A Bi-Objective Model for Environmental Monitoring by Vessels and AUVs

Mahdi Moeini(✉)

ensIIE, SAMOVAR, 1 Place de la Résistance, 91000 Évry-Courcouronnes, France
mahdi.moeini@ensiie.fr

Abstract. In the context of environmental protection, this paper investigates offshore monitoring operations carried out by a primary vessel assisted by multiple autonomous submarines. Specifically, given a depot or port, a fleet consisting of a surface vessel equipped with several autonomous submarines is tasked with monitoring a predefined set of offshore locations. The fleet departs from the depot, completes the monitoring of all locations, and returns to the depot. Monitoring activities are performed through coordinated deployment of the surface vessel and the autonomous submarines, with the vessel acting as a mobile depot for launching and retrieving submarines via synchronized operations. The objective is to minimize total mission time, total energy consumption, or both simultaneously. To address this problem, we propose three mixed-integer linear programming (MILP) models and investigate the role of a bi-objective formulation. The models are implemented in Julia, tested on publicly available benchmark instances, and solved using standard optimization solvers. The results highlight the computational complexity levels as well as the trade-offs between minimizing mission time and reducing energy consumption.

Keywords: Vehicle Routing Problem · Maritime Logistics · Environmental Monitoring · Mixed-Integer Linear Programming · Heuristics

1 Introduction

Autonomous vehicles, e.g., drones, robots, etc., are used since several years for different objectives, e.g., parcel delivery, relief distribution, surveillance, environmental monitoring, etc. [4,11,19]. In this context, the use of autonomous boats, vessels, and submarines have particular importance due to the access restrictions to offshore zones [7,10]. Moreover, since oceans and seas cover about 71% of the Earth's surface, they require an effective protection, which can need frequent monitoring. The monitoring operations may be done for different military or civil purposes, e.g., discovering any potential leak of oil due to damage or sabotage actions. For this purpose, the boats, vessels, and submarines can be equipped with sensors or cameras to find the potential source of pollutants. Herein, an

© The Author(s), under exclusive license to Springer Nature Switzerland AG 2027
O. Gervasi et al. (Eds.): ICCSA 2026, LNCS 16769, pp. 503–519, 2027.
https://doi.org/10.1007/978-3-032-30494-0_32

efficient routing a fleet of maritime vehicles has a particular importance. Indeed, for example, through an optimized route planning, the detection of pollution can be done in a shorter time, which can help directly in environmental protection [20].

In the literature, we find several contributions that study routing a heterogeneous fleet of maritime vehicles for environmental monitoring. In particular, [20] investigates routing a fleet composed of a main vessel equipped with multiple identical submarines that can monitor (visit or cover) a given set of offshore locations in a minimal makespan. In this paper, we expand this concept to investigate alternative objectives, e.g., minimizing energy consumption of the fleet. The new models can be considered as alternative variants of the *multi-visit multi-drone covering salesperson problem with varying coverage* (mCSP-VC), where *varying coverage* refers approaching a location upto a certain distance to be close enough, without reaching the exact location, e.g., to witness, collect information, take pictures, and so on [20].

After presenting the *mixed-integer linear programming* (MILP) formulation of the problems, we solve them by the state-of-the-art solvers using publicly-available test instances and parameters. Moreover, we analyze the results of the models to investigate the benefits of using a bi-objective model taking into account both makespan and energy consumption.

The remainder of the paper is organized as follows. In Sect. 2, we have an overview on the existing literature related the problem studies in this paper. Section 3 is dedicated to the mathematical formulations of the problem. The solution methods are discussed in Sect. 4. The results of our computational experiments are reported in Sect. 5, and the last section is devoted to some concluding remarks.

2 Related Literature and Our Contributions

In terms of scientific research, many contributions have been developed in the context of autonomous vehicles, e.g., for parcel delivery, surveillance, environmental protection, etc. Detailed literature reviews can be found, e.g., in [4,8,20]. In this section, we present a concise review on some scientific articles that are closely related to the objectives of this paper.

In fact, in the literature, we observe different applications of the multi-visit multi-drone covering salesperson problem with varying coverage (mCSP-VC):

(i) Cooperative-based routing problems, where a fleet of autonomous vehicles (e.g., trucks combined with drones or robots) is involved with or without synchronization. In fact, there is a huge amount of research on routing drones [4,11,13], robots, or boats [10]. The general idea consists in routing a fleet of vehicles composed of one or more trucks, possibly equipped with a set of drones/robots. Furthermore, some studies consider a set of lockers or drone stations that are used as micro depots. In the literature, the typical objective consists in routing the vehicles to deliver parcels to all customers in minimal time. Some alternatives objective functions are also studied [19]. The

problems are formulated as MILP models that are then addressed either by a solver, or by an algorithm. In terms of solution methods, there are mostly heuristic or metaheuristic algorithms, but some exact algorithms have been developed too (see, e.g., [4] and references therein).

(ii) Coverage-based routing problems, where the objective consists in reaching a certain proximity of locations is sufficient have been generalized to involve multiple or autonomous vehicles (see, e.g., [2,3,20], and references therein). In particular, in [20], the fleet covering problem has varying coverage feature to visit or cover a given number of locations by a mixed fleet composed of a main vessel and multiple synchronized AUVs in minimal makespan.

