% and 80%, inducing oscillations in the green area of high ecological quality, between 12% and 30%. This behaviour highlights a dynamic feedback between social response and environmental conditions. However, this regime is not sustainable in the long term. After this transient oscillatory phase, the system loses its resilience, the green area of high ecological quality eventually collapses, and the proportion of environmentalists simultaneously declines to zero. This behaviour indicates that, under strong degradation pressure, the region is not able to maintain a stable balance between ecological and social components, leading ultimately to a fully degraded state.

In the corresponding scenario with economic contribution, in the right plot, the introduction of the willingness-to-pay term modifies the long-term behaviour of the system. Although the proportion of environmentalists tends asymptotically to zero, the green area of high ecological quality does not collapse. Instead, it approaches a strictly positive steady state with approximately 25%, indicating the persistence of a non-trivial steady state sustained by the economic contribution.

4.3 Results of LU4 - Nizza Monferrato

A similar qualitative behaviour is observed in Fig. 3, corresponding to LU4 (Nizza Monferrato). In both scenarios considered in the simulations ($s = 0.3$ and $s = 0.7$), the introduction of the economic contribution w_1 consistently improves the long-term outcome of the system.

In particular, the presence of the WTP term enhances the persistence of the green area of high ecological quality, preventing its complete degradation and leading to higher asymptotic levels compared to the baseline cases. This confirms the stabilizing role of the economic component also in regions characterised by different ecological and social conditions.

4.4 Comparative Interpretation Between the LUs Results

The comparison between LU1 and LU4 is also informative. Since the normalized willingness to pay is higher in LU1 than in LU4, the positive effect of the economic term is more pronounced in LU1. This suggests that the response of the model is sensitive not only to the ecological and social parameters, but also, as expected, to the magnitude of the monetary support associated with each landscape unit.

A further insight emerging from this comparison is that the proportion of environmentalists x tends asymptotically to zero in all simulations of Fig. 3. This behaviour can be explained by the structure of the social dynamics: as the ecological state improves (i.e., higher values of v), the incentive for pro-environmental

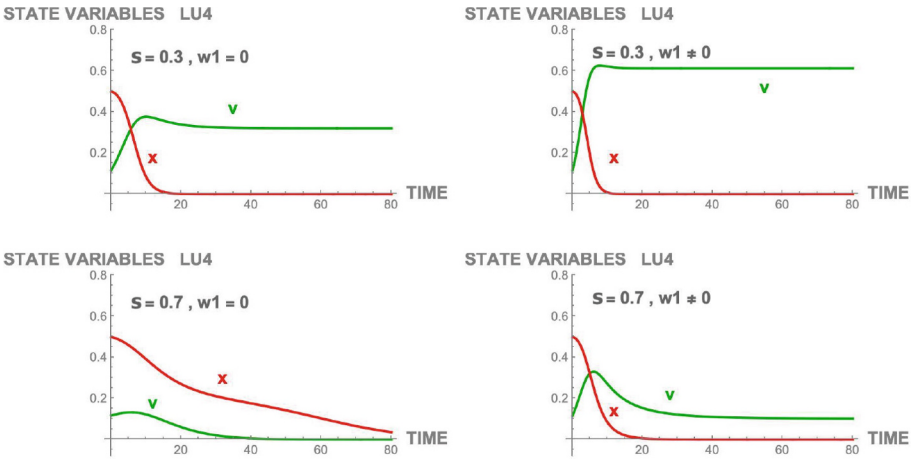

Fig. 3. LU4 - Nizza Monferrato. Asymptotic behaviour of the green area v and the proportion of environmentalists x. The first row corresponds to $s = 0.3$, while the second row corresponds to $s = 0.7$. The first column shows the case without economic contribution ($w_1 = 0$), whereas the second column includes the normalized WTP ($w_1 = 0.337$). The comparison highlights the combined effect of social pressure and economic incentives on the evolution of the system. (Color figure online)

behaviour decreases, leading to a gradual loss of active engagement. From a modelling viewpoint, this indicates that the system evolves towards a regime in which environmental preservation is no longer driven by social pressure, but is instead sustained by structural or economic mechanisms.

4.5 Transition Between Social and Economic Regimes

We now consider a representative hypothetical configuration designed to explore the combined effect of social perception and economic incentives. As a baseline configuration, we consider now a low degradation rate, $s = 0.3$, together with a significantly higher perceived conservation value, $c = 0.6$. This choice reflects a context in which social awareness and environmental sensitivity are strong. Moreover, the degradation pressure is lower than the economic contribution, in the sense that we have $s < w_1$, creating favourable conditions for ecological preservation.

We first analyse the baseline scenario in the left plot of Fig. 4, without economic contribution, $w_1 = 0$. In this case, the system evolves towards a stable configuration where both the green area of high ecological quality and the proportion of environmentalists remain positive. This reflects a socially-driven regime, sustained by the high perceived value of environmental quality.

