Distributionally Robust Optimisation for Prepositioning Wildfire Fighting Resources

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Abstract. We address the problem of prepositioning firefighting resources for wildfire suppression. Given the inherent uncertainty in the location of ignitions and the wind, we formulate a two-stage optimisation model. In the first stage, resource prepositioning decisions are made in anticipation of possible fire outbreaks; in the second stage, once ignitions are realised, the movement of resources to attack positions is determined. Fire spread is modeled using the minimum travel time principle and is represented through a mixed-integer programming model that integrates both fire spread and operational decisions.

To address uncertainty, we propose a two-stage distributionally robust optimisation model which includes a discrete set of scenarios characterised by varying wind directions and ignition locations. A row-and-column decomposition algorithm is developed to efficiently solve the resulting large-scale problem.

Computational experiments are conducted on realistic instances derived from an actual landscape. Rothermel's fire behaviour model is used to estimate the fire potential transmission times. Each instance considers 80 scenarios reflecting different ignition patterns and wind behaviours. The results demonstrate the effectiveness of the proposed approach and highlight the scalability and computational efficiency of the decomposition method.

Keywords: Fire suppression · Distributionally robust optimisation · Two-stages · Decomposition algorithm

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1 Introduction

Wildfires are natural disasters that can have a significant impact on ecosystems which can lead to the loss of lives and material damage. Therefore, their management is a highly relevant issue. Following [1], wildfire management can be divided in three main areas: prevention, detection and management of fire suppression resources. In this paper, we address an operational management of fire suppression resources problem. Although the first operational research approaches to this type of problems are from the early 1960 s [2], only recently a categorized literature review [3] and a survey [4] have been published, supporting the current relevance of the topic.

Operational management of fire suppression resources can be typified in deployment, dispatch and positioning. Deployment decisions are taken before an actual fire is detected and typically amount to defining which resources should be (re)allocated to which bases. Dispatch decisions are taken after an ignition is detected and amount to deciding which resources should be sent to the fire. Lastly, positioning decisions detail where the dispatched resources should attack the fire.

In this paper, we consider a problem that combines the prepositioning and positioning of resources. The decisions of repositioning the resources, which can be seen as deployment decisions but with allocation alternatives other than the bases - for example, roads - are taken before fire is detected. After a fire is detected, the attack positions of the resources are defined. Given the uncertainty involved in the location of the fire (and also on the wind), we derive an optimisation under uncertainty approach based on mixed integer programming (MIP).

Different MIP models have been proposed for the positioning problem. In [5], the minimum travel time principle (MTT, [6]) was used for the first time to model the fire paths and arrival times in a landscape. According to this principle, fire follows the quickest path between any two locations of a landscape. This principle allows to model fire spread as shortest paths in a network and has since been used in software that simulates fire spread (e.g. [7]) and in MIP models for different variants of the positioning problem, for example in [3,8–12].

These are deterministic approaches that, at the expense of increasing the complexity and difficulty of the approach, can be extended to address uncertainty.

Stochastic programming (SP) has been applied to problems related to the one addressed in this paper with Distributionally Robust Optimisation (DRO). For example, in [13], a two-stage model is proposed to minimise the total cost (that includes the cost of the used resources, the expected suppression cost and the net value change). Scenarios are characterised by a set of fires, each with a known perimeter at each time period, and, among other attributes, by the line production rate of the resources. Ntaimo et al. [14] also proposed a two-stage SP model with a similar objective function. A major difference is that the effectiveness of the suppression is assessed by the number of fires that do not receive a standard response, instead of fire containment measures. In [15], a two-

stage MIP model formulates the problem of defining the bases for resources given a set of scenarios, each characterised by the number fires that occur on a day, along with their location and their ignition time. For each scenario, a standard response, i.e. the number of resources required to extinguish the fires is derived. The model minimises the expected value (each scenario has a probability) of fires that do not receive a standard response. A detailed case study with data from California, with a rigorous parametrisation of the model (including the generation of scenarios and resources) is discussed. SP has also been applied in other Wildfire problems, where for defining attacking positions with wind uncertainty [16,17].

A major difference of the proposed approach for prepositioning, besides the use of DRO instead of SP, lies in how fire spread is explicitly modelled. In the proposed approach, fire spread is integrated with decisions related to resource prepositioning and attack positioning. This type of *integrated* approach has also been used in related problems with Robust Optimisation (RO). Mendes and Alvelos [18] proposed a robust optimisation approach to address uncertainty on the number of resources required to suppress fire. Marques et al. [19] addressed the same problem as the one of this work with "pure" robust optimisation.

DRO may be seen as a generalisation that encompasses SP and RO as special cases. While SP assumes that the true probability distribution of the uncertain parameters is known, RO only assumes that the parameters belong to an uncertainty set. In addition, while SP optimises over the expected value, RO evaluates the decisions under the worst-case scenario. In DRO, it is assumed that the probability distribution is unknown but belongs to a set called the ambiguity set. In our case, we use an extension of the one-norm Wasserstein ambiguity set, also known as the Kantorovich ambiguity set. This set is centred on the nominal distribution and includes all distributions whose distance from the nominal distribution does not exceed a parameter ϵ. To measure the distance, we use a transportation problem (where the quantities associated with the origins correspond to the probability mass of given scenarios in the nominal distribution and the quantities in the destination correspond to the probability mass of the final distribution), subject to an additional knapsack constraint that controls the mass to transport. The goal is to determine the solution that minimises the expected cost considering the worst-case probability distribution within the ambiguity set. When the ambiguity set uses the Kantorovich ambiguity set and the support is discrete, Bansal et al. [20] show that two-stage DRO mixed-integer problems can be converted into (larger) computationally tractable single-stage mixed-integer problems. For reviews on DRO, see [21,22].

Given its flexibility, DRO is becoming very popular. To the best of our knowledge, it has never been applied to wildfire management. For a related application with wind uncertainty, see [23].

