# Research on the Gimballed Inertial Navigation System Based on Mathematical Description of Platform Motion

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**Abstract.** This paper deals with such a feature of the mathematical description for a gimballed navigation system as the kinematics of a moving platform with mounted navigation instruments. The measuring part of the studied system includes high-precision rate gyroscopes and pendulum accelerometers. The novelty of the proposed approach lies in the full mathematical description of the platform equipped with navigation instruments in the inertial space. This leads to sufficient improvement in the platform's motion control system, synthesised on the basis of the proposed mathematical description. Systems of coordinates that describe the platform's position in the inertial space are presented. The detailed kinematic equations are derived. The expressions for the platforms' angular rates are developed. The features of the parametric optimization of the platform's motion control system are described. The results of modelling the synthesised motion control system in the form of graphical dependencies and calculation of robustness indices are presented. The obtained results can also be useful for the motion control systems of gyro-stabilized platforms with different observation and information-measuring devices.

**Keywords:** Modelling · Mathematical Description · Motion Control System · Stabilization · Gimballed Navigation System · Parametric Optimization

## 1 Introduction and Literature Review

Gimballed navigation systems are intended for determining the orientation of a moving object in the inertial space with high accuracy due to the use of gyroscopic devices capable of keeping the orientation in the inertial space unchanged relative to a given axis. Moreover, these gyroscopic devices are mounted on the stabilized platform. The stabilization is implemented by a motion control system, for example, a servo system. Modern approaches to the development of stabilized systems for determining course and attitude are characterized by increasing requirements for their functionality, while at the same time being accompanied by the need to reduce design times. It is possible to reconcile such conflicting requirements only if the system is effectively developed at the initial stages of its creation. One of the key means of ensuring such efficiency is the use of modern methods of optimal synthesis, which, in turn, require the formation of a complete mathematical description of the systems under study.

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Earlier, such systems were synthesized on the basis of classical methods of given transfer functions and amplitude-frequency characteristics. Nowadays, modern control methods are used, including LQR synthesis, robust synthesis, adaptive synthesis, and methods based on artificial intelligence.

The studied systems are operated on moving objects in difficult conditions of intensive external disturbances. Given the specifics of the systems under study, one should mention methods of robust structural synthesis. $H_\infty$ synthesis and $\mu$ synthesis significantly depend on the correspondence of the mathematical description to the real system. Here we are talking about the structure of the model. In addition, $H_\infty$ synthesis is based on a generalized model that takes into account transfer weight functions. The definition of such functions is a complex task based on heuristic approaches. This is crucial for the formation of the amplitude-frequency characteristics of the synthesized system (loop-shaping results). $\mu$ synthesis, in addition to the mathematical description of the system, requires mathematical descriptions of uncertainties that significantly affect the results of the synthesis of the motion control system. In other words, the efficiency of robust methods depends essentially on the quality of the system's mathematical description.

When stabilization laws are well known, the acceptable robust approach is the mixed $H_2/H_\infty$ synthesis.

Note that adaptive control systems are resistant to the influence of external disturbances too. Nevertheless, adaptive systems are expensive, complicated to use, and difficult to manufacture. They are practically not used for the platform's stabilization in gimballed navigation systems.

Methods based on artificial intelligence require complex computing units. They are convenient for application in strapdown stabilization systems, where the sequence of rotations in the inertial space due to disturbances is realized by calculations in a computer.

The current stage of developing moving vehicles intended for operation on water requires a radical improvement of navigation and control means of moving objects. The most common such means are considered navigation and stabilization systems based on gyroscopic devices due to their resistance to external influences and a high degree of autonomy [1]. At the same time, serious attention is paid to gyroscopic devices for determining the course of a moving object, primarily gyrocompasses. This is explained by the fact that for reliable and accurate navigation and solving other problems, continuous and fairly accurate information about the current course is necessary.

The development of gimballed systems continues to be relevant when the main goal is to ensure maximum accuracy in autonomous operation or to reduce the duration and cost of creating new systems by using the achievements of previous projects. This trend is especially evident in the field of high-precision systems.

Gimballed inertial navigation systems and features of their application are described in the publication [1]. Such systems are grounded on the inertial navigation [2] and inertially stabilized platform technology [3].

Features of creating models of systems and devices designed to determine the course of moving objects are highlighted in numerous scientific publications.

