# A Convergence Analysis of a 1-Norm Neighborhood Mehrotra-Type Algorithm

A. Teixeira[1,2(✉)] and R. De Almeida[1,2]

[1] Department of Mathematics University of Trás-os-Montes e Alto Douro, 5000-801 Vila Real, Portugal

[2] Centro de Matemática, Universidade do Minho - Polo CMAT-UTAD, Braga, Portugal

{ateixeir,ralmeida}@utad.pt

**Abstract.** Interior-point methods are a class of methods that explore the interior of the feasible region of a problem aiming to reach the optimal solution, contrasting with the simplex method that pursues the same goal by moving, in each iteration, from an extreme point to an adjacent extreme point. In this work, the complexity of a Linear Programming feasible predictor-corrector interior-point variant of the Mehrotra's algorithm using the 1-norm neighborhood is studied. It is proven that the algorithm has polynomial complexity and Q-linear convergence. To achieve these conclusions, some secondary lemmas and propositions are developed and demonstrated. In these results upper bound estimates of both the non linear term in the predictor direction and the non linear term in corrector direction are obtained. Additionally, during the procedure, step size bounds are derived and both the duality gap reduction and the complexity bound are obtained. Within this process, it is proven that, at each iteration, the step size of this 1-norm neighborhood algorithm is bounded by a function depending on the neighborhood parameter and on the dimension of the problem, thus, establishing that it has iteration complexity of $O\left(n^{\frac{3}{2}}\left|\log(\epsilon)\right|\right)$.

**Keywords:** Feasible Mehrotra-type algorithm · Convergence · 1-norm neighborhood

## 1 Introduction

Interior-point methods (IPM), due to Karmarkar [9], have become essential for solving large-scale Linear Programming (LP) problems, offering a robust and efficient alternative to traditional simplex methods [5]. It is worth to recall that while IPM explore the interior of the feasible region of a problem aiming to reach the optimal solution, the simplex method pursues the same goal by moving, in each iteration, from an extreme point to an adjacent extreme point.

One of the IPM variants are the predictor-corrector (PC) methods, that combine a predictor step, which approximates the central path, with a corrector

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step, which restores centrality. The performance of these methods depends on the definition of the central path's neighborhood, which determines how far the algorithm can deviate from centrality without compromising convergence.

The 2-norm neighborhood defines an ellipsoidal region, strongly penalizing individual deviations in the complementary product; thus, forcing the algorithm to maintain a rigid proximity to the central path, even when such a restriction is not necessary to ensure feasibility or progress. The infinity norm neighborhood represents the opposite extreme. By controlling only the largest individual deviation, it creates a highly permissive hypercubic region, allowing some components to deviate significantly from the mean. Although flexibility can be advantageous (in very elongated predictive directions) it can also require stronger corrections. The infinity norm offers maximum directional freedom, but with less overall control over the algorithm's behavior. The 1-norm neighborhood can be an intermediate approach, because it defines a polyhedral region that allows to compensate deviations; some components may deviate further from the mean as long as others remain close to it. Thus, it is effective for degenerate problems, instances where certain variables rapidly approach zero, and situations where the rigidity of the 2-norm limits progress. The polyhedral geometry reduces sensitivity to isolated extreme values and allows for larger prediction steps, often resulting in greater numerical robustness. Considering this, using the 1-norm in the central path neighborhood is a strategy that balances directional flexibility, centrality control, and numerical robustness (qualities essential for addressing the challenges of large-scale problems that drive current research in optimization) and attempting to find algorithms that are more stable and better adapted to the characteristics of real-world problems [15,20].

Mehrotra [13] developed one of the most successful PC IPM implemented, both theoretically and commercially. Several variants of this algorithm have been implemented and studied, namely [1,2,12,14,16,17,21,23,25]. Additionally, some works analyze the complexity of LP algorithms, proving their polynomiality, for e.g., [4,6,8,10,11,19,22,24].

In [3] some feasible PC variants of the Mehrotra's LP algorithm applied to transportation and assignment problems are presented; the special structure of the constraint matrix of the problems is taken into consideration. In [2,18] the authors proved the polynomial complexity of the primal-dual PC methodology using the infinity norm and the 2-norm neighborhood, respectively.