In this paper, we propose a two-stage DRO model based on a discrete set of scenarios. These scenarios are defined by a wind direction and the locations of one or more ignitions. Both factors result from two statistical studies conducted on each factor alone [24]. We consider that the spatial distribution of fuels and

slopes of the landscape under study are known. The problem is to decide where to prepositioning the resources such that the burned area is minimized in the worst-case scenario. The approach can be seen as having static variables (related to the prepositioning decisions) and adjustable variables (related to the decisions of each scenario). Both the number of constraints and the number of adjustable variables depend on the number of scenarios. To solve the problem we propose a row-and-column decomposition algorithm. For a comparison of the different solution approaches for the two-stage DRO problem under different ambiguity sets, see [25].

This paper is organized as follows: we provide a description of the problem in Sect. 2. In Sect. 3, we introduce the DRO model. The row-and-column decomposition algorithm is presented in Sect. 4. The computational experiments conducted to provide insights on the solutions and to evaluate the proposed model and decomposition algorithm are reported in Sect. 5. Finally, in Sect. 6, we provide the conclusions and future research directions.

2 Problem Definition, Notation and a Deterministic Model

We consider the problem of prepositioning a set of available firefighting resources, possibly of different types, in a landscape before a wildfire ignition occurs with the objective of minimising the worst-case burned area, within a given time period, of a set of scenarios, each one characterised by one or more ignitions and a wind speed and direction. After the prepositioning decision is taken and the uncertainty is disclosed (i.e. the actual scenario is known), the resources can be moved from their prepositioning locations to the ones where they interdict fire spread.

In the context of optimisation under uncertainty, the problem considered is a two-stage problem. In the first-stage, the prepositioning of the resources is decided, i.e. static decisions are taken. Then the uncertainty (ignition(s) locations and wind) is revealed. In the second stage, the (adjustable) decisions regarding movement from prepositioning positions to the positions where the resources attack fire are made.

To model this problem, we represent the landscape as a network in which nodes correspond to centroids of hexagons forming a grid, and arcs represent potential fire transmission between adjacent nodes. Each arc has a fire transmission time that corresponds to the time it takes for the fire to reach the destination node of the arc once the fire reached the origin node. In our computational experiments, the fire transmission times of the arcs were derived from the fire behaviour model of Rothermel [26], which accounts for fuels, slopes, and winds to estimate the fire rate of spread under homogeneous conditions. It is worth noting that, in the optimisation model, the fire transmission time is a parameter and therefore can be obtained with other methods.

Based on the fire transmission times, it is possible to model the fire spread by using the minimum travel time principle [6], which states that the time it takes

for the fire to reach a node is the length (duration) of the shortest (quickest) path from an ignition node.

We now introduce the notation used.

The fire network, already introduced, is represented by $G^f(N^f, A^f)$. The set of nodes that are burning at the detection time, e.g. the fire perimeter, is represented by I^f. The fire transmission time between adjacent nodes i and j (i.e. of the arc ij) is represented by c_{ij}. The instant where the burned area is evaluated is denoted by T.

We consider a set of available resource types defined by R. Each resource type, r, $r \in R$, is associated with a graph, represented by $G^r(N^r, A^r)$, where the nodes correspond to potential attack positions and the arcs to potential movements.

The fire and resource nodes are related through a set of pairs (l, i), denoted by A^{rf}, where l is a node of the resource network and i is a node of the fire node. If a unit of resource r is positioned at a node l of its resource network, then all the nodes i of the fire network for which $(l, i) \in A^{rf}$ are attacked. A fire node being attacked means that fire will no longer spread through the outgoing arcs of i. The constant d^r_{kl} is the quickest path duration of resource r between nodes k and l. The number of available resource units of type r is denoted by b^r.

We now introduce a deterministic model that assumes that the prepositioning of the resources is known and provided by parameters $\bar{x}^r_k = 1$ if a resource of type r is prepositioned at node k and $\bar{x}^r_k = 0$, otherwise. If no uncertainty is present, this model provides the optimal attack positions for all resources, given their current location (i.e. assuming fixed prepositions). The following decision variables are defined:

$$t_i \text{ fire arrival time at node } i \qquad i \in N^f;$$

$$z^r_{kl} = \begin{cases} 1, & \text{if the resource of type } r \text{ prepositioned at node } k \text{ is moved to node } l \\ 0, & \text{otherwise} \end{cases} \qquad kl \in A^r, r \in R;$$

$$y_i = \begin{cases} 1, & \text{if node } i \text{ burns before instant } T \\ 0, & \text{otherwise} \end{cases} \qquad i \in N^f.$$

The MIP deterministic model is:

$$\text{Min} \sum_{i \in N^f} y_i \tag{1}$$

s.t.

$$t_i = 0 \qquad i \in I^f \tag{2}$$

$$t_j \leq t_i + c_{ij} + T \sum_{kl \in A^r, li \in A^{rf}} z^r_{kl} \qquad ij \in A^f, r \in R \tag{3}$$

$$d^r_{kl} z^r_{kl} \leq t_i \qquad kl \in A^r, li \in A^{rf}, i \in N^f, r \in R \tag{4}$$

$$\sum_{l \in N^r} z^r_{kl} \leq \bar{x}^r_k \qquad \forall k \in N^r, r \in R \tag{5}$$

$$y_i \geq 1 - t_i/T \qquad i \in N^f \tag{6}$$

$$t_i \geq 0 \qquad i \in N^f \tag{7}$$

$$z^r_{kl} \in \{0,1\} \qquad kl \in A^r, r \in R \tag{8}$$

$$y_i \in \{0,1\} \qquad i \in N^f \tag{9}$$

The objective function (1) minimises the burned area. Constraints (2) define the nodes of the fire initial perimeter, that is, the burning nodes when the fire is detected. Constraints (3) define the fire spread: the fire arrival time at each node is the minimum of the potential fire arrival times from each of the incoming arcs. For a detailed discussion of this type of fire spread modelling, we refer the reader to [10]. A resource in range of the origin of an arc delays the fire transmission from that origin node beyond the time horizon T. Constraints (4) state that a resource can only be moved to a node if the resource manages to reach the node before the fire. Constraints (5) state that a resource can only be moved from a position if it was placed there. There can only be one resource of type r per node. Constraints (6) relate the fire arrival time with the node being burned or not at instant T. The remaining constraints, (7), (8) and (9), define the domain of the decision variables.

