Defining Modeling Priorities for the Seismic Assessment of Minaret Structures: A Geometric Sensitivity Analysis

Sare Nur Avcı[1(✉)], Hany M. Hassan[2], Mariam A. Sallam[3], Gabriele Milani[1], and Marco V. Valente[1]

[1] Politecnico di Milano, 20133 Milan, Italy
sarenur.avci@polimi.it
[2] OGS, Borgo Grotta Gigante 42/C, 34010 Sgonico, Italy
[3] NRIAG, 1 Marsad Street, Helwan, Cairo, Egypt

Abstract. Historical masonry minarets present great diversity in terms of construction materials, geometric details, and dimensions, some of which are demonstrated in post-earthquake observations to affect their seismic performance. Yet numerical seismic analyses often oversimplify their complex geometric features due to data scarcity or computational constraints. This study introduces a systematic geometric sensitivity analysis to quantify the impact of these features on the seismic response and to establish a hierarchy of modeling priorities validated by different analysis methods. Marzouki (Egypt) and Hagia Sophia (Turkey) minarets are selected as case studies for their distinct geometric characteristics and contrasting seismic settings which enable a comparative evaluation. Starting from the original geometries, several modeling scenarios are implemented to isolate the influence of the vertical profile (straight vs. truncated), wall thickness decreases along the height, and the presence of stairs and balconies. To evaluate the influence of these variations on the global capacity and failure modes of the studied minarets, two analytical approaches are followed: Finite Element models subjected to modal and nonlinear static analyses, and Kinematic Limit Analysis. Correlating the modal characteristics, pushover curves, plastic hinge locations, and collapse mechanisms in KLA, the critical structural details and their impact on the seismic vulnerability assessment are identified, allowing to balance numerical accuracy with practical efficiency for heritage conservation.

Keywords: Masonry Minaret · Seismic Risk Assessment · Geometric Sensitivity Analysis · Nonlinear Static Analysis · Kinematic Limit Analysis · Finite Element Modeling · Conservation of Historical Minaret

1 Introduction

Masonry minarets stand as culturally diverse structures within Islamic heritage, representing centuries of architectural and structural development. The architectural diversity through the dimensions and proportions, segment configurations, balcony designs,

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and crowning elements represent a cumulative knowledge of centuries across diverse geographical and cultural contexts. Their value is further shaped by their coping with the long-term effects of the environmental, anthropogenic, and structural challenges. Masonry minarets face severe threats from earthquakes even of those modest ground motion levels. The February 2023 Kahramanmaraş earthquake sequence in Turkey provided extensive documentation, with numerous minarets suffering partial or complete collapse [1–3]. In Cairo, Egypt, the earthquakes in 1847 and 1992 have progressively eroded the city's extensive collection of historical minarets [4–6]. Despite their structural vulnerability and the cultural and religious value these structures represent, specific seismic design and preservation guidelines for minarets are currently absent from many national codes. Establishing rigorous earthquake protection measures should be an urgent priority for the scientific community.

Cairo and Istanbul (Fig. 1) are both renowned as "cities of a thousand minarets," a designation that reflects not merely the numerical abundance of these structures but their profound integration into the urban landscape and cultural identity [5, 7]. The minarets in Cairo and Istanbul display remarkable diversity in form, material, and construction technique [8]. With their different seismic risks, the cities provide a valuable context for comparatively studying the distinct seismic behavior of minarets.

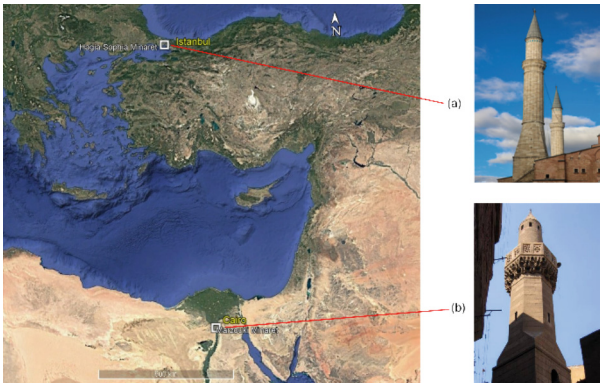

Fig. 1. Location of the two selected minarets. To the right are recent photos of the case study minarets; (a) the Hagia Sophia minaret (Turkey), and (b) the Marzouki minaret (Cairo, Egypt).