In particular, in the publication [4], a simplified model of a navigation system using gyroscopes with dynamic tuning is proposed. Similar models are used to study navigation

contours of course determination and stabilization systems. The models of dynamically tuned gyroscopes are used with some additions for studying the gimballed system in the present paper.

In the paper [5], a mathematical model of a small-sized corrected gyrocompass based on a gyroscope with dynamic tuning is presented, which takes into account the movement of the platform. In contrast to the study given in [5], the present paper considers a system with a developed configuration of inertial measuring devices and three-axis gimbals that ensure the possibility of measuring full information about the location of the moving object in the inertial space.

In the paper [6], models of the gimballed course determination system are presented with a division into modes of precise alignment to the horizon and operation in the course measurement mode. However, a platform with two-axis gimbals is studied in this paper. This approach to considering a gimballed navigation system as a multi-mode one is kept and developed in the present paper, taking into consideration the complication of the gimballed platform with inertial measuring instruments for three-axis construction.

A mathematical description of a system of the corresponding type is presented in the paper [7], but it is limited only to the gyroscopic compass mode, without taking into account the platform motion and without a detailed analysis of the control features. The present paper corrects this approach based on a detailed study of the motion of the gimballed platform.

The main goal of the paper is to study the kinematics of the platform with navigation inertial instruments as a main component of a gimballed inertial navigation system. The results of the study can be used in procedures for the synthesis of the system controlling a gimballed platform motion.

The structure of the paper can be represented in the following way. Section 2 deals with the mathematical description of gimballed inertial navigation system kinematics. Section 3 describes the features of the parametric optimization for the gimballed platform motion control system. The results of modelling and calculating indices of the synthesised motion control system are presented in Sect. 4. Conclusions are given in Sect. 5.

## 2 Mathematical Description of Gimballed Inertial Navigation System Kinematics

The creation of a complete mathematical description of stabilization systems and the determination of the course of dynamic objects will require solving some key tasks, which are listed below:

- It is necessary to recognize the presence of several operating modes, which are differentiated by the warehouse of knowledge, and therefore, are characterized by different features and mathematical descriptions.
- It is important to ensure that the dynamics and kinematics of the platform on which the navigation information sensors are placed is taken into consideration. For this purpose, it is necessary to create consistent models in order to describe the dynamics of the platform and the necessary kinematic relationships.

- It is necessary to consider the modelling and monitoring of the navigation and stabilization contours of the system.
- The analysis of systems may be carried out in accordance with the presence of electronic devices.
- To solve the most significant problems, it is recommended to combine such approaches.

A complete mathematical model of the gimballed navigation system for determining heading and attitude of a moving object is a set of mathematical models that describe the operation of the system in the main modes, including the mode of forward setting to the horizon plane, modes of accurate setting to the horizon plane, gyroscopic compass mode, and gyroscopic azimuth mode.

The system's model in the mode of forward setting to the horizon plane includes the rough alignment of the platform and the alignment of accelerometers. In the mode of precise adjustment to the horizon plane, the mathematical model describes the process of the platform's levelling, the alignment of the gyroscopic vertical based on a dynamically tuned gyroscope, as well as the alignment of pendulum accelerometers.

The most complex models correspond to the gyroscopic compass and gyroscopic azimuth modes. They include equations that describe the platform's dynamics and kinematics, a mathematical description of the gyroscopic vertical, a mathematical description of the gyroscopic compass (gyroscopic azimuth), and mathematical descriptions of accelerometers. The above-mentioned gyroscopic devices are based on such complex units as dynamically tuned gyroscopes. In this case, the peculiarities of controls in the different modes are considered, which are differentiated by the variety of sensors, the main functions, and the ability to achieve accuracy.

The mathematical description of the platform's motion can be obtained on the basis of the theorem about the change in the kinetic moment of a solid body, which will be described below. In this case, it is necessary to completely restore all kinematic correlations of the system. It should be appreciated that such an approach leads to very complex mathematical expressions. This approach can be simplified using a simpler method, which does not account for the difference between the moments of inertia of the different axes of the gimbals.

To extract kinematic relations, it is necessary to introduce the following coordinate systems.

The coordinate system $Ox_0y_0z_0$ is associated with the moving object.