Figure 1 illustrates a solution to the deterministic problem.

3 The Two-Stage Distributionally Robust Optimisation Model

Here we introduce a two-stage distributionally robust optimization (DRO) model. The prepositioning of available resources is the first-stage decision, taken before the uncertainty is revealed (i.e., the location of ignitions and wind direction). The initial locations of the resources in the deterministic model of the previous Section are no longer fixed but can be chosen. When the wildfire ignition occurs, resources are then relocated to prevent fire propagation; these are the second-stage decisions. The resulting program is known as a recourse problem, in which certain decisions can be made after the uncertainty is disclosed. To model uncertainty, we consider a set of scenarios $\Omega = \{\omega_1, \ldots, \omega_n\}$.

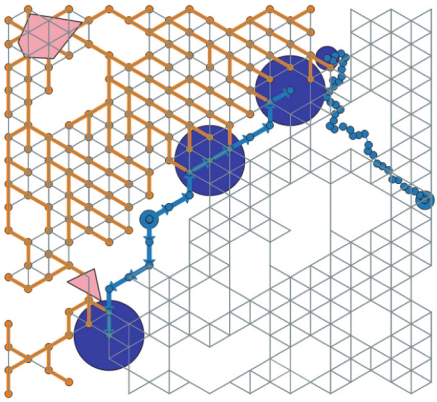

Fig. 1. Illustration of a solution of the deterministic model. Pink polygons are ignition areas. The fire graph is represented by the grey arcs. Orange arcs are arcs in which the fire is effectively transmitted. Blue arcs and nodes correspond to the movement of the resources from their initial positions to their attack positions, in which their attack range is represented by the larger blue circles. All arcs are undirected for clarity. The full resource network is not represented also for clarity. (Color figure online)

The two-stage DRO model can be written as a min-max problem:

$$\min_{x \in X} \left\{ \max_{P \in \mathbb{A}} \sum_{\omega \in \Omega} P_\omega \, Q(x, \omega) \right\},$$

where X is the set of feasible prepositioning locations, $\mathbb{A}$ is the ambiguity set, P_ω is the probability of scenario ω, and $Q(x, \omega)$ represents the recourse cost (i.e., the burned area) given prepositioning decision x and scenario ω.

Next, we discuss in detail the ambiguity set and present the full formulation as a single-stage model.

3.1 The Ambiguity Set

The ambiguity set $\mathbb{A}$ of probability distributions is defined on the finite support Ω. A scenario $\omega \in \Omega$ is represented by the pair $(\alpha_\omega, I_\omega)$, where α_ω is the wind direction and I_ω is the set of ignition locations.

We consider a variant of the Kantorovich ambiguity set which is defined as the set of probability mass distributions that can be obtained from the nominal distribution through a constrained transportation problem.

For a scenario ω represented by $(\alpha_\omega, I_\omega)$ and another scenario ω' represented by $(\alpha_{\omega'}, I_{\omega'})$, let $D^I(\omega, \omega')$ denote the distance between ignitions. This distance is computed using the fire spread model MTT under the average wind conditions between the scenarios. The distance $D(\omega, \omega')$ is defined as:

$$D(\omega, \omega') = a^1 \min\{|\alpha_\omega - \alpha_{\omega'}|, 360 - |\alpha_\omega - \alpha_{\omega'}\}| + a^2 D^I(\omega, \omega'). \quad (10)$$

The coefficients a serve as normalisation values, but may also be assigned user-specific weights. For instance, if we wish to consider only the distance between wind directions, we can set $a^1 = 1$ and $a^2 = 0$.

In this work, we rely only on the maximum value of each distance for normalisation:

$$a^1 = \frac{1}{\max_{\omega,\omega' \in \Omega} \min\{|\alpha_\omega - \alpha_{\omega'}|, 360 - |\alpha_\omega - \alpha_{\omega'}|\}},$$
$$a^2 = \frac{1}{\max_{\omega,\omega' \in \Omega} D^I(\omega, \omega')}.$$

The distance between two probability distributions P and Q is given by:

$$W(P,Q) = \min_{k \in J(P,Q)} \sum_{\omega,\omega' \in \Omega} D(\omega,\omega')\, k_{\omega\omega'},$$

where Q denotes the nominal probability distribution, and $J(P,Q)$ is the set of all couplings (joint probability distributions) with marginals P and Q.

We define our ambiguity set $\mathbb{A}_\epsilon$ such that $W(P,Q) \leq \epsilon$. The ambiguity set is the feasible set of a transportation problem with variables P_ω (probability mass) and $k_{\omega\omega'}$ (transported mass between scenarios). This transportation problem is defined as:

$$\mathbb{A}_\epsilon = \Big\{ \sum_{\omega \in \Omega} \sum_{\omega' \in \Omega} D(\omega,\omega') k_{\omega\omega'} \leq \epsilon, \tag{11}$$
$$\sum_{\omega' \in \Omega} k_{\omega\omega'} = P_\omega, \qquad \omega \in \Omega, \tag{12}$$
$$\sum_{\omega \in \Omega} k_{\omega\omega'} = Q_{\omega'}, \qquad \omega' \in \Omega, \tag{13}$$
$$\sum_{\omega \in \Omega} P_\omega = 1, \tag{14}$$
$$P_\omega \geq 0, \qquad \omega \in \Omega, \tag{15}$$
$$k_{\omega\omega'} \geq 0, \qquad \omega, \omega' \in \Omega \Big\}. \tag{16}$$

Parameter ϵ controls the maximum total transportation amount and, consequently, the size of the ambiguity set. When $\epsilon = 0$, the ambiguity set reduces to the singleton $\{Q\}$ (no transportation), yielding the stochastic program, where a set of scenarios and the probability of their occurrence is known. For sufficiently large ϵ, all probability mass can be transported to a single scenario, resulting in the robust problem (worst-case approach).