The literature on the seismic performance and strengthening of minarets is expanding with most studies focusing on singular case studies by utilizing experimental tests, modal analysis, and numerical simulations. An emerging area of research is the development of qualitative damage parameters, damage assessment forms [9–11], and vulnerability indexes [5, 12] for mosques and minarets. The most common numerical approach adopted for masonry minarets is the macro-modelling in Finite Element method. The Finite Element models of minarets generally include overall slenderness, external dimensions, and material properties. It is found that an adequate representation of the geometry, and the accurate calibration of the material properties of the structure are critical for the reliability of its numerical model [13–15]. Finite Element models frequently omit or simplify internal staircases by modeling the body as a hollow tube. Wall thickness is

sometimes kept constant for simplicity despite actual taper, and balconies are often omitted or modeled only as lumped masses, ignoring their local effects. The impact of different geometric features in the seismic analysis of minarets was investigated in several parametric studies [15–17]. It is also common for diverse masonry structural typologies that complex geometries represent significant structural implications which challenge the validity of simplification strategies [18, 19]. Alternative methods such as the Discrete Element and Applied Element method, and Kinematic Limit Analysis - KLA- present balance between computational complexity and reliability. Limit Analysis is a simple methodology first adopted for masonry structures in the 1960s, with a lower bound approach proposed by Heyman [20]. It develops on three theoretical assumptions: (i) masonry is unable to withstand tensile stress, (ii) it has infinite compressive strength, (iii) sliding between blocks is not allowed. In the upper bound (kinematic) approach, collapse mechanisms are defined in which rigid macro blocks rotate at cylindrical hinges [21]. By the virtual work theorem, a collapse multiplier is found for each mechanism. The minimum collapse multiplier identifies the most likely failure mechanism, which can be compared against the seismic demand for a given zone. The critical step is the correct collapse mechanisms definition which are deduced from the typically observed failure modes in the post-earthquake surveys. Although KLA requires fewer geometric and material parameters than sophisticated numerical approaches, simplifying complex geometries remains a challenge that can be overcome by combination with FE or DE [22–24]. Still, by its simplicity, it is an accessible and reliable tool suitable for rapid preliminary screening of a large inventory of structures under seismic risk. KLA is successfully employed for different slender structure typologies such as towers [25–27], chimneys [28], and pagodas [22]. For minarets, it is adopted in some studies [29, 30] testing certain mechanisms, yet a systematic approach applicable to different typologies and standardized predefined collapse mechanisms are still lacking.

Understanding the seismic behavior of minarets requires careful consideration of their complex geometric features that significantly influence damage patterns as demonstrated by post-earthquake observations [6, 11]. Yet numerical seismic analyses can often oversimplify these characteristics due to data scarcity or computational constraints. This study addresses this gap by introducing a systematic geometric sensitivity analysis in which the individual impacts of these structural characteristics are assessed through controlled modeling scenarios by progressively adding complexity to base models. This is performed through the comparative analysis of the Marzouki minaret in Egypt and Hagia Sophia minaret in Turkey. The investigated features are the vertical profile (straight and truncated), wall thickness changes, presence of internal staircases and the balconies. As a novelty, these impacts are evaluated using diverse different analysis procedures of modal analysis, nonlinear static pushover analysis, and the Kinematic Limit analysis – a method that remains underutilized in the seismic assessment of minarets. The individual influences are quantified in terms of capacity, stiffness, and failure modes. This way, the study establishes a quantitative, data-driven hierarchy of which structural details are most critical for accurate seismic assessment, providing guidance to practitioners in making informed decisions for modeling choices.

2 Historical Minarets and Their Seismic Vulnerability: Insights from Egypt and Turkey

The earliest minarets in Turkey are from the Seljuk period (11th century). They usually have monolithic and tapered bodies designed to withstand lateral wind and seismic loads. In the 16-17th centuries, the classical Ottoman style emerged with a specific scheme composed of the boot, transition segment, body, balcony, upper body, and spire, with a changing number of balconies and bodies. The mina-rets can be isolated from or attached to/built on the mosque structures. The Seljuk minarets consist of brickwork and alternating stone and brick; while starting from the Ottoman period, stone masonry (Kufeki limestone) was more widely adopted [29, 30].

In Egypt, minaret evolution is generally categorized into five periods: the Tulunid (827–904), Fatimid (969–1171), Ayyubid (1171–1250), Mamluk (1250–1517), and Ottoman Turk (1517–1848) period. From the Fatimid period onward, a core morphology consisting of the boot, body, and mabkhara (a top segment in the incense burner shape) segments is observed. In the Ayyubid period, thinner cross sections appeared. The Mamluk period minarets are distinguished by geometric complexity, great shaft variations, and the usage of both brick and stone. During this period, a new top segment called jawsaq was developed as a pavilion on eight columns carrying a bulb. During the Ottoman rule, the Classical Ottoman style was emulated in Egypt, and minarets had more compact and slender profiles. Egyptian minarets are mostly built in limestone, with older examples in brick [5, 8].