The coordinate system $Ox_py_pz_p$ is linked to the platform. It is called the body-axis coordinate system.

The transition from the coordinate system associated with the moving object to the coordinate system associated with the platform is indicated by the subsequent turns. These rotations are implemented around the axis of the outer frame of the platform's gimbal (angular position $\alpha$), around the inner frame of the platform's gimbal (angular position $\beta$), and the platform's rotation (angular position $\gamma$). The designated sequence of turns is shown in Fig. 1.

Note that in such a coordinate system at the initial position, the outer axis of the gimbals rotates together with the longitudinal axis of the moving object, and the inner axis of the gimbals with the lateral axis of the moving object.

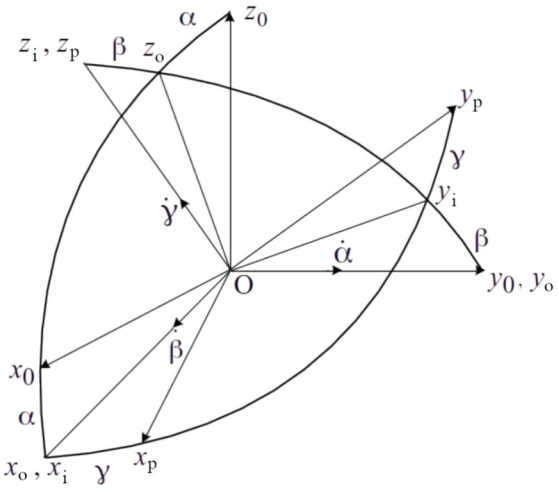


**Fig. 1.** The relative position of the coordinate systems associated with the moving object and the platform.

The equations of motion of the platform can be derived based on the theorem on the change of angular momentum of a rigid body, which is written below [8–10].

$$\begin{aligned}
&\frac{dK_{zp}}{dt} + \omega_{xp}K_{yp} - \omega_{yp}K_{xp} = M_{zp};\\
&\frac{dK_{xi}}{dt} + \omega_{yi}K_{zi} - \omega_{zi}K_{yi} = M_{xi};\\
&\frac{dK_{yo}}{dt} + \omega_{zo}K_{xo} - \omega_{xo}K_{zo} = M_{yo},
\end{aligned} \tag{1}$$

where $K_{xp}$, $K_{yp}$, $K_{zp}$, $\omega_{xp}$, $\omega_{yp}$, $\omega_{zp}$ are the kinetic moments and angular velocities of the platform with the installed gyroscopic devices and pendulum accelerometers; $M_{xp}$, $M_{yp}$, $M_{zp}$ are external moments acting on the rotating platform. These moments are divided into controls and disturbances. Controls are defined by the mode of the gimballed navigation system. Disturbances depend on the type of moving object.

The kinetic moments included in the theorem on the change of angular momentum of a rigid body (1) are calculated using the following formulas describing the kinematics of the platform.

$$\begin{aligned}
K_{xp} &= I_{xp}\omega_{xp};\\
K_{yp} &= I_{yp}\omega_{yp};\\
K_{zp} &= I_{zp}\omega_{zp},
\end{aligned} \tag{2}$$

where $I_{xp}$, $I_{yp}$, $I_{zp}$ are moments of the platform with devices.

The kinematics of the inner gimbal frame can be represented by relationships.

$$\begin{aligned}
K_{xi} &= I_{xi}\omega_{xi} + K_{xp}\cos\gamma - K_{yp}\sin\gamma;\\
K_{yi} &= I_{yi}\omega_{yi} + K_{yp}\cos\gamma + K_{xp}\sin\gamma;\\
K_{zi} &= I_{zi}\omega_{zi} + K_{zp},
\end{aligned} \tag{3}$$

where $K_{xi},\ K_{yi},\ K_{zi}, I_{xi},\ I_{yi},\ I_{zi}, \omega_{xi},\ \omega_{yi},\ \omega_{zi}$ are the corresponding kinetic moments, moments of inertia, and angular velocities of the inner frame of the gimballed platform; γ is the angle of the platform's rotation.

