Multifractal Analysis of Gold as a Hedge Against Geopolitical Risk

Jorge Moreira[1], Rui Pascoal[2], and Humberto Rocha[2(✉)]

[1] University of Coimbra, DMUC, 3001–501 Coimbra, Portugal
[2] University of Coimbra, CeBER, FEUC, 3004–512 Coimbra, Portugal
{ruiapsp,hrocha}@fe.uc.pt

Abstract. This study investigates the efficacy of gold as a hedge against geopolitical risk through a Multifractal Detrended Cross-Correlation Ana- lysis (MF-DCCA). Daily price data of gold, S&P500, DAX40, DXY, Nikkei225, and WTI from 2006 to 2022 is analyzed as well as the performance of these assets during periods of heightened Geopolitical Risk Index. Three hypotheses are tested: whether gold loses its safe haven attributes during times of increased geopolitical risk, whether its ability to offset downside risk is lower than other assets, and whether cross-correlations between assets and geopolitical risk present multifractal properties. The findings suggest that all assets exhibit a long-range correlation with the geopolitical risk index, confirming the presence of multifractality. This study demonstrates significant temporal variations across different market regimes, illustrating the relation of gold with geopolitical risk evolving from the 2008–2009 financial crisis through the COVID pandemic. In the full period analysis, gold reveals more pronounced anti-persistence than most other assets (except S&P500), indicating more rapid mean-reverting behavior in response to geopolitical shocks. This stronger anti-persistence suggests gold functions effectively as a short-term hedge where investors quickly move to gold during crises but is sold once the situation begins to stabilize. The pattern varies across different market regimes. However, gold's behavior during the COVID crisis challenge traditional safe haven narratives. These findings align with recent research showing that gold has a cross correlation with financial markets that may shift significantly during extreme market conditions. This shows that the safe haven character of gold is context-dependent and varies based on the specific nature of the crisis. Overall, this study reveals a more nuanced understanding of gold's relationship with geopolitical risk than traditionally understood, while still affirming its generally superior position relative to other assets during most periods of geopolitical uncertainty.

Keywords: Multifractal Analysis of Asset Returns · Self-Affinity · Geopolitical Risk · Gold

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1 Introduction

The world of finance has always been influenced by various factors, including economic, political, and social conditions. In recent times, the impact of geopolitical risks on the financial market has gained significant attention from researchers and investors alike. Gold, being one of the most sought-after precious metals for the reasons listed below, is no exception to this trend.

Gold has long been considered a safe haven asset [1], meaning that it is often sought-after by investors during times of economic uncertainty or geopolitical risk. This is because gold is seen as a stable store of value and a hedge against inflation, making it an attractive investment option in times of market volatility and it is often used as diversification tool for portfolios by analysts due to its low correlation with other commodities and stocks [2].

When geopolitical risk is high, investors often seek safe havens for their money, and gold can be an attractive option. In times of political instability or military conflict, demand for gold may increase as investors look to protect their wealth. Similarly, geopolitical events can affect the global economy and, in turn, the demand for gold. For example, during times of economic uncertainty, gold may be seen as a more stable and reliable investment than stocks or other financial assets. Additionally, gold is not tied to any particular country or political system, which makes it not suffer as heavily due to regional conflicts, making it a relatively neutral option for investors. These factors can make gold an appealing choice in times of geopolitical risk.

The purpose of this study is to investigate and attempt to test 3 hypothesis. First, if the gold loses its safe haven attributes during periods of increased geopolitical risk. Secondly, if its ability to offset risk during riskier periods is lower than other assets. Lastly, the final hypothesis being tested is if the cross-correlation between the different assets and the geopolitical risk index present multifractal properties. The method being used to test the hypothesis laid out will be a multifractal detrended cross-correlation analysis (MF-DCCA). This method quantifies nonlinear and long-range correlation between two time series. By using MF-DCCA, this study aims to shed light on the intricate connection between gold prices and geopolitical risks and the results of this analysis should provide valuable insights for investors to manage and mitigate the impact of geopolitical risks on their investments. Additionally, the use of multifractal analysis facilitates a comprehensive analysis of the gold market. Unlike traditional methods that assume a linear relationship between two time series, multifractal analysis takes the multifractal nature of financial markets into account. This leads to an accurate representation of the underlying dynamics. If the belief that gold is a safe haven for geopolitical risk and that markets are rational holds true, then the expected result should be that gold maintains its safe haven attributes and that its ability to offset risk should be greater than the other assets. Furthermore, there should not be any multifractal properties in the cross-correlation between the assets and geopolitical risk since that would indicate an inefficient market that could be exploited by rational players.

2 Methodology

Financial markets are institutions and mechanisms that facilitate the buying and selling of financial securities, such as stocks, bonds, and derivatives. These markets play a crucial role in allocating capital and managing risk, and are integral to the functioning of modern economies. To study the behavior of these markets, financial analysts often use financial time series, such as stock prices. These series are often characterized by a high level of volatility and complexity, making them difficult to predict using traditional statistical models.