When the economic contribution is introduced, $w_1 = 0.652$, a clear qualitative change in the system dynamics is observed, as shown in the right plot. This behaviour indicates a transition between two distinct regimes. The improvement

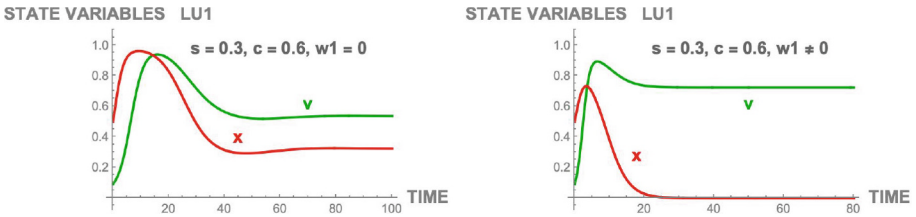

Fig. 4. LU1 – Diano d'Alba. Asymptotic behaviour of the green area v and the proportion of environmentalists x, for a small value of the degradation rate, $s = 0.3$, and a greater value of the perceived conservation value of green areas, $c = 0.6$, Left plot without economic contribution, $w_1 = 0$, and right plot with $w_1 = 0.652$.

of the ecological state, driven by the economic contribution, reduces the social incentive for active environmental engagement. As a consequence, the system transitions from a socially-driven regime, characterised by the coexistence of v and x, to an economically-supported equilibrium in which the preservation of the green area is maintained despite the disappearance of environmentalists.

Remark 3. Overall, these simulations provide a first quantitative indication that the inclusion of a willingness-to-pay component may improve the ecological outcome of the system, especially when combined with favourable social behaviour. From an applied perspective, this supports the interpretation of the model as a useful tool for exploring how ecological conditions, human behaviour, and economic incentives interact in the management of UNESCO wine landscapes, and for guiding planning and intervention strategies.

5 Conclusions and Perspectives

In this paper, a three-dimensional mathematical model for landscape ecology and economics was presented and applied to the Vineyard landscape of Piedmont. The results proved the sensitivity of the mathematical model thanks to the integration of the Willingness to Pay (WTP) as a variable on economic attitude of users together ecological dynamics and human behavior. It can be stated that this extension is a valuable advance for this class of mathematical models in addressing both the current research and policy demands in landscape planning [3,4]. The resulting model preserves the structure of the original framework while introducing an additional nonlinear (logistic) term to the ecological variable v, i.e. the green area with a high ecological quality, allowing for a direct coupling between economic behaviour and environmental dynamics.

The qualitative analysis shows that the inclusion of the WTP parameter w_1 does not alter the fundamental structure of the equilibria, whose existence and stability conditions remain analogous to those of the baseline model. However, w_1 modifies the effective growth term of v, contributing positively through a logistic-type component.

The numerical simulations performed for the selected landscape units indicate that positive values of w_1 can counterbalance the degradation term, leading to higher asymptotic levels of ecological quality. This effect is particularly evident in scenarios characterized by strong anthropogenic pressure (large values of the degradation rate s), where the economic contribution acts as a stabilizing mechanism.

Furthermore, although the WTP parameter does not enter explicitly into the social equation, its indirect influence on v feeds back into the dynamics of x, i.e. proportion of environmentalists, potentially promoting higher values of x itself. The ecological parameters, derived from spatial datasets, and the assumed social parameters provide a coherent, though preliminary, parametrization of the model. Several aspects remain open for further investigation. In particular, the calibration of social parameters based on empirical data would allow for an increase in the model robustness and thus reducing the gap on human behavioral indicators [18].

Other research lines will also address the analysis of an extended model including a rehabilitation term associated with areas with low ecological quality. In this case, to take into account these features, the model is further enriched by incorporating the WTP for the rehabilitation of areas with low ecological quality and for the restoration of green areas. Introducing a new parameter w_2 denoting the normalized monetary quantities representing the WTP for the rehabilitation of areas with low ecological quality, Eq. (2a) relative to the evolution of v, is formulated as:

$$\dot{v} = dv(1-v) - hUv - s(1-x)v + w_1 v(1-v) + w_2(1-v), \tag{8}$$

where the term $w_2(1-v)$ is proportional to the low-quality portion of the green area $1-v$, and represents the contribution of individuals or governmental institutions willing to pay for the rehabilitation of areas with low ecological quality.

Another research avenue is to investigate the full network of coupled landscape units, following the strategy of papers [3,8]. This would make it possible to investigate not only the internal dynamics of each landscape unit, but also the interactions among different units within a broader landscape system. Such developments are expected to provide deeper insight into the role of economic incentives in shaping the long-term behaviour of coupled social-ecological systems. Hence, the launching of a new survey to elicitate the WTP in the same case study is not excluded, after more than one decade from the UNESCO WHL inscription.

In this sense, the presented model can provide predictions of decision-aiding utility for policy design. Regarding its usability for plans of technical-economic affordability (i.e. *Piano di Fattibilità Tecnico-Economica*) of public policy measures, and/or in partnership with private actors, these could benefit from the application of this dynamical model and potentially in completion of other assessment techniques during the screening phase. For example, the model can help policy makers to distinguish between economic support for the conservation of existing high-quality areas and economic support for the restoration or

rehabilitation of degraded areas, even before activating a resource-intensive programming, such as in the case of the localisation of projects nature-based [6]. However, the operational usability of this mathematical model closely depends on data availability (i.e. ecological and economic ones), and currently on the calibration of social parameters that is expected to be more empirical for real policy-oriented applications. In conclusion, the model offers a useful basis for future applications in protected areas, cultural landscapes and UNESCO sites, where conservation, local development and community values are strongly interdependent in influencing the real estate values of properties. In this sense, a Geographically Weighted Regression (GWR) based on Hedonic Prices Mehod (HPM) will deepen these interdependencies [19].

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