The mathematical description of the kinematics of the outer gimbal frame are.

$$\begin{aligned} K_{xo} &= I_{xo}\omega_{xo} + K_{xo}; \\ K_{yo} &= I_{yo}\omega_{yo} + K_{yo}\cos\beta - K_{zo}\sin\beta; \\ K_{zo} &= I_{zo}\omega_{zo} + K_{zo}\cos\beta + K_{yo}\sin\beta, \end{aligned} \tag{4}$$

where $K_{xo},\ K_{yo},\ K_{zo}, I_{xo},\ I_{yo},\ I_{zo}, \omega_{xo},\ \omega_{yo},\ \omega_{zo}$ are the kinetic moments, moments of inertia, and angular velocities of the outer frame of the gimballed platform; β is the angle of the inner frame rotation.

By substituting the expressions for kinetic moments (2) – (4) into the equation of motion of the platform (1), we obtain an equation describing the motion of the platform relative to its own axes

$$I_{zp}\dot{\omega}_{zp} + I_{yp}\omega_{xp}\omega_{yp} - I_{xp}\omega_{xp}\omega_{yp} = M_{zp}. \tag{5}$$

After the conversion of Eq. (5), we will obtain

$$I_{zp}\dot{\omega}_{zp} + \left(I_{yp} - I_{xp}\right)\omega_{xp}\omega_{yp} = M_{zp}. \tag{6}$$

The equations of motion of the platform relative to the inner axis of the gimbals become

$$\begin{gathered} I_{xi}\dot{\omega}_{xi} + I_{xp}\dot{\omega}_{xp}cos\gamma - I_{yp}\dot{\omega}_{yp}sin\gamma + I_{zi}\omega_{yi}\omega_{zi} + I_{zp}\omega_{yi}\omega_{zp} - \\ -I_{yi}\omega_{yi}\omega_{zi} - I_{yp}\omega_{zi}\omega_{yp}cos\gamma - I_{xp}\omega_{zi}\omega_{xp}sin\gamma = M_{xi}. \end{gathered} \tag{7}$$

After conversion of the Eq. (7), we will obtain:

$$\begin{gathered} I_{xi}\dot{\omega}_{xi} + \left(I_{zi} - I_{yi}\right)\omega_{yi}\omega_{zi} + I_{zp}\omega_{yi}\omega_{zp} + \\ +\left(I_{xp}\dot{\omega}_{xp} - I_{yp}\omega_{zi}\omega_{yp}\right)cos\gamma - \left(I_{yp}\dot{\omega}_{yp} + I_{xp}\omega_{zi}\omega_{xi}\right)sin\gamma = M_{xi}. \end{gathered} \tag{8}$$

The equations of motion of the platform relative to the external axis of the gimbal are described as follows:

$$\begin{aligned} &I_{yo}\dot{\omega}_{yo} + I_{yi}\dot{\omega}_{yi}\cos\beta + I_{yi}\dot{\omega}_{yp}\cos\gamma\cos\beta + I_{xp}\dot{\omega}_{xp}\sin\gamma\cos\beta - I_{zi}\dot{\omega}_{zi}\sin\beta - \\ &-I_{zp}\dot{\omega}_{zp}\sin\beta + (I_{xo} - I_{zo})\omega_{xo}\omega_{zo} + I_{xi}\omega_{xi}\omega_{zo} + I_{xp}\omega_{xp}\omega_{zo}\cos\gamma - \\ &-I_{yp}\omega_{zo}\omega_{yp}\sin\gamma - I_{zi}\omega_{xo}\omega_{zi}\cos\beta - I_{zp}\omega_{xo}\omega_{zp}\cos\beta - \\ &-I_{yi}\omega_{xo}\omega_{yi}\sin\beta - I_{yp}\omega_{yp}\omega_{xo}\cos\gamma\sin\beta - I_{xp}\omega_{xo}\omega_{xp}\sin\gamma\sin\beta = M_{yo} \end{aligned} \tag{9}$$