To understand these complex systems better, researchers have turned to mathematical and computational tools, including fractal theory and multifractalism. Fractal theory, first introduced by Benoit Mandelbrot in the 1960 s, is a mathematical framework for studying self-similarity in complex systems [3]. Multifractalism is a further development of fractal theory, which allows for the characterization of complex systems that exhibit self-similarity at multiple scales. In finance, multifractal models have been used to study scaling properties of financial time series, such as stock market indices and exchange rates. These models have been used to gain insights into the underlying dynamics of financial markets and to develop new trading strategies. It is important to mention that not all systems exhibit multifractal behavior. Some systems are monofractal, meaning that they exhibit self-similarity with a single scaling exponent.

By understanding the multifractal nature of financial markets, investors and traders can gain a better understanding of market behavior, and make more informed investment decisions. This multifractal framework has been applied to analyze financial time series and detect patterns in volatility, return and correlation, providing a more comprehensive view of the market. This is demonstrated by the works of Gunay [4], Xu *et al.* [5] and many other researchers, who have used multifractal analysis to gain new insights into the behavior of financial markets.

2.1 Monofractalism

A self-similar stochastic process is a process that is statistically similar to a part of itself. In formal terms, let $(X_t, t \in T)$ be a stochastic process, q a constant that we call the scale exponent and H a constant call the Hurst exponent, then

$$X_{qt} = q^H X_t. \tag{1}$$

The definition is also extended for self-affine time series, which is the basis of fractal analysis. Note that we can get relevant information of a process depending on both the scaling constant and the Hurst exponent. The Hurst exponent is a real number bounded between 0 and 1, with $H < 0.5$ indicating an anti-persistent series, $H > 0.5$ indicating a persistent series and $H = 0.5$ indicating that the series behaves like a random walk, or can be thought of as a non existent persistence. Note that persistence can represent a myriad of definitions that can entail covariance or correlation, but we are using it to the behavior of the series.

For $H > 0.5$, we can say that the process displays long memory, meaning it displays persistency, which will be key for the study presented in this study.

In this study, the focus is on analyzing asset prices, where the data represents the price of an asset. Consequently, the axes of the data, namely the X and Y axes, represent distinct values. In this context, as shown by Kantelhardt [6], self-affinity, which refers to the property of a time series where it exhibits similar patterns at different scales, is what will be used to characterize a stochastic process as a monofractal alongside the definition shown in (1).

2.2 Multifractalism

Expanding upon the definition of monofractalism, we can describe complex systems where a single scaling exponent is not enough to describe the system [7]. For that, we find that we need multiple scaling exponents to provide a thorough analysis of the complex system. In such case, we can define a multifractal system as

$$X_{qt} = q^{h(q)} X_t, \tag{2}$$

where $h(q)$ is the generalized Hurst exponent. This exponent is a value that ranges between 0 and 1 and is calculated by looking at different moments of the time series.

In informal terms, multifractalism can be thought of as an interwoven set of monofractals, where each monofractal represents a different scaling behavior within the overall system. In other words, a multifractal system can be thought of as consisting of multiple monofractals, each exhibiting a particular degree of complexity [6]. This implies that the conclusion that if $h(q)$ is constant, then the above equation simplifies into the definition of a monofractal. In the case of financial time series, each monofractal characterizes the behavior of fluctuation of a particular size.

By adding stricter stationarity as a requirement for the time series, we have the following scaling rule

$$E(|X_t|^q) = c(q)t^{\tau(q)+1}, \tag{3}$$

where $E(|X_t|^q)$ is the q-th moment of the absolute value of the time series, $c(q)$ a function of q, t is the time scale, and $\tau(q)$ is the scaling exponent. With the above equation, two things becomes apparent. First, most of the work will now revolve around the function $\tau(q)$, which is called the scaling function or the Renyi exponent, defined as

$$\tau(q) = h(q)q - 1. \tag{4}$$

Second, for $q > 0$, the larger values of the time series are given more weight. This means that the calculation of the generalized Hurst exponent will be more sensitive to larger fluctuations in the time series. Intuitively this can be seen as having a positive value of q implying that the variable is raised to a positive power, which amplifies the larger values of the variable and de-emphasises the

smaller values. By applying the same interpretation as in the monofractal case, $h(q) > 0.5$ is an indication of a persistent time series, $h(q) < 0.5$ an anti-persistent time series and for $h(q) = 0.5$ an absence of long range correlations [8]. The situation for $q < 0$ is analogous.

Focusing on the interpretation of the scaling function $\tau(q)$, if the time series is a multifractal system, then the scaling function is nonlinear and concave [9]. The function is linear when $h(q)$ is a constant, which means that all the moments of the time series have the same scaling exponent. In this case, the time series is a monofractal, since it has only one fractal scaling exponent.

If we want to characterize the properties of a multifractal system, we look into its spectrum, which we will denote as $f(\alpha)$. One may think of the multifractal as an interwoven set of fractals of different dimensions α, called the Hölder exponent, where $f(\alpha)$ is a measure of their relative strength and frequency. From this concept, we can find powerful tools that will help further in studying the self-similarity in time series [10].

