On Battery Storage Management by Sparse Linear Programming

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Abstract. This paper formulates a problem of optimal battery storage management as a sparse linear programming (LP) problem. We focus on the potential of solving this problem by an augmented Lagrangian (AL) approach. The problem maximizes revenue exploiting volatility of the electricity price. The model deals with equalities to describe the energy balance and contains bounds that reflect the operational limits. We investigate an AL-based algorithm with a first stage for infeasibility detection and an iterative scheme for convergence to a basic feasible solution. Numerical experiments using real electricity price data from Energia XXI on four days (31st October, 1st November, 3rd November, and 5th November 2025) demonstrate fast convergence within 2 to 14 iterations. The daily net revenue for a home situation varies between 0.7690 € and 2.3186 € depending on price volatility.

Keywords: Linear Programming · Battery Storage Management · augmented Lagrangian

1 Introduction

Battery energy storage systems (BESS) provide a solution to deal with variability in renewable energy production. BESS can be used to store excess energy generated during periods of high production and release it when generation is low. This leads to flexibility and stabilizing the provision. Similar observations have been discussed in recent reviews, where optimization of BESS has become central to the integration of renewables in smart grids [2,8].

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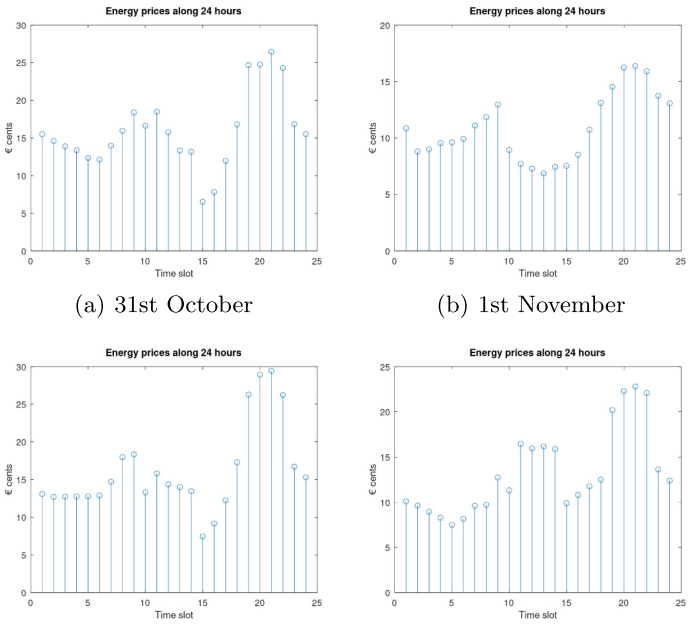
Variable electricity prices allow consumers to sell energy back to the grid when there is surplus and to buy and store when prices are low. Optimization-based strategies, including linear and stochastic formulations, have shown great potential to maximize economic benefits in such environments [1].

Currently, family households are able to buy a home battery, where we focus on a variant that can store 20 kWh. In contracts, the varying price is determined due to an auction system and is known one day in advance for each hour of the day. The dynamic electricity pricing and cyclic behaviour is sketched by the data provided in Fig. 1. This setting allows an individual consumer to buy during low price hours and sell during hours where the price is high. To optimize the revenue, this optimization problem can be captured by a Linear Programming (LP) problem with simple balance constraints and bounds.



(a) 31st October (b) 1st November

(c) 3rd November (d) 5th November

Fig. 1. Electricity prices for different days provided by `www.energiaxxi.com`.

The resulting problem is sparse. Recently, specific LP solution methods have been developed to deal with such sparse problems [5]. One of our questions is to adapt this approach to deal with upper bounds in an augmented Lagrangian setting. Our research investigates the potential of sparse LP formulations to efficiently solve the underlying battery storage optimization problem. Specifically, we focus on the potential of recent advances in sparse solvers and augmented Lagrangian (AL) methods [5,6].

To investigate the question, we introduce the mathematical optimization model in Sect. 2 and place it in the context of literature. We analyze the sparsity of the underlying model in Sect. 3 and show how modern LP solvers can exploit the structure in Sect. 4. Section 5 summarizes our findings.

2 Mathematical Description and Literature Embedding

The operation of management storage systems provides an advantage in revenue in systems where we are dealing with photovoltaic generation systems [7]. Other works have introduced linear and non-ideal battery models that better capture charge/discharge efficiencies and degradation effects without excessive computational burden [4]. The result of recent development in the energy conversion is that governments implement and promote systems in which the consumer is dealing with a varying price during the day in order to stabilize demand.