The inventory of minarets, especially historical masonry examples, is continuously declining over time, which reflects their vulnerability and the cumulative impact of seismic, environmental, and conservation challenges [4, 31]. Taking Cairo as an example, only one Ayyubid minaret style has survived to the date [5]. The vulnerability of minarets against earthquakes depends on several aspects [32]. The brittle nature of masonry material, and its low tensile strength, combined with age and environmental related degradation, are primary contributors to structural damage. Among geometric properties, the exceptional height and slenderness of these structures cause them to act as cantilever beams highly vulnerable to lateral loads and exhibit longer natural periods and larger displacements [33]. The narrow and compact bodies are more vulnerable while vertical tapering can increase their lateral resistance [34]. Wall thickness gradually decreases towards the top to reduce weight, while this causes the axial stress at the above segments to drop significantly. The spiral staircase inside the bodies, if connected to the perimeter walls, contributes to their stiffness [17, 35]. Stiffness variations are observed between different geometries, especially at the transition segment between the boot and the body, which is the location of one of the most common failure modes [7, 36]. The balconies, mabkhara, and top segments create mass irregularities over the height of the structure, disrupting the load distribution [4, 37]. Minaret styles examined in this study rarely have openings (except for the doors, small air slits, and windows). While the openings create concentrated stress zones, they do not affect the global seismic behavior [15] and are mostly ignored in numerical and experimental studies. Some minarets are connected to the mosque wall from the boot segment, which shortens their free height and change their natural period. Moreover, this connection area has a large stress concentration and becomes a weak point [38, 39]. The soil–structure interaction (SSI) can amplify these

effects by increasing displacements and natural periods, affecting both the distribution and ratio of damage [40, 41]. Therefore, cities above soft sediments suffer severe damage during earthquakes due to the amplified ground motion [42].

2.1 Case Study Minarets

The Marzouki minaret is part of the 17th century Marzouq al-Ahmadi Mosque in Cairo (Fig. 2). It is a 27,5 m long stone structure with a height to base width ratio of 8,12. It has a square boot that is connected to the mosque, followed by a transition segment and octagonal main body. On top of the corbelled balcony, there is a tapered cylindrical upper body, and a masonry spire. The material properties for stone masonry with hydraulic lime are defined as Elastic Modulus (E) = 3500 MPa, density (ρ) = 2000 kg/m^3, and Poisson's ratio (ν) 0.2.

The second case study, the Northwest minaret of the Hagia Sophia Mosque in Istanbul was built in the 16th century (Fig. 2). It is a 70,6 m high stone masonry structure. The rectangular boot is connected to the mosque. It is followed by a transition segment and a 20-sided polygonal body, connecting to the upper body through a balcony. The spire is built in wooden frame covered with lead plates. Neglecting the spire, the minaret has a slenderness of 8,26. The material properties are adapted from another studied minaret of Hagia Sophia [43] as E = 8500 MPa, ρ = 2000 kg/m^3, and ν = 0.2.

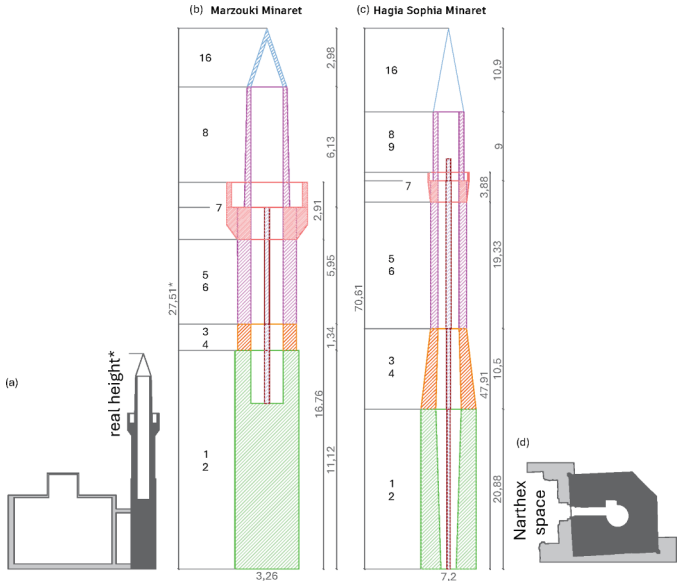

Fig. 2. Marzouki (b) and Hagia Sophia (c) minarets with respective section (a) and plan (d) schemes showing their connection to the mosques.

3 Methodology

3.1 Modelling Scenarios

Following the standardized parametrization proposed in [32], the minaret segment geometries and dimensions are defined. To isolate the influence of the secondary structural features, in total four modelling scenarios are considered, where M and HS refer to Marzouki and Hagia Sophia minarets, respectively: the initial FE models (M1 and HS1) are created respecting the original geometry and the secondary structural features investigated in this study (the truncated vertical profile, wall thickness changes in the perimeter walls and stairwell, presence of helical staircase and balconies), while in the rest of the models, these features are progressively reduced; in MS and HS2, the staircase is removed; in M3 and HS3 the balcony is removed; and in the baseline models (M0 and HS0) all secondary features are removed, (Table 1 and Fig. 3).