With the aim of obtaining an algorithm that balances the smoothness of the 2-norm and the permissiveness of infinity norm, in the present work, a feasible variant of the algorithm in [3], is adapted to use the 1-norm neighborhood and, then, theoretically analyzed. Unlike the neighborhoods based on the 2-norm and the infinite norm, which have been extensively studied in the classical literature on IPM, the 1-norm neighborhood exhibits a polyhedral geometry whose detailed analysis has received less attention. The results obtained here allow for a rigorous clarification of the trade-off between directional flexibility and control of centrality, establishing convergence conditions and complexity bounds that, to the best of our knowledge, have not been systematically developed for

this class of neighborhoods. It is demonstrated that, under assumptions analogous to those considered for Euclidean and infinity norm neighborhoods, the algorithm has polynomial complexity and Q-linear convergence. The step-size bounds derivation, the duality gap reduction, and the complexity bound are also presented, as well as the proofs of some necessary secondary results. This process allowed to prove that the algorithm with the 1-norm neighborhood has iteration complexity of $O\left(n^{\frac{3}{2}}|\log(\epsilon)|\right)$. Additionally, its polyhedral geometry proves particularly well-suited for addressing degenerate situations and large-scale problems, offering a theoretical framework that complements traditional approaches.

In this paper, the algorithm and the fundamentals needed in the remaining of the paper are explained in Sect. 2; the convergence analysis of the algorithm is presented in Sect. 3. Finally, the conclusions are drawn in the last section.

## 2 Preliminaries

The fundamental concepts and notation used throughout this paper will be presented in this section.

Let $A$ be a real matrix $m \times n$ and $b, c$ real column vectors of order $m$ and $n$, respectively. The sets of primal and dual feasible solutions are presented, respectively, by

$$\mathbb{F}_P = \{x \in \mathbb{R}^n : \ Ax = b, \ x \geqslant 0\},$$
$$\mathbb{F}_D = \left\{(u, v) \in \mathbb{R}^m \times \mathbb{R}^n : \ A^T u + v = c, \ v \geqslant 0\right\}.$$

The LP primal problem can be written as

$$\text{(P)} \ \min_{x \in \mathbb{F}_P} \ c^T x,$$

and the dual problem as

$$\text{(D)} \ \max_{(u,v) \in \mathbb{F}_D} \ b^T u.$$

Any strictly feasible solution of this pair of problems belongs to the set

$$\mathbb{F}^\circ = \{(x, u, v) \in \mathbb{F}_P \times \mathbb{F}_D : x \neq 0, \ v \neq 0\}.$$

Here, the complexity of a feasible variant of the predictor-corrector algorithm developed by Bastos and Paixão [3], *Algorithm BP*, will be analyzed. The algorithm begins with the selection of the parameters, $\varepsilon > 0$, $0 < \tau \leqslant 1$ and $0 \leqslant \theta < 1$, as well as an initial point $(x^0, u^0, v^0)$ within the 1-norm neighborhood defined in (1), where $e$ is the column vector of ones,

$$\mathbb{V}_1(\theta) = \left\{(x, u, v) \in \mathbb{F}^\circ : \ \|XVe - \tau\mu e\|_1 \leqslant \tau\theta\mu\right\}. \tag{1}$$

Then, to test if an optimal solution was achieved, the duality gap, given by

$$\mu = \frac{c^T x - b^T u}{\beta(n)}, \tag{2}$$

where $\beta(n) = n^{\frac{3}{2}}$ if $n \in ]5000, +\infty[$ and $\beta(n) = n^2$ otherwise, is compared with the chosen parameter $\varepsilon$. If the duality gap is inferior to $\varepsilon$, then the algorithm stops and an optimal primal-dual solution is attained. Else, to obtain an optimal primal-dual solution, while the duality gap is not inferior to $\varepsilon$ the following computations are performed, where $D_Y = diag(y)$ represents the diagonal matrix of vector $y$.