(iii) offshore routing of maritime vehicles, where a combination of parts (i) and (ii) with some adjustments may be considered have particular importance, either in military or with civil objectives, e.g., tracking pollution, inspecting locations or under-water pipelines [15], or environmental monitoring [14].

In the context of maritime logistics, this paper investigates the use of a mixed fleet of vehicles composed of a main vessel equipped with a set of *autonomous underwater vehicles* (AUVs) (or submarines). The whole fleet starts its mission from a depot, visits or covers a given set of underwater spots, and returns to the depot. The AUVs are synchronized with the main surface vessel, such that each vehicle (main vessel or AUVs) can visit or cover one or multiple locations in a row. In the existing literature, the whole mission should be done in the minimal time [20]. We extend this existing concept through the following contributions:

- We consider alternative objective functions, i.e., minimizing energy consumption, which is aligned with environmental protection objectives. Due to the new objective function, the model becomes nonlinear, which is linearized by a classical technique.
- We introduce and investigate a bi-objective model which combines minimization of makespan and energy consumption.
- We conduct comparative computational study to investigate the role of alternative objective functions, e.g., minimizing energy consumption.

3 The Mathematical Models

In this section, we provide the mathematical formulations for the variants of the *multi-visit multi-drone covering salesperson problem with varying coverage* (mCSP-VC) presented in [20]. The detailed description of the mCSP-VC can be found in [20]; however, for the sake of completeness, we present it concisely in this section, and we use the corresponding notation for the two new models.

We consider a graph $G = (V, A)$, where $V = C \cup \{n + 1\}$ having n nodes $C := \{1, ..., n\}$ that should be visited or covered by either of the vehicles, the node $n + 1$ denotes the depot, and a set of arcs A that connect the nodes. The fleet of vehicles is composed of N_G vessels $L := \{1, ..., N_G\}$ such that each vessel is equipped with N_D identical autonomous submarines, also called *autonomous underwater vehicles* (AUVs).

We assume that the vessels have enough battery to visit all positions and return to the depot. However, each AUV has a limited endurance limit. Moreover, the vessel can launch and retrieve any AUV.

The set $K := \{1, ..., n-1\}$ indicates the index of all potential AUV sorties, where each sortie k in K consist of launch node of an AUV from a node, visiting or covering one or multiple nodes, and returns of the AUV back to the vessel.

When a vehicle, no matter the vessel or an AUV, remains on a node i, the range of covering by the vehicle is defined by the following formula [20]:

$$r_i = min(\theta, \alpha t_i + \beta), \tag{1}$$

where, θ is the maximal covering distance, $\alpha, \beta \geq 0$ are the estimated parameters, and t_i is the time spent by the vehicle on the node i.

A node j is considered as covered if

$$D_{ij} \leq r_i, \tag{2}$$

where, D_{ij} indicates the Euclidean distance between nodes i and j.

The further notation required for the formulation of the problems is summarized in Table 1.

3.1 The MILP Model Minimizing Makespan

In this section, we present the mathematical model of the mCSP-VC with the objective of minimizing the makespan, which is defined by the arrival time of the last vehicle to the depot. Mathematically, the objective function of the mCSP-VC is defined as follows:

$$\text{minimize } f(t) := \max\{t_{n+1}^{G,A}, t_{n+1}^{D,A}\} \tag{3}$$

The makespan defined by $f(t)$ is minimized subject to constraints (4)–(51), formulated as follows:

$$\sum_{k \in K} z_{ik}^{D} + z_i^{C} + z_i^{G} = 1, \forall i \in V \tag{4}$$

$$\sum_{(j,i) \in A} x_{jik} \geq z_{ik}^{D}, \forall i \in V, \forall k \in K \tag{5}$$

$$z_i^{G} = \sum_{(j,i) \in A} y_{ji}, \forall i \in V \tag{6}$$

$$h_{ik}^{R} + \sum_{(i,j) \in A} x_{ijk} = h_{ik}^{L} + \sum_{(j,i) \in A} x_{jik}, \forall i \in C, \forall k \in K \tag{7}$$