Table 1. Modelling scenarios adopted in the current work for Marzouki and Hagia Sophia minarets (where "+" means included element and "−" means subtracted)

Minaret model		Truncated profile & wall thickness changes	Staircase presence	Balcony presence
M0	H0	−	−	−
M1	H1	+	+	+
M2	H2	+	−	+
M3	H3	+	+	−

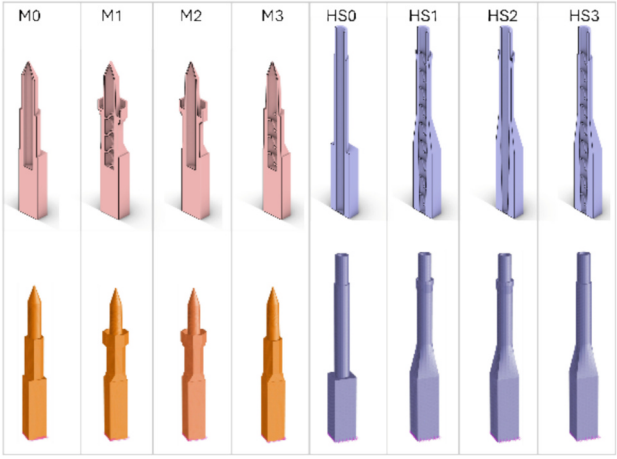

Fig. 3. Four modelling plots and FE mesh models of two case study minarets, where M and HS refer to Marzouki and Hagia Sophia minarets, respectively.

In total, eight FE models are created on Straus7 using brick elements. The masonry was modeled as a homogenous, isotropic continuum. The minarets' connection to the mosques are disregarded and they are considered fixed base structures. The original model's weight for Marzouki Minaret is 3453 kN, and for Hagia Sophia Minaret 29920 kN. For Marzouki minaret, the stairs is about 0,6%, and balcony about 5% of the total weight; while for Hagia Sophia minaret the ratios are 1.3% and 1%, respectively.

4 Sensitivity Analysis

4.1 Modal Analysis Results

To characterize the dynamic behavior shifts, modal analysis is performed on eight models. The performance indicators are the first five periods and corresponding modal participation factors. It is seen that for both Marzouki and Hagia Sophia minarets, the dynamic response is dominated by the first and second translational modes. The identical values between mode pairs demonstrate the bi-axial symmetry in the structures. For Marzouki minaret, the largest difference from the original model (M1) is found in the lack of balcony (M3), in which the fundamental period decreased from 0,54 to 0,487 s (about 10%) (Fig. 4). It indicates that the mass and stiffness impact of the balcony is significant for Marzouki minaret. For Hagia Sophia minaret, the largest difference from the original minaret (HS1) is observed in the baseline model (HS0) with the increase from 0,86 to 1,02 s (about 15%) (Fig. 5), showing that the construction details such as the spiral staircase, balcony and wall changes considerably increase its stiffness compared to the base model.

The modal mass participation in the translational X and Y directions is given in Fig. 6. The vertical component is observed in mode eight with 70% participation for Marzouki and 50 to 60% for Hagia Sophia minarets. In Marzouki minaret, in M1 and M3, the first two modes show a polarized distribution between principal directions. In models without staircases, the participation ratios in X and Y axes are closer, indicating that a diagonal axis of vibration forms when the bracing effect of the stairs, however slightly, is removed. In Hagia Sophia, the polarized distribution is observed in all models, which can be due to its more uniform structure and height that exceeds the effect of staircase against the symmetrical layout more clearly.