We describe an underlying LP problem in (1)-(6) that includes the basics of revenue optimization of a home battery for an individual consumer dealing with a price P_t that varies each hour t of the day and the possibility to return energy to the grid at the same price with an additional cost C.

The variables involved in the model are s_t, that is the amount stored in battery at the end of slot t, b_t is the quantity delivered back to grid in slot t and q_t refers to the quantity taken from grid in slot t. Table 1 resumes the notation and used symbols of the optimization problem.

In the model, (3) represents a balance equation and (2) ensures that the stored capacity is the same at the end and the beginning of the day. Similar formulations with extensions were used in previous studies of off-grid and grid-connected systems [1,3].

$$\max_{s,b,q} \sum_{t=1}^{T} P_t(b_t - q_t) - C \sum_{t=1}^{T} b_t \tag{1}$$

$$\text{s.t.} \quad s_0 = s_{T-1} - b_T + q_T, \tag{2}$$

$$s_t = s_{t-1} - b_t + q_t, \qquad t = 1, \ldots, T-1 \tag{3}$$

$$s_t \in [0, \overline{s}] \tag{4}$$

$$b_t \in [0, \overline{b}] \tag{5}$$

$$q_t \in [0, \overline{q}] \tag{6}$$

The sparse linear programming formulation for battery storage management will be tested using real-time electricity prices obtained from Energia XXI provided by `www.energiaxxi.com` for four days (31st October, 1st November, 3rd November, and 5th November of 2025). Figure 1 shows the electricity P_t prices for each of the day used in the study.

The model parameters for the battery storage management were defined for a home-scale battery system with a storage capacity of $\overline{s} = 20.0$ kWh. The maximum potential of 4.6 kW which should be multiplied by one hour to provide

Table 1. Notation and used symbols

Indices	
t	slot (hour) of the day with fixed price, $t = 1, \ldots, T$
Data	
T	Last time period of the day
C	Cost of delivering back to the grid
P_t	Price of electricity per kWh in slot t
s_0	Stored energy at the beginning and at the end of the day
$\overline{s}$	Battery capacity
$\overline{b}$	Discharging capacity in one slot
$\overline{q}$	Charging capacity in one slot
Variables	
s_t	Amount stored in battery at the end of slot t in kWh
b_t	Quantity delivered back to grid in slot t in kWh
q_t	Quantity taken from grid in slot t in kWh

an upper bound $\overline{q} = 4.6$ kWh on the amount q_t that can be purchased in one hour from the grid. Similarly, the maximum discharging rate of 4.0 kW, which can be obtained from the battery, which translates to $\overline{b} = 4$ kWh as upper bound of what can be sold. Notice that one time slot has a duration of one hour, such that discharging at a rate of b_t during one time slot implies selling b_t kWh. The cost for selling back to the grid is set on $C = 0.02$ €/kWh and the storage at the end of the day is set to $s_0 = 15$.

3 Structure of the Model and Augmented Lagrangian

The presented model has a specific structure and can be reformulated as

$$\max_{y} b^\top y \quad \text{s.t. } Ay = c,\ 0 \leq y \leq \overline{y}.$$

Analyzing the corresponding matrix A, it consists of T rows defined by (3) and (2). The last row fixes the storage again at the end of the day $s_T = s_0$. Each row consists of 4 nonzero values $a_{ij} \in \{1, -1\}$, where the first and last equation have only 3 nonzero values (see Fig. 2). Basically, the matrix has dimension $T \times (3T - 1)$ and the number of non-zeros is $3 + 4(T - 2) + 3$ ($nz = 94$). Therefore, the matrix can be called sparse. Moreover, it is full row rank.

The structure of the resulting constraint matrix A shows significant sparsity, a feature that can be exploited by modern LP solvers and augmented Lagrangian algorithms [6]. Sparse representations are increasingly important for large-scale energy management problems, where thousands of time slots or nodes can be modeled efficiently [2].

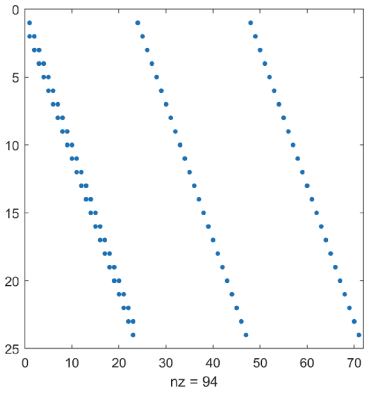


Fig. 2. Nonzero values in matrix A.

In [6], we developed a specific AL algorithm to deal with this type of problems in an efficient way. The advantage is that it has a first stage initialization to discover, if the given LP problem does not have a feasible point generating a so-called certificate of infeasibility of the dual. The second stage is an iterative scheme to generate an approximate basic solution in a finite number of steps [5,6]. The steps of the developed algorithm are depicted in Alg. 1.