Table 1. Notations used for the mathematical models

Sets	
C	Set of all the nodes that must be covered or visited by the AUVs or the vessel.
V	Set of all the nodes, including the depot.
K	Index set of all possible AUV trips.
A	Set of arc among all nodes.
Parameters	
S_G	Speed of vessel.
S_D	Speed of AUVs.
D_{ij}	The Euclidean distance between nodes i and j.
$T_{ij}^G = D_{ij}/S_G$	The travel time of the vessel from node i to j.
$T_{ij}^D = D_{ij}/S_D$	The travel time of an AUV from node i to j.
B	The endurance limit of an AUV.
U	The energy consumption rate of an AUV per unit of time when moving.
$\tilde{U}$	The AUV energy consumption rate per unit of time when staying and taking measurements.
α	Radius expansion rate.
β	Intercept of radius expansion.
θ	Maximum coverage radius.
M	Sufficiently large positive number.
Variables	
x_{ijk}	Binary: Indicates whether the AUV trip k travels along arc $(i,j) \in A$.
y_{ij}	Binary: 1 if the vessel moves from node i to node j for $(i,j) \in A$, 0 otherwise.
z_i^G	Binary: Indicates whether node $i \in V$ is visited by the vessel.
z_{ik}^D	Binary: Indicates whether node $i \in V$ is visited by the AUV trip k.
z_i^C	Binary: Indicates whether node $i \in V$ is covered by staying in another node.
h_{ik}^L	Binary: Indicates whether node $i \in V$ is the launch node of AUV trip k.
h_{ik}^R	Binary: Indicates whether node $i \in V$ is the retrieval node of AUV trip k.
s_{ij}	Binary: 1 if by staying in node j, node i is covered where $(i,j) \in A$.
d_i	Integer: The number of onboard AUV on the vessel upon departure from node $i \in V$.
$t_i^{G,A}$	The vessel's arrival time at node $i \in V$.
$t_i^{G,L}$	The vessel's departure time from node $i \in V$.
$t_i^{D,A}$	The last AUV's arrival time at node $i \in V$.
$t_i^{D,L}$	The last AUV's departure time from node $i \in V$.
p_i^D	The remaining energy of the AUV at the point of departure from node i.
r_i	Radius of coverage at node $i \in C$

$$\sum_{(i,j)\in A} y_{ij} = \sum_{(j,i)\in A} y_{ji}, \forall i \in V \tag{8}$$

$$\sum_{i\in C} y_{i,n+1} = 1 \tag{9}$$

$$\sum_{j\in C} y_{n+1,j} = 1 \tag{10}$$

$$\sum_{(i,j)\in A} x_{ijk} \leq 1, \forall i \in V, \forall k \in K \tag{11}$$

$$\sum_{(j,i)\in A} x_{jik} \leq 1, \forall i \in V, \forall k \in K \tag{12}$$

$$\sum_{k\in K} x_{ijk} + \sum_{k\in K} x_{jik} \leq 1, \forall (i,j) \in A \tag{13}$$

$$z^D_{ik} + z^D_{jk} \geq x_{ijk}, \forall k \in K, \forall (i,j) \in A \tag{14}$$

$$h^L_{ik} \geq x_{ijk} + z^D_{jk} - z^D_{ik} - 1, \forall k \in K, \forall (i,j) \in A \tag{15}$$

$$h^R_{jk} \geq x_{ijk} + z^D_{ik} - z^D_{jk} - 1, \forall k \in K, \forall (i,j) \in A \tag{16}$$

$$\sum_{k\in K} h^L_{ik} \leq M z^G_i, \forall i \in V \tag{17}$$

$$\sum_{k\in K} h^R_{ik} \leq M z^G_i, \forall i \in V \tag{18}$$

$$h^R_{ik} + h^L_{ik} \leq 1, \forall i \in V, \forall k \in K \tag{19}$$

$$\sum_{i\in V} h^L_{ik} = \sum_{i\in V} h^R_{ik}, \forall k \in K \tag{20}$$

$$\sum_{i\in V} h^R_{ik} \leq 1, \forall k \in K \tag{21}$$

$$d_{n+1} + \sum_{j \in C} \sum_{k \in K} x_{n+1,j,k} = N_D \tag{22}$$

$$d_i + \sum_{k \in K} \sum_{(l,j) \in A} x_{ljk} \geq d_j + \sum_{k \in K} \sum_{(j,l) \in A} x_{jlk} - M(1 - y_{ij}), \forall (i,j) \in A \tag{23}$$

$$d_i + \sum_{k \in K} \sum_{(j,n+1) \in A} x_{j,n+1,k} \geq N_D - M(1 - y_{i,n+1}), \forall i \in C \tag{24}$$

$$p_i^D \geq B - M(1 - z_i^G), \forall i \in V \tag{25}$$

$$p_j^D \leq p_i^D - UT_{ij}^D - \tilde{U}(t_j^{D,L} - t_j^{D,A}) + M(3 - x_{ijk} - z_{ik}^D - z_{jk}^D), \forall k \in K, \forall (i,j) \in A \tag{26}$$

$$p_j^D \leq p_i^D - UT_{ij}^D - \tilde{U}(t_j^{D,L} - t_j^{D,A}) + M(3 - x_{ijk} - h_{ik}^L - z_{jk}^D), \forall k \in K, \forall (i,j) \in A \tag{27}$$

$$p_i^D \geq UT_{ij}^D - M(2 - x_{ijk} - h_{jk}^R), \forall k \in K, \forall (i,j) \in A \tag{28}$$

$$B \geq UT_{ij}^D - M(2 - x_{ijk} - h_{ik}^L), \forall k \in K, \forall (i,j) \in A \tag{29}$$

$$t_j^{G,A} \geq t_i^{G,L} + T_{ij}^G - M(1 - y_{ij}), \forall (i,j) \in A \tag{30}$$

$$t_i^{G,L} \geq t_i^{G,A} - M(1 - z_i^G), \forall i \in C \tag{31}$$

$$t_i^{G,L} \geq t_i^{D,A} - M(1 - h_{ik}^R), \forall k \in K, \forall i \in C \tag{32}$$

$$t_j^{D,A} \geq t_i^{D,L} + T_{ij}^D - M(1 - \sum_{k \in K} x_{ijk}), \forall (i,j) \in A \tag{33}$$