As shown in Fig. 1, the expressions for the angular velocities of the platform can be presented in the following form:

for the outer gimbals frame

$$\begin{aligned} \omega_{xo} &= \omega_{x0}\cos\alpha - \omega_{z0}\sin\alpha, \\ \omega_{yo} &= \dot{\alpha} + \omega_{y0}, \\ \omega_{zo} &= \omega_{z0}\cos\alpha + \omega_{x0}\sin\alpha, \end{aligned} \tag{10}$$

for the inner gimbals frame

$$\begin{aligned}\omega_i &= \dot{\beta} + \omega_{xo},\\ \omega_{yi} &= \omega_{yo}\cos\beta + \omega_{zo}\sin\beta,\\ \omega_{zi} &= \omega_{zo}\cos\beta - \omega_{yo}\sin\beta,\end{aligned} \tag{11}$$

for the platform

$$\begin{aligned}\omega_{xp} &= \omega_{xi}\cos\gamma + \omega_{yi}\sin\gamma,\\ \omega_{yp} &= \omega_{yi}\cos\gamma - \omega_{xi}\sin\gamma,\\ \omega_{zp} &= \dot{\gamma} + \omega_{zi},\end{aligned} \tag{12}$$

where $\omega_{x0},\ \omega_{y0},\ \omega_{z0}$ are components of the object's velocity; $\omega_{xp},\ \omega_{yp},\ \omega_{zp}$ are components of the platform's velocity; $\omega_{xi},\ \omega_{yi},\ \omega_{zi}$ are projections of the platform on the axis of the coordinate system associated with the internal axis of the gimbal; $\omega_{xo},\ \omega_{yo},\ \omega_{zo}$ are projections of the platform on the axis of the coordinate system associated with the external axis of the gimbals.

After the substitution of relationships (6), (8), (9) in expressions (10) – (12), the equations of motion of the platform can be presented in a complete form. Due to the complexity and cumbersomeness of the transformations, only the equation of motion of the platform relative to its own axis is given in the work. To achieve the maximum possible simplification, a special case of the motion of an object with a constant course, in the small-angle regime, was considered. The resulting equation has the following form:

$$\begin{aligned}&\ddot{\gamma} + \tfrac{\dot{\alpha}}{I_{zp}}\omega_{x0} + \tfrac{I_{yp}-I_{xp}}{I_{z\Pi}}[cos2\gamma\left(\dot{\alpha}\omega_{x0} + \dot{\beta}\omega_{z0} + \dot{\beta}\omega_{y0} + \omega_{z0}\omega_{x0} + \omega_{y0}\omega_{x0}\right)+\\ &+0.5sin2\gamma\left(2\dot{\alpha}\omega_{y0} + \omega_{y0}^2 - 2\dot{\beta}\omega_{x0} - \omega_{x0}^2 + 2\omega_{x0}\omega_{z0}\alpha + 2\omega_{y0}\omega_{z0}\beta\right)] = M_{zp}.\end{aligned} \tag{13}$$

The projections of the angular velocity of an object can be determined using the following relationship:

$$\omega_0 = \omega_{0'} + \overset{''}{\omega_0}, \tag{14}$$

where $\omega_{0'}$ is the velocity of the initial reference frame $O\xi\eta\zeta$; $\omega_0''$ is the velocity of the body relative to this reference frame.

The position of the body relative to the initial reference frame, which is determined by rotations through angles $\psi_o,\ \vartheta_o,\ \varphi_o$, is shown in Fig. 2. In accordance with the representation given in Fig. 2, the formulas for velocity projections $\omega_{0'}$ as a component in the expression (14) can be presented in the following form:

$$\begin{aligned}\omega_{x0'} &= -\dot{\psi}_o\cos\vartheta_o\sin\varphi_o + \dot{\vartheta}_o\cos\varphi_o\\ \omega_{y0'} &= \dot{\varphi}_o + \dot{\psi}_o\sin\vartheta_o,\\ \omega_{z0'} &= \dot{\psi}_o\cos\vartheta_o\cos\varphi_o + \dot{\vartheta}_o\sin\varphi_o\end{aligned} \tag{15}$$

To complete the full mathematical description, it is necessary to represent the relationships for the description of the angular rate using parameters defined in the system of coordinates associated with the moving object.