1. The predictor direction $(\Delta x^a, \Delta u^a, \Delta v^a)$ that satisfies

$$\begin{cases} AD_{\triangle X}e = 0 \\ A^T D_{\triangle U}e + D_{\triangle V}e = 0 \\ D_V D_{\triangle X}e + D_X D_{\triangle V}e = \tau\mu e - D_X D_V \end{cases} . \tag{3}$$

2. The corrector direction $(\Delta x, \Delta u, \Delta v)$ that satisfies

$$\begin{cases} AD_{\triangle X}e = 0 \\ A^T D_{\triangle U}e + D_{\triangle V}e = 0 \\ D_V D_{\triangle X}e + D_X D_{\triangle V}e = \tau\mu e - D_X D_V - D_{\triangle X^a} D_{\triangle V^a} e \end{cases} . \tag{4}$$

3. The primal-dual step lengths, $(\alpha_p, \alpha_d)$, as the pair of the maximum positive value components, $\alpha_1$ and $\alpha_2$, such that

$$(x + \alpha_1 \Delta x,\ u + \alpha_2 \Delta u,\ v + \alpha_2 \Delta v) \in \mathbb{V}_1(\theta).$$

4. The next iterate

$$(x^+, u^+, v^+) = (x, u, v) + 0.9995(\alpha_p \triangle x, \alpha_d \triangle u, \alpha_d \triangle v).$$

In this paper, the following index sets, concerning the components of the predictor direction, are also considered

$$\mathbb{I} = \{1, 2, \cdots, n\},$$
$$\mathbb{I}_+ = \{i \in \mathbb{I} : \triangle x_i^a \triangle v_i^a > 0\}, \qquad \mathbb{I}_- = \{i \in \mathbb{I} : \triangle x_i^a \triangle v_i^a < 0\}.$$

In order to study the convergence of *Algorithm BP*, some results are presented and proved in the next section.

## 3 Convergence Analysis

An upper bound for the non linear term involving the predictor direction used by *Algorithm BP* is obtained in Lemma 1 and an upper bound for the norm concerning the non linear term of the corrector direction of this algorithm is given by Proposition 1. Moreover, to make this work self contained, some previously published results are also presented, namely Lemmas 2, 3 and 4.

**Lemma 1.** *Let* $(\triangle x^a, \triangle u^a, \triangle v^a)$ *be the predictor direction of Algorithm BP. Then, for all* $i \in \mathbb{I}_+$, $\theta \in [0, 1[$ *and* $\tau \in ]0, 1]$,

$$\triangle x_i^a \triangle v_i^a \leqslant \frac{1}{4} \frac{\theta^2}{(1-\theta)} \tau\mu.$$

*Proof.* From the third condition of (3), one gets

$$\frac{\triangle x_i^a}{x_i} + \frac{\triangle v_i^a}{v_i} = \frac{\tau\mu}{x_i v_i} - 1. \tag{5}$$

Since

$$0 \leqslant \left(\frac{\triangle x_i^a}{x_i} - \frac{\triangle v_i^a}{v_i}\right)^2 = \left(\frac{\triangle x_i^a}{x_i}\right)^2 + \left(\frac{\triangle v_i^a}{v_i}\right)^2 - 2\frac{\triangle x_i^a \triangle v_i^a}{x_i v_i},$$

then

$$\left(\frac{\triangle x_i^a}{x_i}\right)^2 + \left(\frac{\triangle v_i^a}{v_i}\right)^2 \geqslant 2\frac{\triangle x_i^a \triangle v_i^a}{x_i v_i}. \tag{6}$$

Therefore, using

$$\left(\frac{\triangle x_i^a}{x_i} + \frac{\triangle v_i^a}{v_i}\right)^2 = \left(\frac{\triangle x_i^a}{x_i}\right)^2 + \left(\frac{\triangle v_i^a}{v_i}\right)^2 + 2\frac{\triangle x_i^a \triangle v_i^a}{x_i v_i}$$

$$\geqslant 4\frac{\triangle x_i^a \triangle v_i^a}{x_i v_i}$$

equality (5) and inequality (6), one has

$$\frac{\triangle x_i^a \triangle v_i^a}{x_i v_i} \leqslant \frac{1}{4}\left(\frac{\tau\mu}{x_i v_i} - 1\right)^2$$

$$= \frac{1}{4}\left(\frac{\tau\mu - x_i v_i}{x_i v_i}\right)^2.$$

Since

$$|x_i v_i - \tau\mu| \leqslant \sum_{i=1}^{n} |x_i v_i - \tau\mu| = \|XVe - \tau\mu e\|_1 \leqslant \tau\theta\mu,$$

then follows

$$\triangle x_i^a \triangle v_i^a \leqslant \frac{1}{4}\frac{|\tau\mu - x_i v_i||\tau\mu - x_i v_i|}{x_i v_i} \leqslant \frac{1}{4}\frac{(\tau\theta\mu)^2}{x_i v_i}.$$