$$t_i^{D,L} \geq t_i^{D,A} - M(1 - \sum_{k \in K} z_{ik}^D), \forall i \in C \tag{34}$$

$$t_i^{D,L} \geq t_i^{G,L} - M(1 - h_{ik}^L), \forall k \in K, \forall i \in V \tag{35}$$

$$t_i^{D,L} \leq t_i^{G,L} + M(1 - h_{ik}^L), \forall k \in K, \forall i \in V \tag{36}$$

$$M \sum_{a \in C: a \leq i} z_{ak}^D \geq \sum_{a \in C: a \leq i} \sum_{b \in K: b \geq k} z_{ab}^D, \forall k \in K, \forall i \in C \tag{37}$$

$$r_j \leq \alpha(t_j^{D,L} - t_j^{D,A}) + \beta + M(1 - \sum_{k \in K} z_{jk}^D), \forall j \in C \tag{38}$$

$$r_j \leq \alpha(t_j^{G,L} - t_j^{G,A}) + \beta + M(1 - z_j^G), \forall j \in C \tag{39}$$

$$r_j \geq D_{ij} - M(1 - s_{ij}), \forall i \in V, \forall j \in C : (i,j) \in A \tag{40}$$

$$s_{ij} \leq \sum_{k \in K} z_{jk}^D + z_j^G, \forall i \in V, \forall j \in C : (i,j) \in A \tag{41}$$

$$\sum_{(i,j) \in A} s_{ij} \geq 1 - M(1 - z_i^C), \forall i \in C \tag{42}$$

$$\sum_{(i,j) \in A} s_{ij} \leq M z_i^C, \forall i \in C \tag{43}$$

$$x_{ijk} \in \{0,1\}, \forall (i,j) \in A, \forall k \in K \tag{44}$$

$$y_{ij}, s_{ij} \in \{0,1\}, \forall (i,j) \in A \tag{45}$$

$$z_i^G, z_i^C \in \{0,1\}, \forall i \in V \tag{46}$$

$$z_{ik}^D, h_{ik}^L, h_{ik}^R \in \{0,1\}, \forall i \in V, \forall k \in K \tag{47}$$

$$d_i \in \mathbb{N}^0, \forall i \in V \tag{48}$$

$$t_i^{G,A}, t_i^{G,L}, t_i^{D,A}, t_i^{D,L} \geq 0, \forall i \in V \tag{49}$$

$$0 \leq p_i^D \leq B, \forall i \in V \tag{50}$$

$$0 \leq r_i \leq \theta, \forall i \in C \tag{51}$$

The objective function (3) minimizes the makespan. Constraints (4)–(21) concern routing the fleet of the vehicles. Constraints (22)–(29) control the battery consumption and levels. Constraints (30)–(43) take care of the synchronization as well as the operational range of the vehicles. Finally, constraints (44)–(51) define the domain of the decision variables (for details, refer to [20]).

3.2 The MILP Model Minimizing the Consumed Energy

As an alternative objective, we are interested in minimizing the energy consumption during the trajectories, i.e., the total energy that vehicles consume while moving from one point to another. For this purpose, the objective function (3) is replaced by (52), in which the parameter κ is used as a coefficient to measure the rate of the energy consumption in the vessel versus the AUVs.

$$\begin{aligned}\text{minimize } g(x,y,z) := & \sum_{(i,j)\in A} \kappa \cdot U \cdot T^G_{ij} \cdot y_{ij} + \sum_{k\in K} \sum_{(i,j)\in A} U \cdot T^D_{ij} \cdot x_{ijk} \\ & + \sum_{k\in K} \sum_{j\in C} \cdot (t^{D,L}_j - t^{D,A}_j) \cdot z^D_{jk}\end{aligned} \tag{52}$$

Since the objective function (52) is nonlinear, we linearize it by introducing new variables as follows:

$$w^D_{jk} := (t^{D,L}_j - t^{D,A}_j) \cdot z^D_{jk} \quad : \quad \forall k \in K, \forall j \in C \tag{53}$$

Using the new variables, the objective function (52) becomes as follows:

$$\text{minimize } \sum_{(i,j)\in A} \kappa \cdot U \cdot T^G_{ij} \cdot y_{ij} + \sum_{k\in K} \sum_{(i,j)\in A} U \cdot T^D_{ij} \cdot x_{ijk} + \sum_{k\in K} \sum_{j\in C} w^D_{jk} \tag{54}$$

Moreover, to complete the formulation of the model, we add the following constraints to the model.

$$w^D_{jk} \leq M z^D_{jk}, \quad : \quad \forall k \in K, \forall j \in C \tag{55}$$

$$w^D_{jk} \geq 0, \quad : \quad \forall k \in K, \forall j \in C \tag{56}$$

$$w^D_{jk} \leq t^{D,L}_j - t^{D,A}_j, \quad : \quad \forall k \in K, \forall j \in C \tag{57}$$

$$w^D_{jk} \geq (t^{D,L}_j - t^{D,A}_j) - M(1 - z^D_{jk}), \quad : \quad \forall k \in K, \forall j \in C \tag{58}$$

The constraints of the model includes all constraints (4)–(51) as well as constraints (55), (56), (57), and (58).