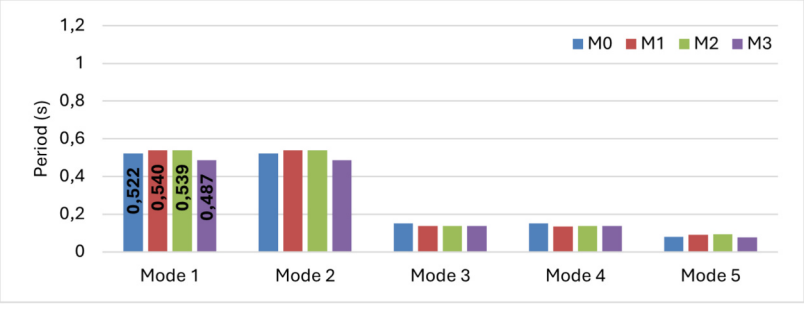

Fig. 4. Natural Periods across the first five modes of Marzouki minaret

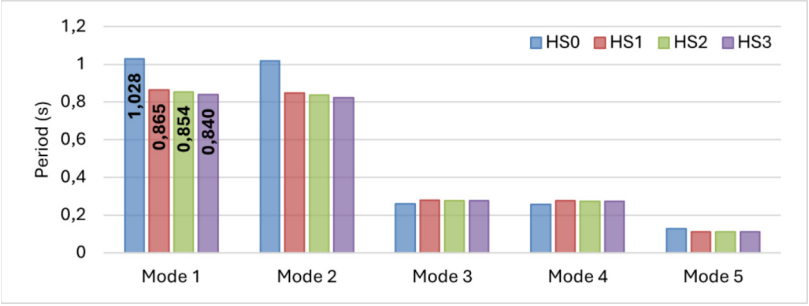

Fig. 5. Natural Periods across the first five modes of Hagia Sophia minaret

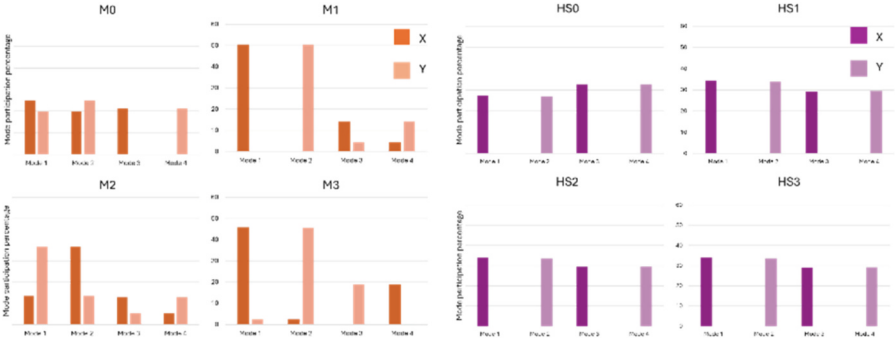

Fig. 6. Mode participation percentage in the principal directions for Marzouki and Hagia Sophia minarets

4.2 Kinematic Limit Analysis Results

Four of the five collapse mechanisms defined by [32] is utilized to manually perform the Kinematic Limit analysis on minaret models (Fig. 7). The tensile strength of the masonry is ignored according to the no-tension assumption. The load distribution according to the first mode shape of the original models is considered.

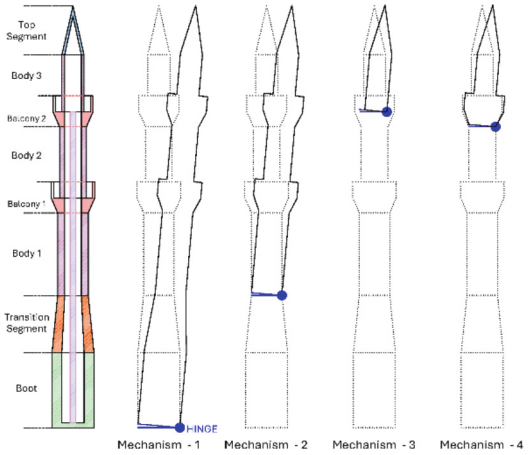

Fig. 7. Predefined collapse mechanisms [32] represented on a generic minaret: Mechanism 1 - Full collapse from the base; Mechanism 2 – Transition Segment collapse; Mechanism 3 – Upper Body collapse; Mechanism 4 – Balcony collapse

For Marzouki minaret, it is seen that for all minaret models, mechanism 1 triggered, which is global overturning from the base (Fig. 8). As expected, removing the balcony affects mechanism 4 occurring on that level significantly – for M3, by 0,091 g and of about 19%. The stairs have a small mass; therefore, their removal does not have a considerable effect. In mechanism 2 the cumulative impact has a strong influence since the wall changes are prominent around the Transition Segment.