The distribution separation problem, a variant of the well-known transportation problem, is:

$$\max_{P \in \mathbb{A}} \sum_{\omega \in \Omega} P_\omega\, Q(x,\omega) \tag{17}$$
$$\text{s.t. } (11) - (16),$$

where x is fixed from the first-stage problem.

3.2 DRO Model

The mixed-integer programming (MIP) model that addresses the robust prepositioning problem includes static (first-stage) decision variables defining the prepositions. The parameters $\bar{x}_k^r$ of the deterministic model of the previous Section become decision variables:

$$x_k^r = \begin{cases} 1, & \text{if a resource of type } r \text{ is prepositioned at node } k, \\ 0, & \text{otherwise,} \end{cases} \quad k \in N^r,\ r \in R.$$

To model uncertainty, we consider a set of scenarios Ω. Each scenario $\omega \in \Omega$ represents a set of ignition locations, wind speed, and direction. For each scenario, we define the following adjustable (second-stage) decision variables:

t_i^ω : fire arrival time at node i in scenario ω, $\quad i \in N^f$;

$$z_{kl}^{\omega r} = \begin{cases} 1, \text{ if a resource of type } r \text{ prepositioned at node } k \\ \quad \text{is moved to node } l \text{ in scenario } \omega, \\ 0, \text{otherwise,} \end{cases} \quad kl \in A^r,\ r \in R;$$

$$y_i^\omega = \begin{cases} 1, & \text{if node } i \text{ burns before time } T \text{ in scenario } \omega, \\ 0, & \text{otherwise,} \end{cases} \quad i \in N^f.$$

To formulate a complete MILP, we use the well-known epigraph reformulation to remove the inner maximization problem. We introduce an auxiliary variable θ representing the worst-case expected burned area:

$$\theta = \max_{P \in \mathbb{A}} \sum_{\omega \in \Omega} P_\omega \left(\sum_{i \in N^f} y_i^\omega \right). \tag{18}$$

The full model is:

$$\min \theta \tag{19}$$

s.t.

$$\theta \geq \sum_{\omega \in \Omega} P_\omega \left(\sum_{i \in N^f} y_i^\omega \right), \qquad P \in \mathbb{A}, \tag{20}$$

$$\sum_{k \in N^r} x_k^r \leq b^r, \qquad r \in R, \tag{21}$$

$$t_i^\omega = 0, \qquad i \in I^f, \omega \in \Omega, \tag{22}$$

$$\sum_{k \in N^r} z_{kl}^{\omega r} \leq 1, \qquad r \in R, \omega \in \Omega, \tag{23}$$

$$t_j^\omega \leq t_i^\omega + c_{ij}^\omega + T \sum_{kl \in A^r : l = i} z_{kl}^{\omega r}, \qquad ij \in A^f, r \in R, \omega \in \Omega, \tag{24}$$

$$d_{kl}^r z_{kl}^{\omega r} \leq t_l^\omega, \qquad kl \in A^r, li \in A^{rf}, r \in R, \omega \in \Omega, \tag{25}$$

$$\sum_{l \in N^r} z_{kl}^{\omega r} = x_k^r, \qquad k \in N^r, r \in R, \omega \in \Omega, \tag{26}$$

$$y_i^\omega \geq 1 - t_i^\omega / T, \qquad i \in N^f, \omega \in \Omega, \tag{27}$$

$$t_i^\omega \geq 0, \qquad i \in N^f, \omega \in \Omega, \tag{28}$$

$$x_k^r \in \{0, 1\}, \qquad k \in N^r, r \in R, \tag{29}$$

$$z_{kl}^{\omega r} \in \{0, 1\}, \qquad kl \in A^r, r \in R, \omega \in \Omega, \tag{30}$$

$$y_i^\omega \in \{0, 1\}, \qquad i \in N^f, \omega \in \Omega. \tag{31}$$

Constraints (20) assure that all the distribution of probabilities of the ambiguity set are taken into account. These constraints, together with the objective function (19), assure the burned area in the worst case distribution of probabilities is minimized.

Note that (20) represents an infinite family of constraints. To solve the model, we use an exact decomposition algorithm that considers only a subset of $\mathbb{A}$. Initially, this subset, denoted by A is set to the singleton $A = \{Q\}$. Then, in each iteration, for a given first-stage solution, we solve the transportation problem to check for violated inequalities; when found, the corresponding probability distribution is added to A.

Constraints (21) limit the number of resource units of each type. Constraints (22) define the nodes in the initial fire perimeter (burning nodes at detection time). Constraints (26) ensure that a resource can only move from a node (second-stage) if it is initially there (first stage).

The following constraints follow the deterministic model directly, but are repeated for each scenario: Constraints (23) limit the number of resources of type r in a node to one, in each scenario. Constraints (24) model fire spread in each scenario. Constraints (25) ensure that a resource can only move to a node if it arrives before the fire in each scenario. Constraints (27) link fire arrival time

with whether a node burns before time T. The remaining constraints define variable domains.