By EC-mCSP-VC, we denote the mathematical model of the energy consumption minimizing mCSP-VC, which minimizes (54) subject to (4)–(51) and (55)–(58).

3.3 The Bi-Objective Model: Minimizing Makespan and Energy Consumption

From a bi-objective point of view, we can be interested in minimizing simultaneously makespan and energy consumption to investigate the relation and impact of each objective. This is motivated by searching for an optimal allocations and routing of heterogeneous fleet (vessel and AUVs) to decide which locations and in which order should be visited/covered by which vehicle type to minimize not only the mission time but also the energy consumption, which is meaningful when different vehicle types have different level of energy consumption.

For this purpose, we consider a third model, denoted by Bi-mCSP-VC, in which the objective function is composed of (3) and (54) that should be minimized simultaneously. The constraints of the model are the same as those of the second model, i.e., all constraints (4)–(51) as well as constraints (55), (56), (57), and (58).

Bi-mCSP-VC

$$\text{minimize } (f(t), g(x, y, z)) \tag{59}$$

$$\text{s.t. } (4) - (51) \text{ and } (55) - (58).$$

4 Methodology

We can use any standard MILP solver, e.g., Gurobi [6], to solve small instances of the single-objective MILP models presented in Sect. 3. Moreover, we present a simplified, yet efficient, version of the matheuristic presented in [20] to solve MILP models minimizing the makespan or energy consumption. In the scientific literature, heuristic, metaheuristic, and matheuristic algorithms are known as suitable choices for addressing complex optimization problems (see, e.g., [5,9,12,13,16–18,20], and references therein).

In the following, we present the description of the algorithm that we used to solve the single-objective models.

Algorithm 1 presents the pseudo-code of the matheuristic. The algorithm starts from a simple TSP tour in which each location is visited exactly once by the main vessel. Afterwards, a sequence of operators are applied to integrate the AUVs and to improve the quality of the solution. In the following, we describe different components of the algorithm.

Once the initial solution, i.e., a TSP tour, is available, the main loop of the algorithm begins and repeats a given number of iterations. Throughout this loop, first a random removal operator is applied by which either a given number of random locations are removed from the route, or a location is selected form a distance-based ordered list. Then, a randomly selected insertion operator is performed with the objective of inserting a vessel or AUV to accomplish a visit or cover a location. Two insertion operators are selected randomly: *covered insertion*

and *greedy insertion*. The former one checks if the removed location could be covered by a vehicle. The latter operator verifies all possibilities to see if an insertion in the vessel or AUV route gives a better solution to perform the operation.

The obtained solution is passed to the *one-step* sub-procedure (line 7 of Algorithm 1). This step of the algorithm, along with the new solution, takes a given number of parameters, in particular an upper bound of the objective value of the current solution, then builds a restricted MILP model in which some parts of the model are fixed. More precisely, the solution generated by the vessel's and AUVs' routes are the fixed parts of the model to which an upper bound is set. Afterwards, the restricted MILP model is solved by the MILP solver under a shortened time limit. If the restricted model is feasible, its solution is then evaluated versus the current best solution to replace it if the new one has a better objective value.

Algorithm 1. The matheuristic for the MILP models presented in Section 3.

1: Read the instance and initialize the parameters
2: Generate a TSP tour s
3: **for** iterations $\leq$ maximum number of iterations **do**
4: Select randomly a removal operator and apply it
5: Select randomly a insertion operator and apply it
6: Compute an upper bound obj_{ub}
7: Apply the *one-step* procedure to generate s_{new}
8: **if** s_{new} is better than the current best solution s_c **then**
9: $s_c \leftarrow s_{new}$
10: **end if**
11: **end for**
12: **return** s_c

Figure 1 gives sample visualizations of the solutions provided by the models minimizing makespan and energy consumption (with $\kappa = 2$). As we observe, when it is about minimizing the total mission time, parallel dispatching of vehicles is used, while minimizing the energy consumption avoid such a case.

5 Computational Experiments

This section is dedicated to the description of the computational experiments and their numerical results.

5.1 Settings of the Experiments

The MILP models presented in Sect. 3 are solved by the existing solvers and tested on publicly available instances with up to 10 nodes. More precisely, the

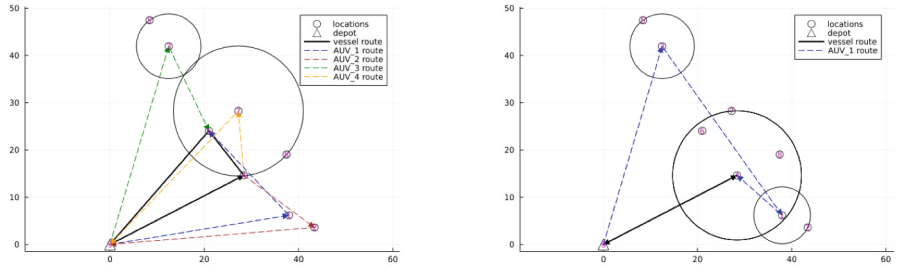

Fig. 1. Sample results of two models on a same instance. Left: the sample output of the model minimizing the makespan. Right: the sample output of the model minimizing the energy consumption with $\kappa = 2$.