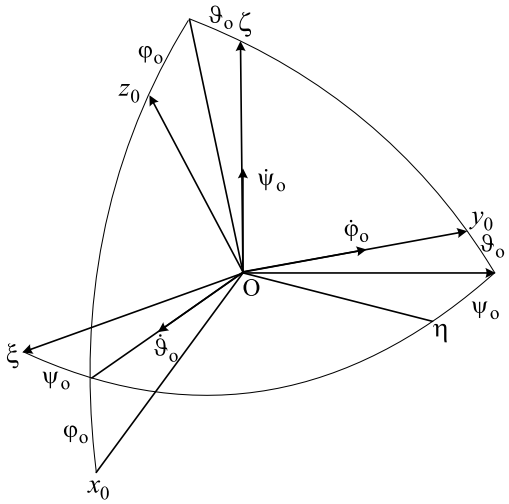


**Fig. 2.** Location of the object relative to the initial navigation coordinate system.

The motion of an object relative to the initial coordinate system $\omega_0''$ as a component of the expression (14) can be described using the following expressions:

$$\begin{aligned}
\omega_{x0}'' &= -\frac{V_{xp}}{R_1}\sin k - \frac{V_{yp}}{R_2}\cos k - \Omega\cos\varphi\sin k;\\
\omega_{y0}'' &= \frac{V_{xp}}{R_1}\cos k + \frac{V_{yp}}{R_2}\sin k + \Omega\cos\varphi\cos k;\\
\omega_{z0}'' &= \frac{V_{xp}}{R_1}\mathrm{tg}\phi - \dot{k} + \Omega\sin\varphi,
\end{aligned} \tag{16}$$

where: Ω is the speed of the Earth's motion; φ is the geographical latitude of the point; $R_1 = \frac{a\cos\varphi}{\sqrt{1-e^2\sin^2\varphi}}$ is the principal radius of curvature of the Earth's ellipsoid in the plane perpendicular to the meridian plane; $R_2 = \frac{a(1-e^2)}{(1-e^2\sin^2\varphi)^{3/2}}$ is the principal radius of curvature of the Earth's ellipsoid in the meridional plane; a, b are the semi-major and minor axes of the Earth's ellipsoid; $e = \frac{\sqrt{a^2-b^2}}{a}$ is the magnitude of the eccentricity; k is the direction of motion of the object.

Other features of the full mathematical description of the studied system, including mathematical models of gyroscopic devices, accelerometers, and electronic units, are presented in the publication [11].

## 3 Features of Parametric Optimization

To develop a computational algorithm for parametric optimization of the system, it is necessary to take into account its specific characteristics. In the case of the stabilized attitude and heading system, these features are as follows. First of all, it is a static second-order system, which necessitates the need to perform its minimum implementation before

optimizing the parameters. A study of the system matrix of the model of this system shows that the values of its elements differ by approximately three orders of magnitude. Given this, it is advisable to use a balanced implementation of the model. As parameters to be optimized, it is advisable to choose the transmission coefficients that determine the control laws. The parametric optimization process is implemented in two consecutive stages. The first stage involves the direct implementation of parameter optimization using the appropriate models developed using the advanced tools of the Control System Toolbox and Simulink Toolbox. The second stage involves the verification of the obtained results using a mathematical model that reflects all the nonlinear properties inherent in real systems and, if possible, includes models of all components that are part of the system under study.

The essence of the parametric optimization of the studied system lies in ensuring resistance to the internal and external disturbances. Such an approach can be based on the so-called mixed approach when the optimization criterion using $H_2$, $H_\infty$ norms is used [12–15]. The important component of the optimization criterion is the penalty function, which checks the stability of the motion control system during the optimization procedure. The algorithm of the parametric disturbances in application to the studied systems, taking into consideration the necessity of the precise stabilization of the platform with measuring instruments, is represented in publications [16, 17]. The synthesis of systems of another class, which are resistant to disturbances are given in publication [18–21]. These paper describe features of the robust control for ground and aerial vehicles. Some general aspects proposed in these publications were useful for developing the algorithm of the robust optimization for studied in the present paper system assigned for operation on marine vehicles with the necessity of realizing precise navigation measurements.

## 4 Modelling Results

The modelling of the studied system has been carried out in MATLAB environment and software, which is well adapted to solving problems associated with creating complex control systems, including those resistant to disturbances based on robust and adaptive approaches [22, 25]. These books are well known among scientists, which deal with the optimal synthesis of motion control systems of the wide class. They include many useful examples that show possibilities of MATLAB in realization of automatic control systems of the wide class.

The application of MATLAB to a successful solution of structural robust control for stabilization observation equipment on unmanned aerial vehicles is represented in papers [26, 27]. The present paper deals with parametric robust optimization. Nevertheless, ideas represented in [26, 27] were taken into consideration for creating the algorithm of platform stabilization in conditions of uncertainty. Examples of modelling for ground-moving vehicles, drive control systems, and aerial stochastic dynamic objects with unspecified dynamics are carried out in [28, 29]. Some similarity in the control of these objects and the studied moving platforms allowed us to improve some aspects of modelling.