4 Decomposition Algorithm

In [25], several solution approaches were compared to solve DRO problems with finite supports. The study concludes that a solution approach should be selected according to the ambiguity set characteristics. These solution approaches comprise the dualization approach, which is the simplest to apply but may not be suitable to instances that are simultaneously hard and have large number of scenarios; the row-and-column generation generalizes the same approach for Robust Optimization, and it is suitable exactly when the ambiguity set approximates the RO case, and the L-shape is the best approach for the harder instances with many scenarios. As in fire suppression, preventing the worst-case scenarios is critical, we use the row-and-column approach. This algorithm is an exact decomposition method to solve the two-stage DRO model. It iteratively alternates between solving a master problem (where only few scenarios are considered) and a separation problem to identify the worst-case probability distribution. The corresponding cut (20) and the scenarios with positive probability in that cut are added to the master problem. The algorithm is provided in Algorithm 1.

Algorithm 1 Dynamic decomposition algorithm for the DRO model

Initialize $i \leftarrow 0$, $UB \leftarrow \infty$, $LB \leftarrow -\infty$, $Gap \leftarrow \infty$, and $\mathcal{A} \leftarrow \{Q\}$;
Build the initial master problem MR_0 (including only first-stage variables and constraints);
Solve MR_0; obtain x' and LB;
while $Gap > tol$ **and** $i \leq maxiter$ **do**
  Solve the transportation problem (17) using x' and $R(x', \omega)$;
  Let P_ω be the resulting probabilities;
  if $UB > \sum_{\omega \in \Omega} P_\omega R(z', \omega)$ **then**
    $UB \leftarrow \sum_{\omega \in \Omega} P_\omega R(z', \omega)$;
  end if
  $i \leftarrow i + 1$;
  Extend MR_{i-1} to MR_i by adding:

- the inequality $\theta \geq \sum_{\omega \in \Omega} P_\omega R(x', \omega)$,
- for each ω with $P_\omega > 0$, the corresponding second-stage variables $y_i^\omega, z_{kl}^{\omega r}, t_i^\omega$ and constraints (22)–(31).

  Solve MR_i; update x' and LB;
  Compute $Gap = 100 \times (UB - LB)/UB$;
end while

5 Computational Experiments

The computational tests were performed on an 8th Gen Intel i7 CPU running at 1.8 GHz. The optimization models were implemented in Python using the Gurobi solver (version 10.0.2) and pyO3F-a Python framework developed within the project "An optimization framework for reducing forest fire" [27,28]. This framework enables the definition of fire and resource networks from an actual landscape, given a specified resolution and fire spread scenarios that depend on fuels, slopes, and wind conditions.

The experiments were conducted on a landscape surrounding Baião, a municipality in northern Portugal. The study area spans approximately 20 km by 18 km (around 360 km^2). A resolution of 1000 m was used, corresponding to the distance between adjacent nodes in the hexagonal grid representing the landscape.

For all tests, the initial fire perimeters were obtained by simulating fire spread during 30 min, and the time horizon was set to 120 min.

Only one type of resource was considered, with speed varying according to terrain: from 5 km/h on paths unsuitable for vehicles to from 60 km/h on primary and secondary roads (using the Open Street map categorizations). Each resource unit had a suppression radius of 1000 m, meaning it suppresses fire on any fire node within this distance.

For each instance, a preprocessing phase was conducted reducing the number of nodes in the fire and resource networks. In the fire network we remove every node whose quickest path from the ignition is greater than T. For the resource network, we created a grid and kept only the nodes closest to the intersections.

5.1 Instances and Scenarios Generation

The classification of instances is based on the assumption that relevant information, such as weather forecasts and estimated ignition counts, is available on the day prior to operations. This enables a preliminary assessment of the risk level for the upcoming day. Instances are defined according to these two key parameters, wind intensity and the expected number of ignitions, and on the number of available resources.

Six instance types are identified as follows:

1. **High Risk - Very Dangerous**
   - Wind speed: 60 km/h (very strong)
   - Number of ignitions: 4 (high)
   - Instance set: `60_4`
2. **Moderate-to-High Risk - Dangerous**
   - Wind speed: 60 km/h (very strong)
   - Number of ignitions: 1 or 2 (low)
   - Instance sets: `60_1` and `60_2`
3. **Moderate-to-High Risk - Dangerous**
   - Wind speed: 30 km/h (strong)
   - Number of ignitions: 4 (high)

   - Instance set: `30_4`
4. **Moderate Risk - Less Dangerous**
   - Wind speed: 30 km/h (strong)
   - Number of ignitions: 1 or 2 (low)
   - Instance sets: `30_1` and `30_2`

For each set of instances we generate 80 scenarios, that differ on the wind direction and on the ignition location(s):

- For instances with one ignition (labelled `60_1` and `30_1`): four wind directions and 20 combinations of two ignition locations.
- For instances with two ignitions (labelled `60_2` and `30_2`): four wind directions and 20 combinations of two ignition locations.
- For instances with four ignitions (labelled `60_4` and `30_4`): four wind directions and 20 combinations of four ignition locations.

5.2 Computational Results

Table 1 and Table 2 summarise the computational results obtained with Algorithm 1 for all instance sets under wind speeds of 30 km/h and 60 km/h, respectively, and different values of ϵ. Each instance corresponds to a combination of wind direction, number of ignitions and number of available resources, with increasing levels of risk and uncertainty captured by ϵ. The tables present the same information. Column "Inst" gives the name of the instance. The instance is identified by the wind speed, number of ignitions and number of resources. Hence, the case where no resources are employed amounts to a fire spread simulation. Column "ϵ" specifies the parameter used to control the size of the ambiguity set. Columns "LB" and "UB" give the lower and upper bound obtained by Algorithm 1, respectively, while column "Gap" computes the corresponding gap $Gap = 100 \times (UB - LB)/UB$. Column "$\#S$" gives the number of scenarios added. The three columns below Time give the running time, in sec., spent with the master problem (column "M"), subproblem (column "S") and the transportation problem (column "T"). Finally, the number of iterations is given in column "Iter".