Due to

$$(1-\theta)\tau\mu \leqslant x_i v_i \leqslant (1+\theta)\tau\mu$$

one gets

$$\triangle x_i^a \triangle v_i^a \leqslant \frac{1}{4}\frac{(\tau\theta\mu)^2}{x_i v_i} \leqslant \frac{1}{4}\frac{\theta^2}{(1-\theta)}\tau\mu. \tag{7}$$

Henceforth, the condition $\triangle x^{aT}\triangle v^a \geqslant 0$ will be considered for the predictor direction of *Algorithm BP*.

In order to prove Proposition 1 the following result (transcribed from [20, Lemma 5.3 (p.88)] and herein called Lemma 2), presenting an upper bound estimate for $\|D_U D_V e\|_2$, will also be needed.

**Lemma 2.** *Let $u, v \in \mathbb{R}^n$ with $u^T v \geqslant 0$. Then,*

$$\|D_U D_V e\|_2 \leqslant 2^{-\frac{3}{2}} \|u+v\|_2^2.$$

An upper bound for the norm of the non linear term of the corrector direction of *Algorithm BP* is derived in the next result.

**Proposition 1.** *Let $(x,u,v) \in \mathbb{V}_1(\theta)$ be the current iterate of Algorithm BP and $(\triangle x, \triangle u, \triangle v)$ be the corrector direction of Algorithm BP. Then,*

$$\|D_{\triangle V} D_{\triangle X} e\|_1 \leqslant C(\theta)\, \tau\mu,$$

*for $\triangle x^{aT}\triangle v^a \geqslant 0$, and $C(\theta) = \dfrac{n^{\frac{3}{2}}}{18\sqrt{2}(1-\theta)^3}$.*

*Proof.* Lets start by multiplying the third equation of (4) by $(D_X D_V)^{-\frac{1}{2}}$ and, thus, obtain

$$\begin{aligned}(D_X)^{-\frac{1}{2}}(D_V)^{\frac{1}{2}}\triangle x + (D_V)^{-\frac{1}{2}}(D_X)^{\frac{1}{2}}\triangle v = \\ = (D_X D_V)^{-\frac{1}{2}}\left(\tau\mu^k e - D_{\triangle X^a} D_{\triangle V^a} e - D_X D_V e\right).\end{aligned} \tag{8}$$

Taking the first term of (8) and considering

$$M = (D_V)^{-\frac{1}{2}}(D_X)^{\frac{1}{2}},$$

one has

$$(D_X)^{-\frac{1}{2}}(D_V)^{\frac{1}{2}}\triangle x + (D_V)^{-\frac{1}{2}}(D_X)^{\frac{1}{2}}\triangle \mathrm{v} = M^{-1}\triangle x + M\triangle v.$$

Now, let

$$w_1 = M^{-1}\triangle x, \quad w_2 = M\triangle v, \quad W_1 = \mathrm{diag}(w_1), \quad W_2 = \mathrm{diag}(w_2).$$

Using Lemma 2 and the fact that $\|\cdot\|_2 \leqslant \|\cdot\|_1 \leqslant \sqrt{n}\,\|\cdot\|_2$, follows

$$\begin{aligned}
\|W_1W_2e\|_1 &= \|D_{\triangle V}D_{\triangle X}e\|_1 \\
&\leqslant \sqrt{n}\ \|D_{\triangle V}D_{\triangle X}e\|_2 \\
&\leqslant 2^{-\frac{3}{2}}\sqrt{n}\ \|w_1+w_2\|_2^2 \\
&= 2^{-\frac{3}{2}}\sqrt{n}\ \left\|(D_XD_V)^{-\frac{1}{2}}\left(\tau\mu e - D_{\triangle X^a}D_{\triangle V^a}e - D_XD_Ve\right)\right\|_2^2 \\
&\leqslant 2^{-\frac{3}{2}}\sqrt{n}\left(\left\|(D_XD_V)^{-\frac{1}{2}}\left(\tau\mu e - D_XD_Ve\right)\right\|_2\right. \\
&\qquad \left. + \left\|(D_XD_V)^{-\frac{1}{2}}D_{\triangle X^a}D_{\triangle V^a}e\right\|_2\right)^2. \qquad (9)
\end{aligned}$$