For Hagia Sophia minaret, the collapse mechanism 1 is triggered in all scenarios (Fig. 9). The staircase leads to only a slight change in all mechanisms (HS2). The biggest difference from the original model is observed in mechanism 4, in the lack of balcony and wall geometry changes (HS0)– a decrease by 0,028 g and of about 6%. This change in the collapse multiplier, as indicated by the results in Figs. 8 and 9, is less prominent than in Marzouki minaret since the balcony in Hagia Sophia makes a smaller contribution to its mass.

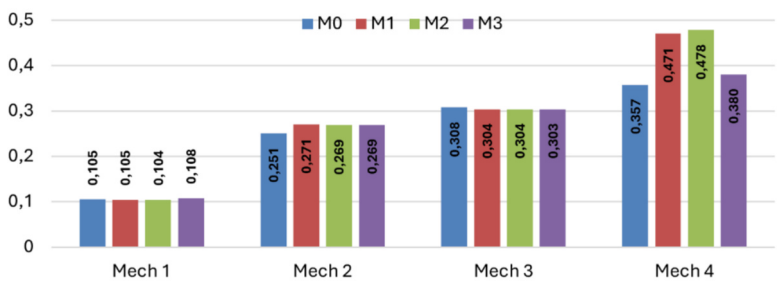

Fig. 8. Collapse multiplier values for Marzouki minaret

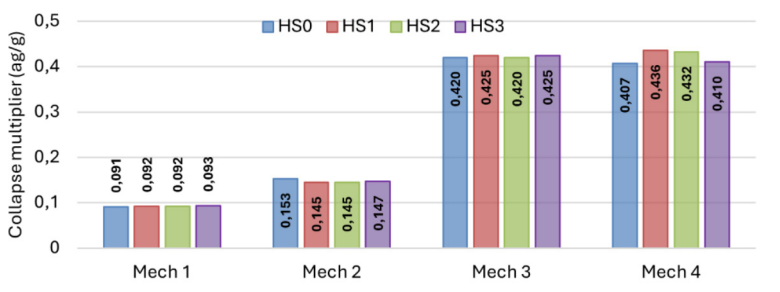

Fig. 9. Collapse multiplier values for Hagia Sophia minaret

4.3 Nonlinear Static (Pushover) Analysis Results

To optimize computing time and energy while investigating the effect of the secondary structural parameters, the NLSA is applied on the two models of the Marzouki minaret: M0 and M1. The structural response was examined under constant gravity loading and a lateral force on the X direction applied with a target load of 0.2 g, proportional to the mass of the structure. For material nonlinearity, Mohr-Coulomb criteria were adapted with the cohesion value of 0.12 MPa and friction angle (φ) of 30°.