Overall, the model achieved zero optimality gaps across nearly all experiments, indicating that the solution approach consistently identified optimal or near-optimal solutions within acceptable computational limits. This is particularly relevant given the complexity of the problem.

For the instances with wind speed 30 km/h (Table 1), the runtimes (sum of columns M, S, and T) ranged from a few seconds up to a total of more than 4000 sec. in the most demanding case with 8 iterations. The hardest instances are those with one ignition and one resource. Increasing the number of resources makes such instances easier. As ϵ increases, fewer scenarios are activated (see $\#S$ column) since, as explained above, the probability mass concentrates in fewer scenarios. In all cases the number of iteration is very small.

Table 1. Results of the dynamic DRO model of the instances with wind speed 30 km/h.

						Runtime			
Inst	ϵ	LB	UB	Gap	$\#S$	M	S	T	Iter
30_1_1	0	5.585	5.585	0.0	31	682	101	17	4
30_1_1	0.1	6.678	6.678	0.0	18	541	111	19	4
30_1_1	0.2	7.0	7.0	0.0	15	3811	244	37	8
30_1_1	0.3	7.0	7.0	0.0	13	1859	190	32	7
30_1_1	0.4	7.0	7.0	0.0	12	1740	186	31	7
30_1_1	0.5	7.0	7.0	0.0	12	1157	139	24	6
30_1_3	0	5.368	5.368	0.0	21	13	27	5	1
30_1_3	0.1	6.638	6.638	0.0	7	5	37	5	1
30_1_3	0.2	7.0	7.0	0.0	3	1	26	4	1
30_1_3	0.3	7.0	7.0	0.0	3	2	30	4	1
30_1_3	0.4	7.0	7.0	0.0	3	2	33	5	1
30_1_3	0.5	7.0	7.0	0.0	3	1	24	4	1
30_1_10	0	5.368	5.368	0.0	21	11	26	4	1
30_1_10	0.1	6.638	6.638	0.0	7	4	34	5	1
30_1_10	0.2	7.0	7.0	0.0	3	2	29	5	1
30_1_10	0.3	7.0	7.0	0.0	3	2	28	5	1
30_1_10	0.4	7.0	7.0	0.0	3	1	26	4	1
30_1_10	0.5	7.0	7.0	0.0	3	2	32	5	1
30_2_1	0	12.187	12.187	0.03	24	713	118	16	3
30_2_1	0.1	15.2	15.2	0.0	5	124	121	17	3
30_2_1	0.2	16.0	16.0	0.0	2	15	75	12	2
30_2_1	0.3	16.0	16.0	0.0	2	16	89	12	2
30_2_1	0.4	16.0	16.0	0.0	2	12	60	10	2
30_2_1	0.5	16.0	16.0	0.0	2	14	67	11	2
30_2_3	0	10.553	10.553	0.0	22	15	35	5	1
30_2_3	0.1	12.408	12.408	0.0	10	8	43	6	1
30_2_3	0.2	13.0	13.0	0.0	3	3	41	6	1
30_2_3	0.3	13.0	13.0	0.0	3	3	44	6	1
30_2_3	0.4	13.0	13.0	0.0	3	2	34	5	1
30_2_3	0.5	13.0	13.0	0.0	3	2	36	5	1
30_2_10	0	10.553	10.553	0.0	22	19	38	5	1
30_2_10	0.1	12.408	12.408	0.0	10	10	50	7	1
30_2_10	0.2	13.0	13.0	0.0	3	3	53	6	1
30_2_10	0.3	13.0	13.0	0.0	3	2	42	6	1
30_2_10	0.4	13.0	13.0	0.0	3	2	36	5	1
30_2_10	0.5	13.0	13.0	0.0	3	2	30	5	1
30_4_1	0	25.396	25.396	0.0	22	486	84	26	2
30_4_1	0.1	29.915	29.915	0.0	11	95	81	25	2
30_4_1	0.2	31.637	31.637	0.0	6	56	84	25	2
30_4_1	0.3	32.0	32.0	0.0	2	20	95	28	2
30_4_1	0.4	32.0	32.0	0.0	2	17	95	26	2
30_4_1	0.5	32.0	32.0	0.0	2	23	110	29	2
30_4_3	0	20.222	20.23	0.039	24	71	87	25	2
30_4_3	0.1	23.484	23.484	0.0	11	27	93	26	2
30_4_3	0.2	25.0	25.0	0.0	3	9	98	27	2
30_4_3	0.3	25.0	25.0	0.0	3	5	80	24	2
30_4_3	0.4	25.0	25.0	0.0	3	6	110	29	2
30_4_3	0.5	25.0	25.0	0.0	3	8	121	33	2
30_4_10	0	19.976	19.976	0.0	22	20	47	13	1
30_4_10	0.1	23.285	23.285	0.0	10	9	43	12	1
30_4_10	0.2	24.927	24.927	0.0	3	4	44	13	1
30_4_10	0.3	25.0	25.0	0.0	2	2	48	13	1
30_4_10	0.4	25.0	25.0	0.0	2	3	50	14	1
30_4_10	0.5	25.0	25.0	0.0	2	3	52	14	1

Table 2. Results of the dynamic DRO model of the instances with wind speed 60 km/h.