Lets start by estimate

$$\left\|(D_XD_V)^{-\frac{1}{2}}\left(\tau\mu e - D_XD_Ve\right)\right\|_2.$$

Since $(x,u,v)\in\mathbb{V}_1(\theta)$, then

$$\|D_XD_Ve-\tau\mu e\|_2 \leqslant \|D_XD_Ve-\tau\mu e\|_1 \leqslant \theta\tau\mu$$

and $\min\limits_i\{x_iv_i\}\geqslant(1-\theta)\tau\mu$, therefore, one gets

$$\begin{aligned}
\left\|(D_XD_V)^{-\frac{1}{2}}\left(\tau\mu e - D_XD_Ve\right)\right\|_2 &\leqslant \left(\sum_{i\in\mathbb{I}}\frac{1}{x_iv_i}\right)^{\frac{1}{2}}\|D_XD_Ve-\tau\mu e\|_2 \\
&\leqslant \frac{\sqrt{n}\theta\tau\mu}{\sqrt{(1-\theta)\tau\mu}}. \qquad (10)
\end{aligned}$$

To estimate

$$\left\|(D_XD_V)^{-\frac{1}{2}}D_{\triangle X^a}D_{\triangle V^a}e\right\|_2$$

one uses Lemma 1 and $\min\limits_i\{x_iv_i\}\geqslant(1-\theta)\tau\mu$, and obtains

$$\begin{aligned}
\left\|(D_XD_V)^{-\frac{1}{2}}D_{\triangle X^a}D_{\triangle V^a}e\right\|_2 &= \left(\sum_{i\in\mathbb{I}}\frac{(\triangle x_i^a\triangle v_i^a)^2}{x_iv_i}\right)^{\frac{1}{2}} \\
&\leqslant \frac{\sqrt{n}\theta^2\tau\mu}{4(1-\theta)\sqrt{(1-\theta)\tau\mu}}. \qquad (11)
\end{aligned}$$

Applying (10) and (11) in (9), one gets

$$\|D_{\triangle V} D_{\triangle X} e\|_1 \leqslant 2^{-\frac{11}{2}} n^{\frac{3}{2}} \frac{\left(4\theta - 3\theta^2\right)^2}{(1-\theta)^3} \tau\mu.$$

Since $0 \leqslant \theta < 1$, the function $g(\theta) = \left(4\theta - 3\theta^2\right)^2$ has a maximum at $\theta = \frac{2}{3}$, then

$$0 \leqslant g(\theta) = \left(4\theta - 3\theta^2\right)^2 \leqslant \frac{16}{9}.$$

Therefore,

$$\|D_{\triangle V} D_{\triangle X} e\|_1 \leqslant n^{\frac{3}{2}} \frac{1}{18\sqrt{2}(1-\theta)^3} \tau\mu.$$

To obtain the relation between the duality measure of the next iterate of *Algorithm BP* with the current one the next result, proved in [18], will be used.

**Lemma 3.** *Consider $\mu$ defined in (2). Then,*

$$\mu^+ = \Big(1 - \delta(\lambda)\Big)\mu$$

*where*

$$\delta(\lambda) = \lambda \left(1 - \frac{n}{\beta(n)} + \frac{\triangle v^{aT} \triangle x^a}{v^T x}\right).$$

The previous results allows to obtain an upper bound for the steplength taken by *Algorithm BP*, as shown in the following theorem.

**Theorem 1.** *Consider that the current iterate of Algorithm BP, $(x, u, v)$, is in $\mathbb{V}_1(\theta)$. Suppose that $(\triangle x^a, \triangle u^a, \triangle v^a)$ is the solution of (3) and that $(\triangle x, \triangle u, \triangle v)$ is the solution of (4). Then, the maximum step size, $\lambda$, that keeps the next iterate of Algorithm BP, $(x^+, u^+, v^+)$, in $\mathbb{V}_1(\theta)$, satisfies*

$$\lambda \leqslant \frac{9\sqrt{2}(4\theta - 3\theta^2)(1-\theta)^2}{2n^{\frac{3}{2}}}, \qquad k \geqslant 0, \quad n \geqslant 5.$$

*Proof.* The aim, in this proof, is to find the maximum steplength $\lambda \geqslant 0$ that guarantees $(x^+, u^+, v^+) \in \mathbb{V}_1(\theta)$.