Algorithm 1. Augmented Lagrangian

Require: A: matrix; c, $\overline{y}$, b: vectors; $\rho_0 > 0$: penalty parameter
$\omega > 1$: penalty updating factor; ϵ: tolerance; maxiter

$y_0 \in \operatorname{argmin}_{y \in [0,\overline{y}]} \frac{1}{2}(c - Ay)^\top (c - Ay)$
$d_0 = -(c - Ay_0)$
if $\|d_0\| \geq 10^{-6}$ **then** ▷ no feasible point exists
  return d_0
else ▷ y_0 is feasible, but not necessarily basic solution
  $k = 0$ ▷ Use y_0 as starting value for the AL optimization
  repeat
    $y_{k+1} \in \operatorname{argmin}_{y \in [0,\overline{y}]} \left(-b^\top y - d_k^\top (c - Ay) + \frac{\rho_k}{2}(c - Ay)^\top (c - Ay)\right)$
    $d_{k+1} = d_k - \rho_k(c - Ay_{k+1})$
    $\rho_{k+1} = \omega \rho_k$ ▷ update ρ
    $k = k + 1$
  until $\|y_k - y_{k-1}\| < \epsilon$ or $k >$maxiter
return y_k

4 Results and Discussion

The developed Algorithm 1 is an adaptation of the augmented Lagrangian method described in [6], in order to handle upper bounds (4)-(6) on the decision variables. The stopping criterion, based on the Euclidean norm, uses the

tolerance $\epsilon = \sqrt{\varepsilon_{\text{mach}}} = \sqrt{2^{-52}} \approx 1.49 \times 10^{-8}$ and the maximum number of iterations is set to maxiter = 120. According to [6], the initial value of the penalty parameter was set to $\rho = 1$, and is updated using $\omega = 3$.

The optimization in line 8 of Alg. 1 was implemented using the FMINCON solver from MATLAB®, which provides both flexibility in constraint handling and efficiency in convergence for sparse problems. The active set solver takes care of efficiently handling the bounds on the variables. We provide the gradient analytically to the solver. The termination tolerance was set to 10^{-12} for both y and the AL function value.

We provide Alg. 1 with a vector of zeros (zero dual infeasible point) as starting point for the solver in line 1. The results have been compared with the results of the LP solver LINPROG of MATLAB®. The reached objective function values are provided in Table 2.

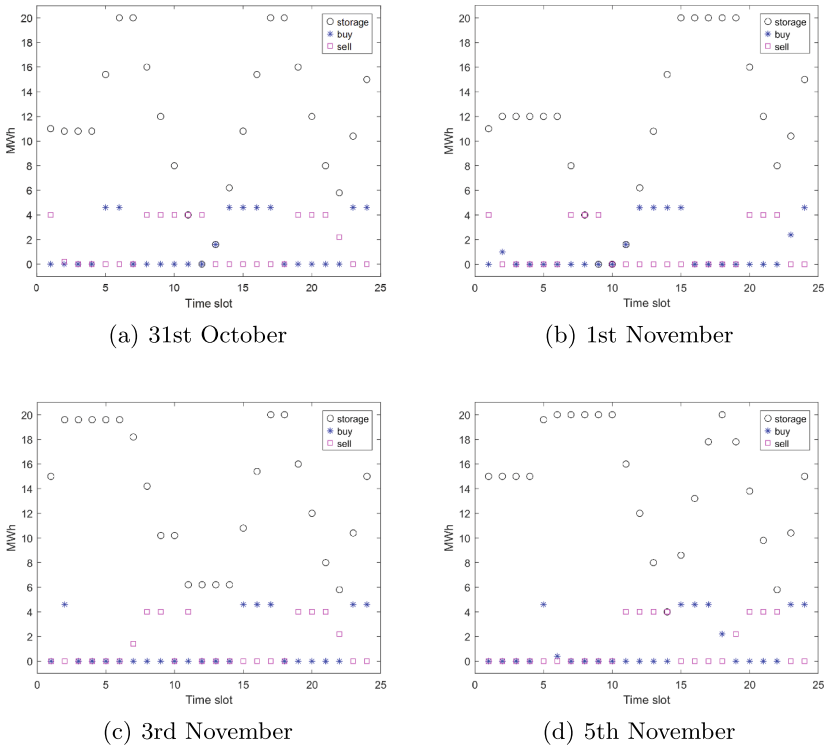
Table 2. Objective function value of LP and result of Alg. 1. The number of iterations (Iter) of Alg. 1 and the total number of function evaluations (Nr fev.) running the solver.