						Runtime			
Inst	ϵ	LB	UB	Gap	$\#S$	M	S	T	Iter
60_1_1	0	4.379	4.233	0.0	24	19	23	4	1
60_1_1	0.1	5.0	5.0	0.0	8	4	23	4	1
60_1_1	0.2	5.0	5.0	0.0	8	4	24	4	1
60_1_1	0.3	5.0	5.0	0.0	8	4	22	4	1
60_1_1	0.4	5.0	5.0	0.0	8	10	52	5	1
60_1_1	0.5	5.0	5.0	0.0	8	4	23	4	1
60_1_3	0	4.241	4.233	0.202	24	12	24	4	1
60_1_3	0.1	5.0	5.0	0.0	8	3	22	4	1
60_1_3	0.2	5.0	5.0	0.0	8	3	23	4	1
60_1_3	0.3	5.0	5.0	0.0	8	3	39	4	1
60_1_3	0.4	5.0	5.0	0.0	8	7	34	7	1
60_1_3	0.5	5.0	5.0	0.0	8	3	22	4	1
60_1_10	0	4.233	4.233	0.0	24	9	23	4	1
60_1_10	0.1	5.0	5.0	0.0	8	3	23	4	1
60_1_10	0.2	5.0	5.0	0.0	8	3	22	4	1
60_1_10	0.3	5.0	5.0	0.0	8	5	29	5	1
60_1_10	0.4	5.0	5.0	0.0	8	3	24	4	1
60_1_10	0.5	5.0	5.0	0.0	8	3	22	4	1
60_2_1	0	9.431	9.431	0.0	26	240	58	9	2
60_2_1	0.1	11.0	11.0	0.0	6	41	74	11	2
60_2_1	0.2	11.0	11.0	0.0	5	38	96	15	3
60_2_1	0.3	11.0	11.0	0.0	5	44	97	14	3
60_2_1	0.4	11.0	11.0	0.0	5	32	78	13	3
60_2_1	0.5	11.0	11.0	0.0	5	31	73	13	3
60_2_3	0	8.387	8.387	0.0	22	13	28	4	1
60_2_3	0.1	9.744	9.744	0.0	8	5	36	5	1
60_2_3	0.2	10.0	10.0	0.0	3	1	34	5	1
60_2_3	0.3	10.0	10.0	0.0	3	3	47	7	1
60_2_3	0.4	10.0	10.0	0.0	3	1	27	4	1
60_2_3	0.5	10.0	10.0	0.0	3	1	31	4	1
60_2_10	0	8.387	8.387	0.0	22	15	35	5	1
60_2_10	0.1	9.744	9.744	0.0	8	6	41	5	1
60_2_10	0.2	10.0	10.0	0.0	3	1	33	5	1
60_2_10	0.3	10.0	10.0	0.0	3	1	30	4	1
60_2_10	0.4	10.0	10.0	0.0	3	1	27	4	1
60_2_10	0.5	10.0	10.0	0.0	3	1	27	4	1
60_4_1	0	18.935	18.935	0.0	23	83	32	12	1
60_4_1	0.1	21.714	21.714	0.0	8	19	37	12	1
60_4_1	0.2	22.0	22.0	0.0	5	8	31	12	1
60_4_1	0.3	22.0	22.0	0.0	5	11	35	12	1
60_4_1	0.4	22.0	22.0	0.0	5	7	30	12	1
60_4_1	0.5	22.0	22.0	0.0	5	8	32	12	1
60_4_3	0	16.086	16.094	0.054	23	74	67	24	2
60_4_3	0.1	18.767	18.767	0	9	36	133	47	4
60_4_3	0.2	19.0	19.0	0.0	5	8	69	24	2
60_4_3	0.3	19.0	19.0	0.0	5	12	82	26	2
60_4_3	0.4	19.0	19.0	0.0	5	10	74	24	2
60_4_3	0.5	19.0	19.0	0.0	5	11	84	26	2
60_4_10	0	15.504	15.504	0.0	22	14	34	12	1
60_4_10	0.1	18.498	18.498	0.0	7	5	34	12	1
60_4_10	0.2	19.0	19.0	0.0	2	4	38	22	1
60_4_10	0.3	19.0	19.0	0.0	2	1	42	13	1
60_4_10	0.4	19.0	19.0	0.0	2	1	34	12	1
60_4_10	0.5	19.0	19.0	0.0	2	2	36	12	1

For the instances with wind speed 60 km/h (Table 2), the computational times are generally smaller and more stable than in the 30 km/h case. Most instances were solved within a few seconds, with only a few reaching higher runtimes, particularly those involving four ignitions and three resources, where up to four iterations were required. As with the previous case, increasing the number of resources tends to make the instances easier to solve, and the number of activated scenarios (#S) decreases as ϵ grows, confirming the expected concentration of probability mass in fewer representative scenarios. Optimality gaps remain equal to zero in all cases, showing that the solution method maintains its robustness and efficiency under higher wind speed conditions.

Overall, the dynamic DRO model demonstrated strong computational performance and stability across all tested configurations, requiring only a few iterations to converge. The proposed approach efficiently handles different levels of uncertainty, while reaching optimality within reasonable computational effort.

From the operational point of view, we can observe that in most instances increasing the number of resources has no meaningful impact. For example, in instances 30_1 employing more than one resource has no impact for the robust case, i.e., for $\epsilon = 0$. While in the worst-case of four ignition, case 30_4, there is a reduction on the burned area when 3 resources are employed instead of 1, but there is no further improvement by increasing the number of resources.

6 Conclusions

This work proposed a two-stage Distributionally Robust Optimisation model for the prepositioning of firefighting resources under uncertain ignition locations and wind conditions. The approach incorporates an ambiguity set based on the Wasserstein metric, allowing the model to hedge against uncertain weather conditions and location of ignitions. To efficiently solve the resulting problem, we developed an exact decomposition algorithm that generates second-stage variables and constraints only for scenarios with positive probability, significantly improving the scalability of the instances solved.

The computational experiments demonstrated that the proposed algorithm is computationally efficient by solving the tested instances to optimality or near optimality, always using a small number of iterations, confirm the suitability of the proposed approach for addressing complex wildfire resource allocation problems under uncertainty.

From an operational perspective, the DRO approach provides stable and interpretable solutions that remain effective under a wide range of uncertainty realisations. Moreover, it provides insights not only on the place to allocate the resources but also on the impact on the number of resources on the fire suppression.