As $\triangle x^{aT} \triangle v^a \geqslant 0$, it is assured that $\mathbb{I}_+ \neq \emptyset$. Thus, consider $i \in \mathbb{I}_+$. Then,

$$x_i^+ v_i^+ = (1-\lambda) x_i v_i + \lambda\tau\mu - \lambda \triangle x_i^a \triangle v_i^a + \lambda^2 \triangle x_i \triangle v_i.$$

To satisfy the condition $(x^+, u^+, v^+) \in \mathbb{V}_1(\theta)$, lets consider $x_i^+ v_i^+ - \tau\mu^+$ and Lemma 3. Therefore, one gets

$$x_i^+ v_i^+ - \tau\mu^+ = (x_i v_i - \tau\mu)\left(1 - \lambda\left(1 + \frac{\triangle x_i^a \triangle v_i^a}{x_i v_i - \tau\mu} - \lambda\frac{\triangle x_i \triangle v_i}{x_i v_i - \tau\mu}\right.\right.$$

$$\left.\left. - \frac{\tau\mu}{x_i v_i - \tau\mu}\left(1 + \frac{\triangle v^{aT}\triangle x^a}{v^T x} - \frac{n}{\beta(n)}\right)\right)\right).$$

Hence, in order to obtain the maximum $\lambda$, such that $(x^+, u^+, v^+) \in \mathbb{V}_1(\theta)$, consider

$$\frac{\triangle x_i^a \triangle v_i^a}{x_i v_i - \tau\mu} - \lambda\frac{\triangle x_i \triangle v_i}{x_i v_i - \tau\mu} - \frac{\tau\mu}{x_i v_i - \tau\mu}\left(1 + \frac{\triangle v^{aT}\triangle x^a}{v^T x} - \frac{n}{\beta(n)}\right) \geqslant$$

$$\geqslant \frac{\triangle v^{aT}\triangle x^a}{v^{kT} x^k} - \frac{n}{\beta(n)},$$

consequently, obtaining

$$\triangle x_i^a \triangle v_i^a + (x_i v_i)\left(-\frac{\triangle v^{aT}\triangle x^a}{v^T x} + \frac{n}{\beta(n)}\right) - \tau\mu \geqslant \lambda\triangle x_i \triangle v_i,$$

which allows to get

$$\triangle x_i^a \triangle v_i^a + x_i v_i - \tau\mu \geqslant \lambda\triangle x_i \triangle v_i. \tag{12}$$

Now, using Lemma 1, one has

$$\triangle x_i^a \triangle v_i^a + x_i v_i - \tau\mu \leqslant \frac{1}{4}\frac{\theta^2}{(1-\theta)}\tau\mu + \theta\tau\mu$$

and, using Proposition 1, one can obtain

$$\triangle x_i^a \triangle v_i^a + x_i v_i - \tau\mu \geqslant \lambda\frac{n^{\frac{3}{2}}}{18\sqrt{2}(1-\theta)^3}\tau\mu \geqslant \lambda\triangle x_i \triangle v_i.$$

Therefore,

$$\lambda \leqslant \frac{9\sqrt{2}(4\theta - 3\theta^2)(1-\theta)^2}{2n^{\frac{3}{2}}}.$$

In order to prove Theorem 2, which indicates an upper bound for the number of iterations in which *Algorithm BP* stops and allows to conclude that it has polynomial complexity, Theorem 3.2 of [20, p. 61] (here transcribed and renamed as Lemma 4) is required.