MILP models are implemented in Julia [1]. The single-objective models are solved by the standard solver Gurobi (with time limit of 1800 seconds), while the bi-objective model is solved by Jump package of Julia. For the parameters of the models, we use the same values that we find in the literature [20]. Specifically, $B = 400.0$ (the endurance limit of an AUV), $U = 2.0$ (the energy consumption rate of an AUV per unit of time when moving), $\tilde{U} = 1.0$ (AUV energy consumption rate per unit of time when staying and taking measurements), $\alpha = 1.5$ (the radius expansion rate), $\beta = 0.0$ (the intercept of radius expansion), $\theta = 15.0$ (the maximum coverage radius), $S_D = 1.25$ (speed of AUVs), $S_G = 1.0$ (speed of vessel), $ND = 4$ (the number of AUVs).

Finally, for the model minimizing the energy consumption as well as the bi-objective model, we set the value of $\kappa = 2$, which indicates that the vessel consumes twice as an AUV per unit traveled distance.

5.2 Numerical Results

The following figures visualize the results of solving the three presented models for different instances and the corresponding parameters as described in Sect. 5.1.

On Figs. 2, the left-side figure shows obtained by the solver, i.e., the values of the consumed energy by the vehicles measured by the models. The focus of the first model (mCSP-VC) is on minimizing the makespan while the other models take into account the energetic (and consequently, ecologic) impact of the vehicles too. Consequently, the mCSP-VC model gives results that are not so ecological as the results provided by the two other models. Moreover, the values obtained by the bi-objective model coincide with the values obtained though the model minimizing the energy consumption, i.e., EC-mCSP-VC. In these experiments, when required, the value of κ is set to 2. The right-side figure shows similar experiments, but the single-objective models are solved by the metaheuristic presented in Sect. 4. As we observe in Figs. 2, the metaheuristic algorithm provides high-quality solutions for both the mCSP-VC as well as the EC-mCSP-VC models.

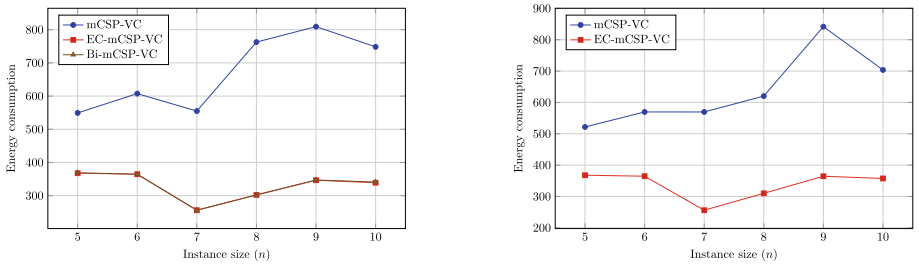

Fig. 2. Energy consumption values measured by three models obtained by using the solvers (Left) versus the results of the matheuristic (Right). In case of using the solvers, the values obtained by the bi-objective model coincide with the values obtained though the model minimizing the energy consumption, where we set $\kappa = 2$. The matheuristic is designed and used for solving only the single-objective models.

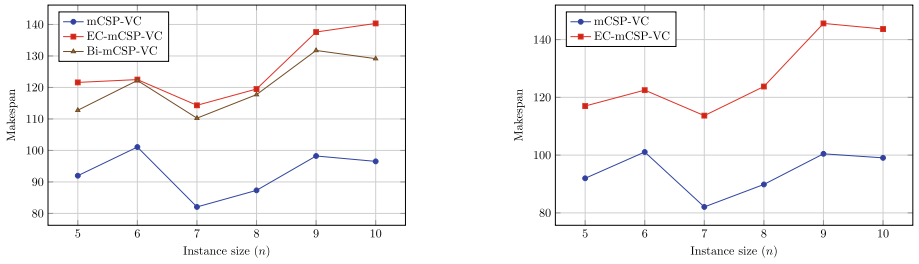

Fig. 3. Makespan values measured by three models obtained by using the solvers (Left) versus the results of the matheuristic (Right). In the corresponding models, we set $\kappa = 2$. The matheuristic is designed and used for solving only the single-objective models.

On Figs. 3, the left-side figure presents the results of the solvers in addressing the three models. In these results, the focus on the values of the makespan. The focus of the first model is on minimizing the makespan which is not fully the case of the other models, in particular the bi-objective model Bi-mCSP-VC considers a combination of makespan and energy consumption. Thus, as we observe, there is a big gap between the values obtained by the model minimizing the makespan versus the others. In these experiments, whenever required, the value of κ is set to 2. On the right-side figure, we present the results of the similar experiments where the single-objective models are addressed by the metaheuristic presented in Sect. 4. Similar to the case of Figs. 2, we observe also in Figs. 3 that the metaheuristic algorithm provides high-quality solutions.