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References

1. Martell, D.L.: A review of recent forest and wildland fire management decision support systems research. Curr. Forestry Rep. **1**, 128–137 (2015)
2. Martell, D.L.: A review of operational research studies in forest fire management. Can. J. For. Res. **12**(2), 119–140 (1982)
3. Mendes, A.B., Alvelos, F.P.: Iterated local search for the placement of wildland fire suppression resources. Eur. J. Oper. Res. **304**(3), 887–900 (2023)
4. Granda, B., León, J., Vitoriano, B., Hearne, J.: Decision support models and methodologies for fire suppression. Fire **6**(2) (2023)
5. Hof, J., Omi, P.N., Bevers, M., Laven, R.D.: A timing-oriented approach to spatial allocation of fire management effort. Forest Sci. **46**(3), 442–451 (2000)
6. Finney, M.A.: Fire growth using minimum travel time methods. Can. J. For. Res. **32**(8), 1420–1424 (2002)
7. Finney, M.A., et al.: An overview of FlamMap fire modeling capabilities. In: Fuels Management—How to Measure Success: Conference Proceedings, USDA Forest Service (2006)
8. Wei, Y., Rideout, D.B., Hall, T.B.: Toward efficient management of large fires: a mixed integer programming model and two iterative approaches. Forest Sci. **57**(5), 435–447 (2011)
9. Belval, E.J., Wei, Y., Bevers, M.: A mixed integer program to model spatial wildfire behavior and suppression placement decisions. Can. J. For. Res. **45**(4), 384–393 (2015)
10. Alvelos, F.: Mixed integer programming models for fire fighting. In: Gervasi, O., et al. (eds.) ICCSA 2018. LNCS, vol. 10961, pp. 637–652. Springer, Cham (2018). https://doi.org/10.1007/978-3-319-95165-2_45
11. Harris, M.G., Forbes, M.A., Taimre, T.: Logic-based Benders decomposition for wildfire suppression. Comput. Oper. Res., 106392 (2023)
12. Alvelos, F., Marto, M., Mendes, A.: Dispatching, positioning and routing resources for wildfire initial attack. Networks **86**(1), 80–102 (2025)
13. Hu, X., Ntaimo, L.: Integrated simulation and optimization for wildfire containment. ACM Trans. Model. Comput. Simul. **19**(4), 1–29 (2009)
14. Ntaimo, L., Gallego Arrubla, J.A., Stripling, C., Young, J., Spencer, T.: A stochastic programming standard response model for wildfire initial attack planning. Can. J. Forest Res. **42**(6), 987–1001 (2012)

15. Lee, Y., Fried, J.S., Albers, H.J., Haight, R.G.: Deploying initial attack resources for wildfire suppression: spatial coordination, budget constraints, and capacity constraints. Can. J. For. Res. **43**(1), 56–65 (2013)
16. Belval, E.J., Wei, Y., Bevers, M.: A stochastic mixed integer program to model spatial wildfire behavior and suppression placement decisions with uncertain weather. Can. J. For. Res. **46**(2), 234–248 (2016)
17. Belval, E.J., Wei, Y., Bevers, M.: Modeling ground firefighting resource activities to manage risk given uncertain weather. Forests **10**(12), 1077 (2019)
18. Mendes, A.B., Alvelos, F.P.: A robust optimisation approach for the placement of forest fire suppression resources. Int. Trans. Oper. Res. **32**(3), 1312–1342 (2025)
19. Marques, F., Agra, A., Alvelos, H., Xambre, A.R., Alvelos, F.: A robust approach for the prepositioning of resources for wildfire suppression. In: Alvelos, F., Martins, I., Rocha, A.M.A.C. (eds) Advances in Optimization and Wildfire. Springer, Cham (2026). https://doi.org/10.1007/978-3-032-03108-2_10
20. Bansal, M., Huang, K.-L., Mehrotra, S.: Decomposition algorithms for two-stage distributionally robust mixed binary programs. SIAM J. Optim. **28**(3), 2360–2383 (2018)
21. Lin, F., Fang, X., Gao, Z.: Distributionally robust optimization: a review on theory and applications. Numer. Algebra, Control Optim. **12**(1), 159–212 (2022)
22. Rahimian, H., Mehrotra, S.: Frameworks and results in distributionally robust optimization. Open J. Math. Optim. **3** (2022)
23. Agra, A., Cerveira, A.: Wind farm layout optimization under uncertainty. TOP **32**, 202–223 (2024)
24. Alvelos, H., Xambre, A.R., Marques, F., Agra, A., Alvelos, F.: Modelling wind behaviour for the development of scenarios in the context of wildfire spread. In F. Alvelos, I. Martins, and A. Rocha (eds.), Advances in Optimization and Wildfire: Proceedings of the First Optimization and Wildfire Conference, Springer, Cham (2026). https://doi.org/10.1007/978-3-032-03108-2_7
25. Agra, A.: Two-stage distributionally robust optimization with a finite support. Comput. Oper. Res. **183**, 107142 (2025)
26. Rothermel, R.C.: A mathematical model for predicting fire spread in wildland fuels. Intermountain Forest and Range Experiment Station, Forest Service, USA (1972)
27. Alvelos, F.: pyO3F – a python framework for wildfire-related optimization, Part I: Design and fundamentals. In: Alvelos, F., Martins, I., Rocha, A. (eds.), Advances in Optimization and Wildfire: Proceedings of the First Optimization and Wildfire Conference. Springer, Cham (2026). https://doi.org/10.1007/978-3-032-03108-2_3
28. Alvelos, F.: pyO3F – a python framework for wildfire-related optimization, Part II: usage. In: Alvelos, F., Martins, I., Rocha, A. (eds.), Advances in Optimization and Wildfire: Proceedings of the First Optimization and Wildfire Conference, Springer, Cham (2026). https://doi.org/10.1007/978-3-032-03108-2_4