**Lemma 4.** *Let $\epsilon \in ]0,1[$ be given. Suppose that our algorithm for solving the Karush-Kuhn-Tucker conditions associated with the standard primal-dual pair of Linear Programming problems generates a sequence of iterates that satisfies*

$$\mu^+ \leqslant \left(1 - \frac{\delta}{n^\omega}\right)\mu$$

*for some positive constants $\delta$ and $\omega$. Suppose too that the starting point $(x^0, u^0, v^0)$ satisfies*

$$\mu^0 \leqslant \frac{1}{\epsilon^\kappa}$$

*for some positive constant $\kappa$. Then there exists an index $K$ with*

$$K = O\left(n^\omega \left|\log(\epsilon)\right|\right)$$

*such that*

$$\mu \leqslant \epsilon \quad \text{for all } \kappa \geqslant K.$$

An upper bound for the number of iterations in which *Algorithm BP* stops is next established.

**Theorem 2.** *Let $\epsilon \in ]0,1[$ and $\triangle x^{a^T} \triangle v^a \geqslant 0$. Algorithm BP stops after at most $O(n^{\frac{3}{2}} |\log(\epsilon)|)$ iterations with a solution for which $x^T v \leqslant \epsilon$.*

*Proof.* Consider $\mu$ define as in (2) and Lemma 3. In order to obtain an upper bound for $\mu^+$ a lower bound for $\delta(\lambda)$ must be found. Since $\triangle x^{a^T} \triangle v^a \geqslant 0$, and $n\sqrt{n} \leqslant \beta(n) \leqslant n^2$, then

$$\delta(\lambda) \geqslant \lambda\left(1 - \frac{1}{\sqrt{n}}\right).$$

Using Theorem 1, for $n \geqslant 5$ and

$$\lambda = \frac{9\sqrt{2}(4\theta - 3\theta^2)(1-\theta)^2}{2n^{\frac{3}{2}}},$$

one gets

$$\mu^+ \leqslant \left(1 - \lambda\left(1 - \frac{1}{\sqrt{n}}\right)\right)\mu \leqslant \left(1 - \frac{9\sqrt{2}(4\theta - 3\theta^2)(1-\theta)^2}{4n^{\frac{3}{2}}}\right)\mu.$$

Finally, applying Lemma 4 the proof is complete.

This last result allows to conclude that this feasible version of the algorithm developed by Bastos and Paixão that uses the 1-norm neighborhood, *Algorithm BP*, is Q-linear convergent.

## 4 Conclusions

The analysis of the performance of interior-point algorithms continues to rely predominantly on their numerical effectiveness, since, in practice, worst-case complexity bounds tend to be overly conservative and not very representative of the behavior observed in real-world instances. Nevertheless, the rigorous study of asymptotic complexity remains a central element in theoretical research in this field. Several authors have demonstrated that the existence of a guarantee that the number of iterations grows in a controlled manner with the problem size is not merely an academic result, but a relevant indicator of a method's structural robustness [2,7,10–12,14,16,17,19,21–25]. An algorithm that satisfies such a guarantee offers greater predictability and stability in the face of adverse instances, reducing the probability of severe performance degradation. Thus, although practical efficiency is an important criterion when comparing methods performances, theoretical bounds that ensure polynomial computational effort constitutes an important complement that reinforces confidence in the algorithm's overall reliability and its ability to maintain controlled behavior even in extreme scenarios.

In this work, it is proved that, at each iteration of the *Algorithm BP*, the step size computed is bounded by $\frac{9\sqrt{2}(4\theta-3\theta^2)(1-\theta)^2}{2n^{\frac{3}{2}}}$, where $\theta \in [0,1[$, therefore, obtaining an $O\left(n^{\frac{3}{2}}|\log(\epsilon)|\right)$ iteration complexity bound; thus, proving that the feasible version of the 1-norm neighborhood algorithm developed by Bastos and Paixão in [3] has polynomial iteration complexity and is Q-linear convergent. In this sequel, it can be concluded that the obtained results show that it could be worth to implement the proposed variant of the Bastos and Paixão algorithm and analyze the results of its application to transportation and assignment class of problems.

Finally, it is important to observe that in this paper the authors concentrate on the theoretical aspects of the 1-norm neighborhood, specifically focusing on the convergence and complexity analysis of the algorithm under analysis. However, although no computational results have been included, the authors recognize that numerical evaluation of the algorithm is important to illustrate the practical impact of different geometric choices in the neighborhood of the central path. Thus, as future work, they intend to further study the algorithm, not only to improve its convergence results, but also to proceed with its implementation, conducting numerical experiments to compare the 1-norm neighborhood approach with the ones that use classical neighborhoods.

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