For the studied system, simulation in different modes of the gimballed inertial navigation system has been carried out. The results of modelling optimal control laws in

the mode of preliminary horizon reduction, performed using a mathematical model approximated to the real gimballed system motion, are shown in Figs. 3 and 4. The obtained optimal control laws are based on a stochastic computational procedure taking into consideration the complex motion of the platform with the kinematics described by Eqs. (13), (15), (16).

A comparison of the stability of optimized and non-optimized systems to the action of disturbances in the stabilization mode is shown in Fig. 4.

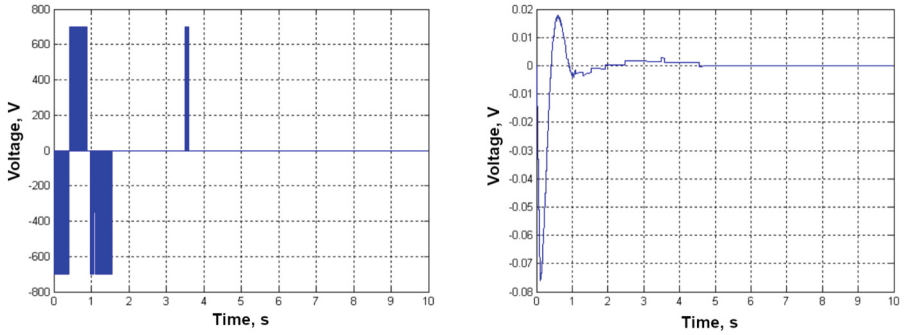


**Fig. 3.** Output signals of the pulse-width modulator (a) and calculating unit (b).

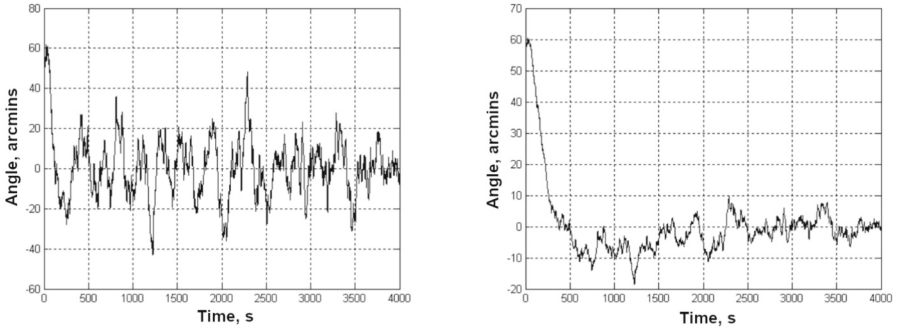


**Fig. 4.** Transient processes of changing angles of deviation from the horizontal plane in a perturbed non-optimized system (a) and optimised (b) systems.

It is obvious that the optimized system in real operating conditions shows better stabilization accuracy (see Fig. 4b). A characteristic feature of parametric optimization of the stabilized attitude and course determination system is the use of $H_2$, $H_\infty$ norms for both channels with the weighting coefficients, which are taken into account on equal terms.

The corresponding norms of the synthesized system are 0.1224 and 0.0054, respectively. These values (sufficiently less than 1) prove the low root mean square error and high robustness (stability to disturbances) of the synthesised system, correspondingly.

Consider the results of modelling in the mode of setting to the meridian. The process of parametric optimization of the studied system is accompanied by obtaining a minimal and balanced implementation of the model. In this mode, the $H_2$, $H_\infty$ norms, which characterize the accuracy and robustness of the system, were improved sufficiently from 7.24 to 0.78 and from 2.81 to 0.08, respectively. These values prove the high efficiency of the parameter optimization from the point of view of high accuracy achievement (defined by the low value of $H_2$ norm) and disturbance suppression (defined by the low value of $H_\infty$ norm). The high resistance to the disturbance for the synthesised system is explained also by the additional means directed to achieving the same goal, for example, application of the integral correction ensuring realization of the undisturbed gyroscopic vertical.