The pushover curve of the original model (M1) indicates that, at the damage initation, the top displacement of 33 mm under 0.08 g loading – resulting in 297 kN base shear force - is expected (Fig. 10). For M0, the top displacement of 38 mm under 0.11 g loading - corresponding to 342 kN base shear force - is observed (Fig. 10). In both cases, the principal stress and strain distribution demonstrate that a plastic hinge formes in the lower to mid section of the boot (Figs. 11 and 12), while the maximum stress at the base of the minaret increases in the baseline model. Between two models, the maximum stress and strain values show only a slight difference of 4% and 0,2%, respectively.

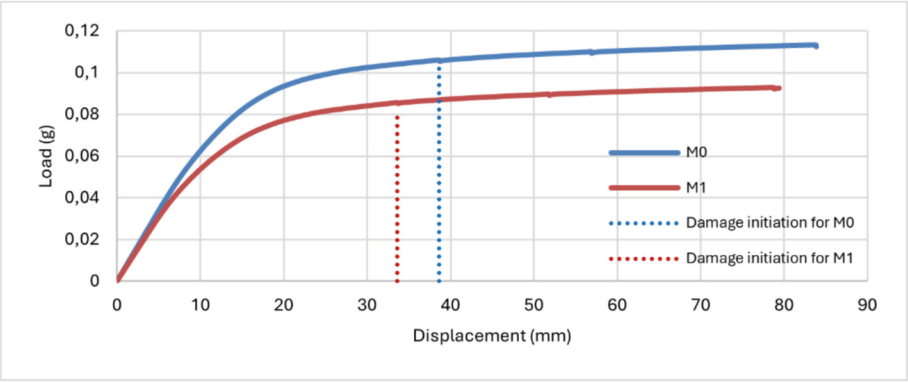

Fig. 10. Pushover curves of M0 and M1

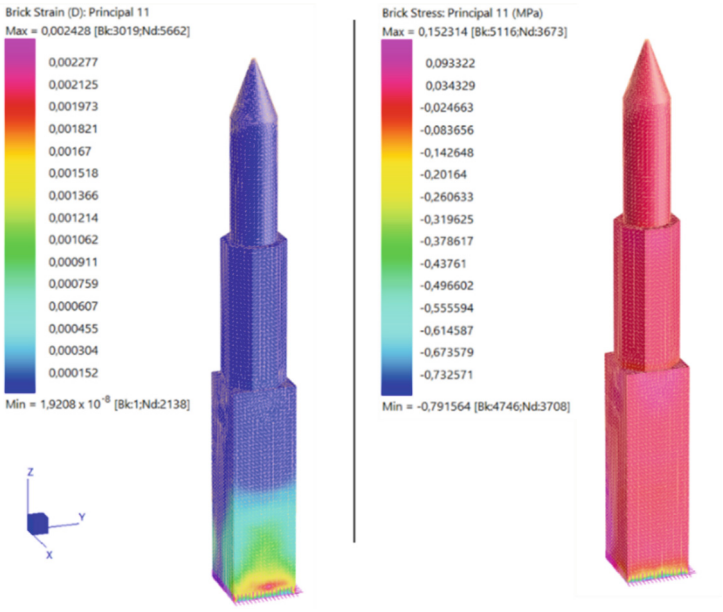

Fig. 11. Principal stress and strain graphs of M0 model

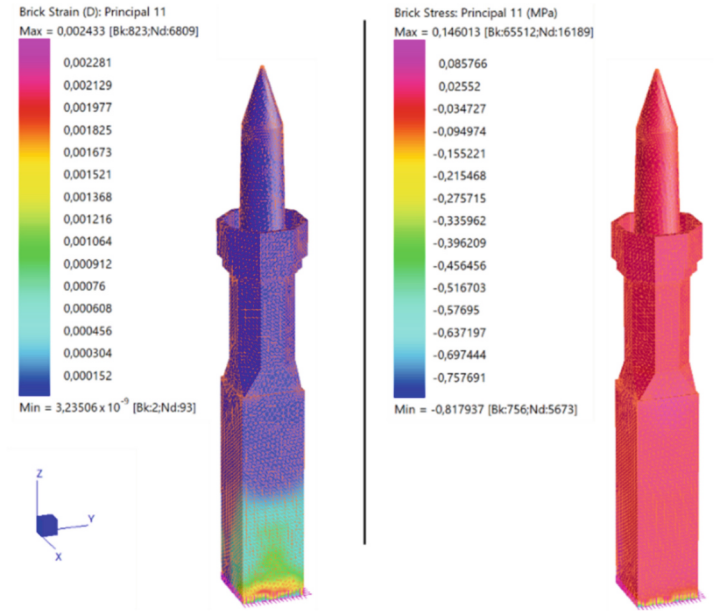

Fig. 12. Principal stress and strain graphs of M1 model

5 Discussion and Conclusion

In this study, the impact of the secondary structural features of minarets on their seismic assessment is investigated utilizing different analytical approaches. For this aim, two case study minarets from Turkey and Egypt are selected which have different architectural features and ratios, and seismic settings. The secondary features that are usually omitted or simplified in the literature on minarets, namely the impact of balcony, staircase, and vertical profile and wall thickness changes along the perimeter walls, are isolated in four modelling scenarios.

In Figs. 13, 14, 15 and 16 the change in performance indicators due to removing the secondary features are given where the negative values on the horizontal axis can be read as the overestimation of the minaret's strength. Modal analysis performed on the minaret model shows that, for a shorter structure like Marzouki minaret, removal of the relatively large mass of the balcony from the model drops the fundamental period between 10–14%. Moreover, the stairs have a bracing effect that significantly changes the distribution of mass participation ratio across principal directions. For the higher Hagia Sophia minaret, this distribution changes slightly between models, while a considerable increase (between 15–20%) in the period values from the baseline model occurs (Figs. 13 and 14). Secondly, the performed kinematic limit analysis reveals that the triggered mechanism does not change while the collapse multiplier values vary by the modelling scenario. In Marzouki minaret, the balcony has a prominent impact of 24% in mechanism 4. Hagia Sophia minaret has several cross-section changes at boot and transition segment, therefore the removal of all investigated secondary elements including the wall features results in a 5% increase in mechanism 2 and 6% decrease in mechanism 4 (Figs. 15 and 16). Lastly, as mentioned above, nonlinear static pushover analyses performed on the baseline and original models of Marzouki minaret show that the base shear load and top displacement of the minaret is overestimated in the former. In terms of the lateral acceleration triggering the plastic hinge formation, the difference is 0.3 g (13%). The change in stress and strain values overall is insignificant.

Overall, the collapse mechanisms demonstrated in kinematic limit analysis and nonlinear static analysis coincide with the commonly observed failure modes of minarets in both countries. In addition, the structural contribution by the staircases, wall thickness changes, and the balcony effect stated in literature are validated [34, 35, 37, 44].