Figure 4 shows the computation time required by the solvers and the matheuristic in addressing the three models. As we observe, solving the energy consumption model (EC-mCSP-VC) is fast, and using a heuristic approach doesn't seem to be necessary. Indeed, most of the tested instances are solved in less than one minute by the solver Gurobi. In this regard, the matheuristic

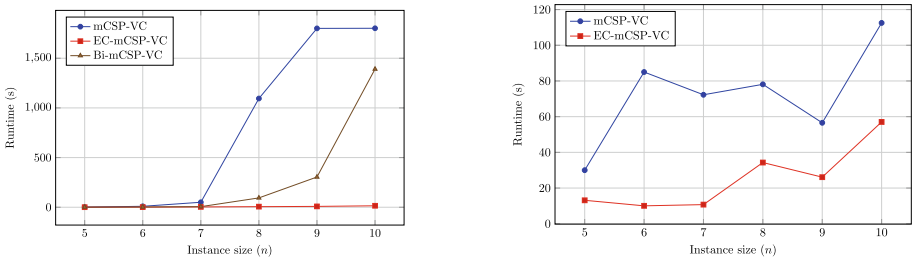

Fig. 4. Required computation time for solving the single- and bi-objective models. The figure on the left side corresponds to the results of the solvers, and the right-side figure illustrates the computation time of the matheuristics for solving the single-objective models.

finds high-quality solutions, but in many cases, it requires longer computation time than the solver.

However, when dealing with the bi-objective model, solving the model remains time consuming, even though it is still less challenging that the model minimizing the makespan, i.e., the mCSP-VC.

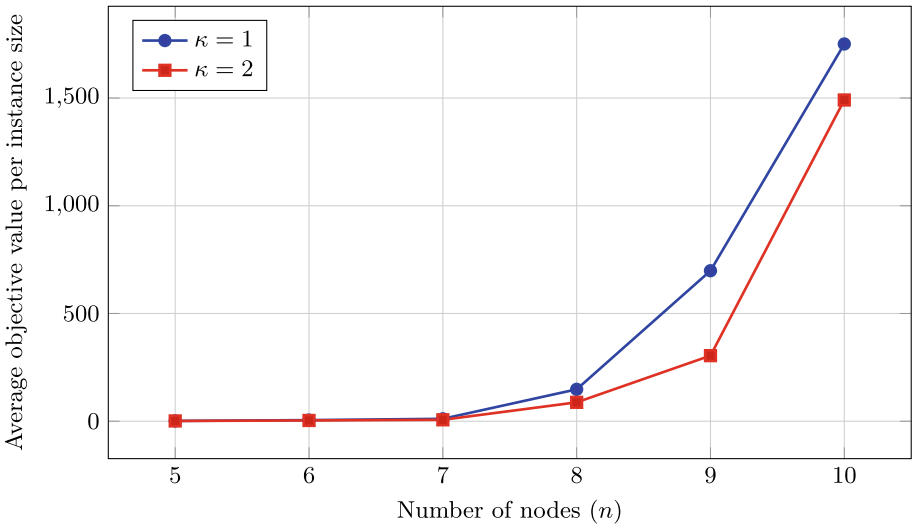

Fig. 5. The bi-objective model Bi-mCSP-VC: Objective value as a function of the problem size n for two values of κ.

Figure 5 illustrates the computation time required to solve the bi-objective model for $\kappa \in \{1, 2\}$. As we observe the computation time increases fast and the growing rate of computation time is reduced by increasing the value of κ.

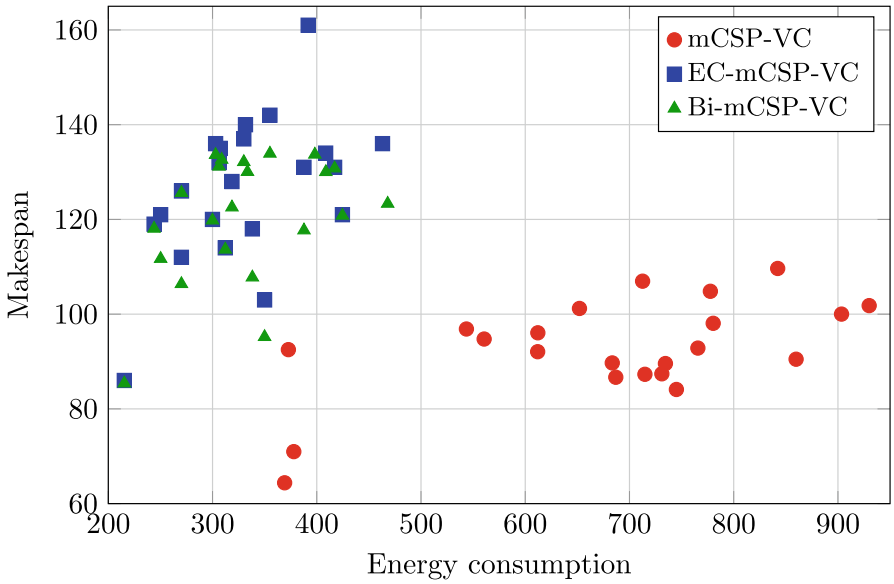

Fig. 6. Benchmarking makespan versus energy consumption levels provided by the solutions of the single- and b-objective models.

Finally, Fig. 6 provides an overview on the results of the models. More precisely, the figure shows how the objective values of the three models are scattered in a comparison of makespan values versus energy consumption levels. As expected, the single-objective models give quite different measures of makespan and energy consumption. Meanwhile, the bi-objective model provides a compromise between these objectives, and sometimes its values coincide with those of the single-objective model minimizing the energy consumption.