It should be noted that, in accordance with the purpose of the studied type of system, the most significant component of the control law is the one responsible for the accelerated drive to the meridian. It is advisable to include restrictions on this coefficient in the penalty function, since this allows you to control the speed of the process of bringing to the meridian. At the same time, reducing the time of bringing to the meridian is accompanied by an increase in the oscillations and norms of the system. The results of parametric optimization for the limiting values of this coefficient are shown in Fig. 5. It shows the transient process in the azimuth angle for the minimum (a) and maximum (b) values of the appropriate coefficient. In both cases, the stability margin in amplitude is 49.6 dB, and in phase, 76°. At the same time, there is a significant difference in the time of alignment to the meridian: 943 s and 601 s, respectively.

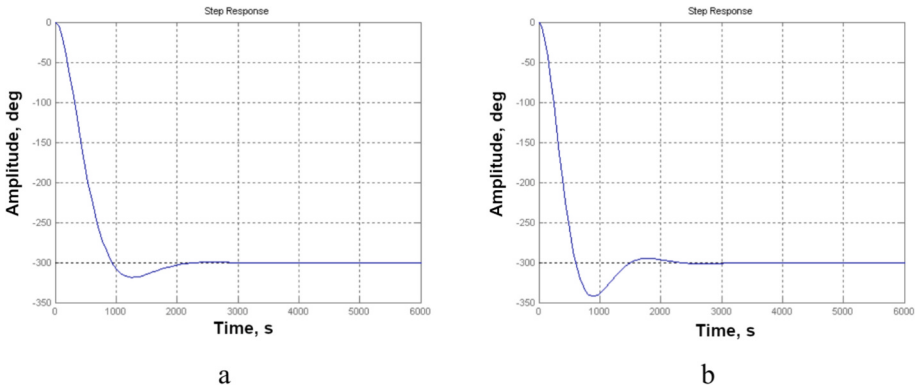


**Fig. 5.** Transients in the mode of setting to the meridian in the usual and accelerated modes.

The features of using MATLAB computing platform for the synthesis and modelling of the motion control systems studied in this paper are given in the books [22–24]. During modelling, Control System Toolbox and Robust Control Toolbox have been used. The full descriptions of these Toolboxes are given in the User's Guides [25, 30].

## 5 Discussion and Conclusions

The research aims to improve the operating characteristics of the gimballed inertial navigation systems using the detailed mathematical description of the stabilized platform kinematics for the synthesis of the high-precision gimballed navigation system.

The main achievement is taking into consideration the gimballed platforms' motion in the mathematical description of the gimballed navigation systems. This leads to sufficient improvement of the platform's stabilization and, hence, navigation accuracy. The obtained results can be useful for the creation of robust control systems in many areas of application of moving objects, including aerial, marine, and ground ones. Nevertheless, the main contribution of the obtained results can be achieved for the development of precision navigation systems.

Finally, it is possible to represent the following conclusions.

In the presented paper, the mathematical description of the kinematics for the gimballed inertia navigation system is presented. The coordinate systems for the complex motion of the platform with navigation instruments, which ensure its motion by three degrees of freedom, are represented. The equation of the platform's motion is represented. Expressions of the platform's angular rates are derived. The developed mathematical description of the complex motion of the gimballed platform with navigation instruments such as gyroscopic devices and accelerometers allows us to take into consideration the platform's complex motion of the platform and increase the accuracy of the motion control system synthesised based on the proposed mathematical description.

The features of the parametric optimization of motion control systems resistant to internal and external disturbances are generalized and described. The appropriate optimization criterion is analysed.

The modelling results, taking into consideration different aspects of the multi-mode gimballed inertial navigation system, such as previous levelling, stabilization, and setting to the meridian, are given. The representation of transient processes and calculation of $H_2$, $H_\infty$ norms is given.

The presented results can be useful for applications that require precision stabilization based on the possibility of controlling a moving platform with highly accurate navigation instruments. At the same time, the obtained results can be applicable for the precision stabilization of various measurement and observation devices (antennas, cameras, and so on), which nowadays are widely used on moving objects of different types.

The future work will be connected with a similar mathematical description of the gimballed system assigned for the stabilization of the observation equipment mounted on moving objects operated in difficult, disturbed conditions. The emphasis will be given on the instrumentation of the ground-moving objects.

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