Day	Objective function LP	Objective function AL	Iter	Nr fev.
31st October	2.213558000000001	2.213557999999553	2	570
1st November	0.7690440000000003	0.7690439963065097	14	2467
3rd November	2.318600000000000	2.318599999872144	13	2092
5th November	1.765136000000000	1.765135999999051	3	649

The numerical results obtained with the energy values for each day are summarized in Table 2. The LP solution provides the optimum for the four price instances (days) and the resulting objective ranges from 0.7690 to 2.3186 €, corresponding to the net revenue achieved by the battery storage management of charging and discharging cycles. What can be observed is that the AL approach provides practically the same solution as the LP solver. It requires between 2 and 14 iterations, and fewer than 2500 function evaluations for all cases.

These results confirm the expected correlation between price volatility and achievable profit. On days with larger fluctuations in the hourly price P_t, the optimizer exploits opportunities for arbitrage - charging during low-price hours and discharging during peaks. For instance, the highest objective (2.3186 €) was obtained on 3rd November, when intraday price differences exceeded 0.10 €/kWh. The lowest return (0.7690 €) occurred on 1st November, when the price curve is more flat which provides less advantage in terms of providing revenue.

Figure 3 illustrates the energy movement and the storage profiles along the four days. The battery shows a cyclic feature, returning to its initial charge s_0 at the end of the day, to balance the daily storage. When the price rises, the system discharges energy to the grid (magenta curve). On the contrary, during off-peak hours, it draws power (blue curve) to replenish the storage.



(a) 31st October (b) 1st November

(c) 3rd November (d) 5th November

Fig. 3. Outcome of the model, in black the status of the battery, in blue the amount taken from the grid (bought) and in magenta the amount sold back to the grid. (Color figure online)

This behavior corresponds to the optimal scheduling reported in similar studies on LP-based and stochastic energy management frameworks [1,3]. The complexity of the model formulation just depends on the temporal resolution T. This means we can increase the density going for instance from an hour base to a 15 min base as is planned among Europe without increasing significantly the computational effort [2].

The numerical results show that the augmented Lagrangian method is able to find a feasible and near-optimal solution. Notice that the accuracy in this context can be in the order of euro cents. We found that the method requires a small number of iterations. Compared to classical simplex or interior-point solvers, the AL framework provides an interesting alternative. Notice that it is a first order method that does not depend on matrix inversions, see [5].

Notice that the described model and results focus on realistic price variation and bounds on operational activity to describe a real-world household battery behavior. The framework can be extended easily to include non-ideal characteristics such as round-trip efficiency, degradation cost, or temperature-dependent capacity as suggested in [1,4].

From a practical point of view, the described procedure shows that one can use sparse LP-based optimization for home or microgrid energy management systems. In such a practical environment, computation time and memory efficiency are relevant. Recent reviews [8] discuss that such formulations represent a promising step toward fully automated, cost-aware battery control in the context of dynamic electricity markets.

5 Conclusions

This study investigated the application of sparse linear programming and augmented Lagrangian methods to the battery storage management problem under variable electricity prices. The described model intends to optimize revenue in the context of a household-scale battery system. The idea is to exploit variability in real-world prices dynamics to maximize energy revenue by buying when the price is low and returning energy when the price is high. Due to the European trading system, these prices are known one day beforehand.

The numerical experiments show how a daily policy of buying and returning can be generated fast by a sparse LP formulation allowing future higher temporal discretization. We experimented with multiple daily price profiles. The model successfully adapted charging and discharging schedules to exploit market arbitrage, achieving higher profits on days with stronger price volatility.

One of our questions was whether an AL algorithm can handle effectively the upper-bound constraints in an all equality LP. The equalities describe the balance in the daily energy cycle, where we return to the initial storage at the end of each optimization horizon. We found that the AL approach being a first order method is efficient with respect to memory management and facilitates the use of iterative solvers. This shows that the AL approach can be a viable alternative to traditional simplex or interior-point methods in large-scale or embedded optimization contexts [5,6]. These findings are in line with recent findings in BESS optimization, where computational scalability and real-time applicability are becoming interesting challenges [2,8].

An interesting extension for future research of the current model to include the electricity consumption from the battery, efficiency losses, and temperature-dependent performance. The latter has shown to significantly affect long-term economic outcomes [1,4]. Another challenge is to include stochastic or predictive elements to represent uncertain renewable generation and price forecasts in the model to have a better robustness [3].

Overall, the results support the conclusion that sparse linear programming, combined with an augmented Lagrangian approach, provides a computationally efficient framework for battery storage optimization problem.

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