6 Conclusion

In this paper, we introduced two new MILP models that deal with minimizing energy consumption of a heterogeneous fleet composed of a vessel and AUVs with the mission of visiting or covering a set of locations. These are new variants of an existing model [20], named mCSP-VC. In particular, we present a bi-objective model that minimizes simultaneously makespan and energy consumption. We solved all models either by a solver or a matheuristic. The results of computational experiments on publicly available instances are reported. Moreover, we observe that the models provide solutions that can be quite different from the solution of the mCSP-VC.

The future research directions consist in developing efficient algorithms for the bi-objective model as well as extending the models for multiple vessels. The research in these directions is in progress, and the results will be reported in the future.

Acknowledgements. The author would like to thank Mr. Grégoire BROUARD for his assistance and help as well as the anonymous reviewers for their generous and constructive feedback.

References

1. Bezanson, J., Edelman, A., Karpinski, S., Shah, V.B.: Julia: a fresh approach to numerical computing. SIAM Rev. **59**(1), 65–98 (2017)
2. Biswas, A., Babaee Tirkolaee, E., De Narayan, L., Yu, V.F., Pal, T.: On multi-objective multi-coverage covering salesman problem. Soft. Comput. **29**, 6005–6022 (2025)
3. Di Placido, A., Archetti, C., Cerrone, C., Golden, B.: The generalized close enough traveling salesman problem. Eur. J. Oper. Res. **310**(3), 974–991 (2023)
4. Enes Cengiz, C.Y., Kahraman, H.T.: A literature review of drone-truck routing problems: challenges and future research directions. RAIRO Oper. Res. **59**(5), 3169–3205 (2025)
5. Goeke, D., Moeini, M., Poganiuch, D.: A variable neighborhood search heuristic for the maximum ratio clique problem. Comput. Oper. Res. **87**, 283–291 (2017)
6. Gurobi: Gurobi Optimizer Reference Manual (2019)
7. Liu, Y., Xia, J., Xu, Z.: Drone scheduling optimization for continuous sea area monitoring. Transp. Res. Part E Logist. Transp. Rev. **208** (2026). https://doi.org/10.1016/j.tre.2026.104701
8. Macrina, G., Pugliese, L.D.P., Guerriero, F., Laporte, G.: Drone-aided routing: a literature review. Transp. Res. Part C Emerg. Technol. **120**, 102762 (2020)
9. Moeini, M.: Optimizing airline pilots training plans: a mixed integer linear programming approach. Commun. Comput. Inf. Sci. **2144**, 289–301 (2024)
10. Moeini, M.: Routing a truck and its autonomous supporting vehicles in humanitarian relief distribution. In: 26th congrès de la Société Française de Recherche Opérationnelle et d'Aide à la Décision (ROADEF), pp. 1–2. ROADEF (2025)
11. Moeini, M., Le, D.T.D.: Multiple traveling salesman problem with a drone station: using multi-package payload compartments. Commun. Comput. Inf. Sci. **2144**, 226–237 (2024)
12. Moeini, M., Schermer, D., Wendt, O.: Cascading-tree algorithm for the 0-1 knapsack problem (in memory of Heiner Müller-Merbach, a former president of IFORS), pp. 1–11 (2024)
13. Moeini, M., Wendt, O., Schummer, M.: A bi-objective routing problem with trucks and drones: minimizing mission time and energy consumption. In: Gervasi, O., et al. (eds.) Computational Science and Its Applications – ICCSA 2023 Workshops, ICCSA 2023. LNCS, vol. 14106, pp. 291–308. Springer, Cham (2023). https://doi.org/10.1007/978-3-031-37111-0_21
14. Niu, H., Adams, S., Lee, K., Husain, T., Bose, N.: Applications of autonomous underwater vehicles in offshore petroleum industry environmental effects monitoring. J. Can. Pet. Technol. **48**(5), 12–16 (2009)
15. Sahoo, A., Dwivedy, S.K., Robi, P.: Advancements in the field of autonomous underwater vehicle. Ocean Eng. **181**, 145–160 (2019)
16. Schermer, D., Moeini, M., Wendt, O.: Algorithms for solving the vehicle routing problem with drones. In: Nguyen, N.T., Hoang, D.H., Hong, T.-P., Pham, H., Trawiński, B. (eds.) ACIIDS 2018. LNCS (LNAI), vol. 10751, pp. 352–361. Springer, Cham (2018). https://doi.org/10.1007/978-3-319-75417-8_33

17. Schermer, D., Moeini, M., Wendt, O.: A hybrid VNS/Tabu search algorithm for solving the vehicle routing problem with drones and en route operations. Comput. Oper. Res. **109**, 134–158 (2019)
18. Schermer, D., Moeini, M., Wendt, O.: A matheuristic for the vehicle routing problem with drones and its variants. Transp. Res. Part C Emerg. Technol. **106**, 166–204 (2019)
19. Schermer, D., Moeini, M., Wendt, O.: The drone-assisted traveling salesman problem with robot stations. In: Proceedings of the 53rd Hawaii International Conference on System Sciences, pp. 1308–1317 (2020)
20. Torabi, P., Hemmati, A.: A synchronized vessel and autonomous vehicle model for environmental monitoring: mixed integer linear programming model and adaptive matheuristic. Comput. Oper. Res. **183**, 108389 (2025)