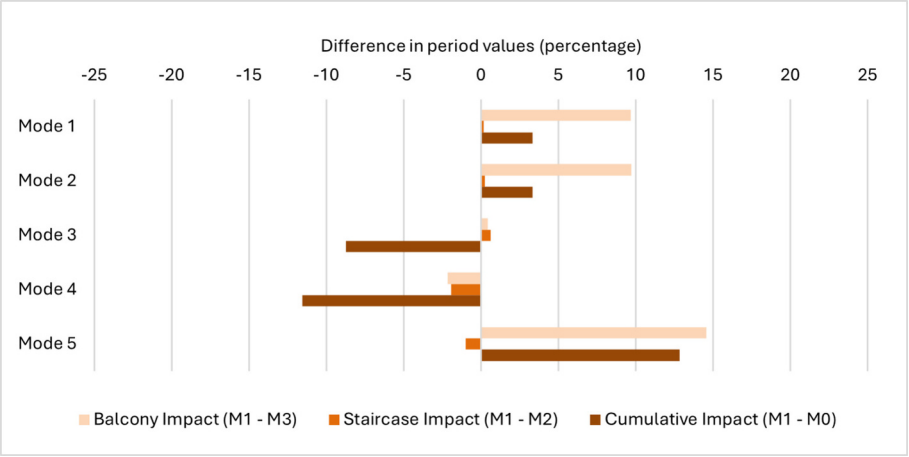

Fig. 13. Impact of secondary features in period values for Marzouki minaret

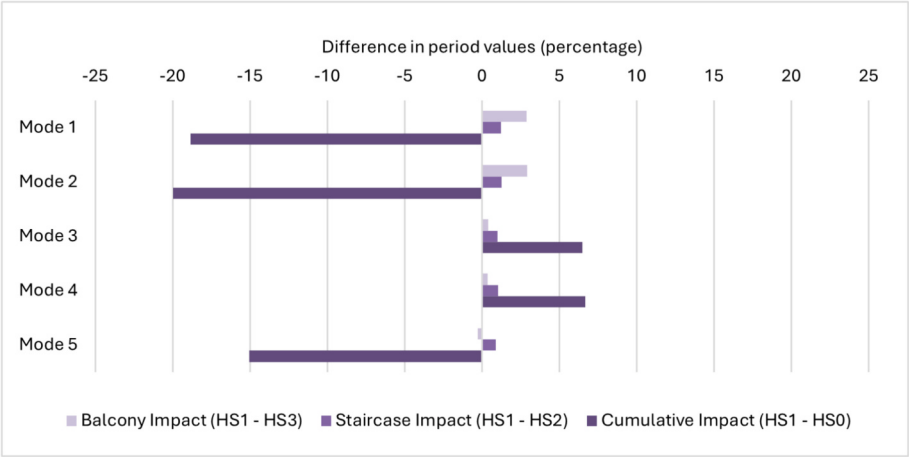

Fig. 14. Impact of secondary features in period values for Hagia Sophia minaret

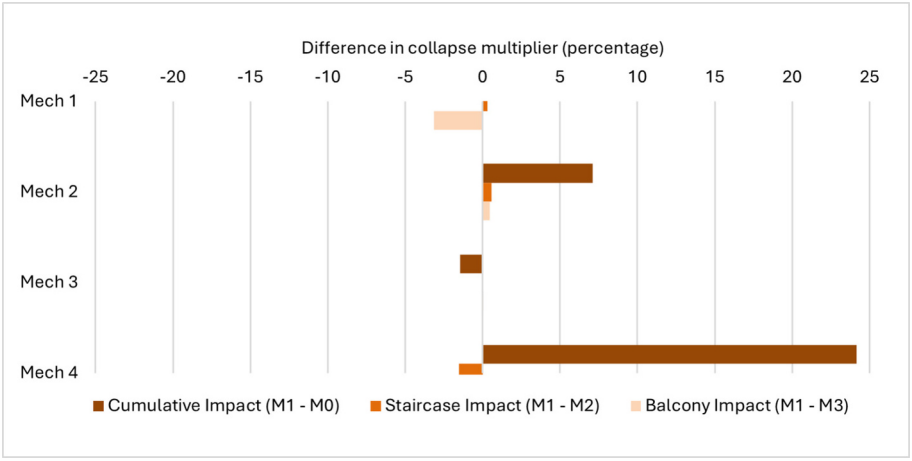

Fig. 15. Impact of secondary features in collapse multiplier values for Marzouki minaret

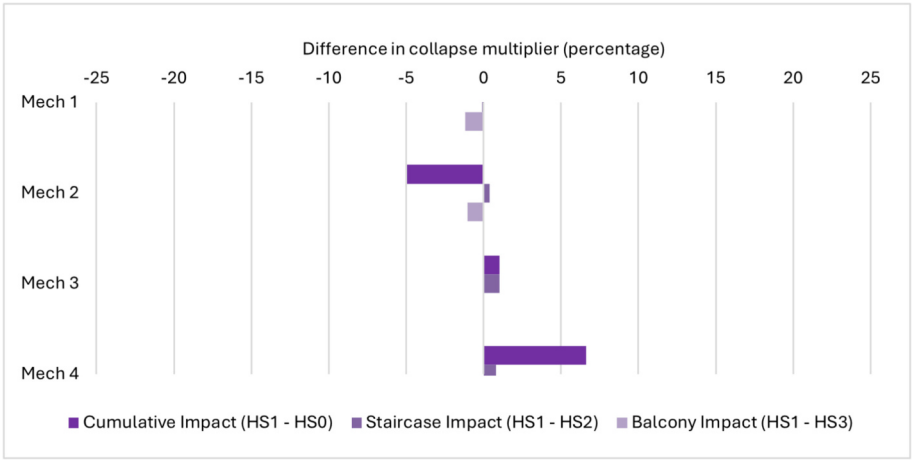

Fig. 16. Impact of secondary features in collapse multiplier values for Hagia Sophia minaret

In the end, the study shows that the secondary structural features of minarets which are often omitted in numerical analyses have impact on the expected seismic performance of the structure. It is a significant remark since, for instance, regarding the modal analysis, the varying fundamental period values could result in inconsistency during comparisons with the response spectrum; and asymmetry observed in mass participation ratios could determine whether to integrate higher mode effects in the analyses. The Kinematic Limit Analysis results in terms of collapse multiplier vary for a shorter minaret like Marzouki minaret, although, for both minarets, it does not change the expected mechanism with the minimum value. Here it should be noted that for Hagia Sophia minaret, load distribution pattern involving the higher mode effects could exhibit this variation more clearly, a

consideration which should be investigated in future studies. In the nonlinear static pushover analyses, it is clearly observed that the pushover curve changes significantly between modelling scenario, and the baseline model overestimates the capacity of the structure.

Future studies can incorporate larger and more diverse benchmark cases from real and virtual minarets. In the current study, in the pushover analysis phase, due to computational and time cost of the approach, only two models representing the two edge cases of one minaret could be examined. For a more detailed view, all cases of two minarets can be investigated. In addition, proved to be effective on their seismic performance, minarets' connection to the mosque can be involved in the analyses. Lastly, the material properties can be parametrically studied to reflect the masonry behavior, which can be utilized to better implement the no-tension material assumption on Finite Element software.

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