2.3 Development of Multifractal Analysis in Financial Markets

The application of multifractal analysis to financial markets has its roots in the work of Mandelbrot, who demonstrated that price changes exhibit scaling properties inconsistent with normal distributions [3].

Multifractal analysis extends beyond traditional financial theories by characterizing complex systems through local scaling properties that vary across different regions of the system. Unlike monofractal analysis, which assumes uniform scaling properties described by a single scaling exponent (the Hurst exponent), multifractal analysis reveals how scaling behavior changes across different magnitudes of fluctuations, providing deeper insights into market dynamics. This is particularly important, because financial markets exhibit different statistical properties during periods of stability versus periods of crisis, which cannot be adequately captured by conventional models assuming stationarity and Gaussian distributions [11].

The multifractal approach offers several key advantages for financial analysis: (1) it captures both short and long-range dependence in time series; (2) it characterizes the degree of heterogeneity in scaling behavior; (3) it provides measures of market complexity and inefficiency; and (4) it enables the identification of structural changes in market behavior that may precede significant events [12]. These capabilities make multifractal analysis especially valuable for investigating safe haven assets like gold, whose behavior during market stress often deviates significantly from normal conditions.

Methodological advances have been substantial. Peng *et al.* introduced the original Detrended Fluctuation Analysis (DFA), providing a foundational method for analyzing long-range correlations in non-stationary time series while eliminating trends [13]. Building upon this work, Kantelhardt *et al.* developed the MF-DFA method [7], extending DFA to analyze multifractal properties of non-stationary time series by examining how fluctuation functions scale differently

for various q-order statistical moments, effectively capturing the multifractal spectrum.

Further methodological developments saw Podobnik and Stanley introduce the Detrended Cross-Correlation Analysis (DCCA), which extends DFA to measure cross-correlations between two non-stationary time series [14]. Subsequently, Zhou advanced this approach by developing the Multifractal Detrended Cross-Correlation Analysis (MF-DCCA), which combines the multifractal approach with cross-correlation analysis, enabling the study of how cross-correlations between different time series vary across multiple time scales and fluctuation magnitudes [15].

While existing literature has established both gold's safe haven properties and the presence of multifractal characteristics in financial markets, there remains a significant gap in understanding how these properties interact specifically during periods of heightened geopolitical risk. Most studies have focused either on gold's safe haven properties or on market multifractality, but few have combined these approaches to analyze their joint dynamics during geopolitical events. The application of MF-DCCA to study these relationships offers potential new insights into market dynamics that traditional correlation analyses might miss.

2.4 Multifractal Formalism

The multifractal formalism is a mathematical framework used to understand complex systems that show multiple fractal scaling exponents. In this particular formalism, we will be basing upon the works of Salat *et al.* where one quantifies the self-similarity present in a system at different scales [10]. The analysis can be approached through two main methods: moment methods and histogram methods. To effectively compute the multifractal spectrum, the moment method is employed by raising the probability measure p_i to different moments q. This technique forces only one value of α for each moment q to make a significant contribution to the total value of the measure. In contrast, histogram methods focus on counting occurrences at different levels of aggregation, with box-counting being the most commonly implemented approach, which uses square grids of increasing resolutions to analyze the data. Alternative aggregation methods include ball neighborhoods, gliding boxes, and regular unit shapes.

In this framework, a grid of unit r is used to cover a domain D and a phenomenon occurring N times in D. The probability measure is represented by μ and p_i is defined as the number of times the phenomenon occurs in the ith box divided by the total number of times the event occurs in D, represented as:

$$p_i := \frac{N_i}{N} = \int_{i^{th}\ box} d\mu(x). \tag{5}$$

This verifies $p_i \sim r^{\alpha_i}$, where α_i is the Hölder dimension calculated as

$$\alpha_i = \lim_{r \to 0} \frac{\log(\mu_i(r))}{\log(r)}. \tag{6}$$

The number of times α occurs in each interval $[\alpha_i, \alpha_i + d\alpha_i]$ is represented by $N(\alpha_i)$ and verifies:

$$N(\alpha_i) \sim \rho(\alpha_i) r^{-f(\alpha_i)} d\alpha_i. \tag{7}$$

The moment method is used to compute f. By raising p_i to different moments, p_i^q, at each q one single value of α is forced to contribute to the total value of the measure. This is represented by:

$$Z(q) := \sum_i p_i^q \sim \sum_i r^{\alpha_i q} \sim \int_\alpha N(\alpha) r^{\alpha q} \sim \int \rho(\alpha) r^{\alpha q - f(\alpha)} d\alpha, \tag{8}$$

where $Z(q)$ is a partition function. For small enough r, the value of $Z(q)$ is almost entirely determined by the α that minimizes $\tau(q) = \alpha q - f(\alpha)$. This minimal value of α is defined as $\alpha(q)$ and can be calculated using the Legendre transform:

$$\begin{aligned} \alpha(q) &= \frac{d\tau(q)}{dq} \\ f(\alpha(q)) &= \alpha(q) q - \tau(q). \end{aligned} \tag{9}$$

Since q represents the moments order of the distribution, the influence of q on the nature of the fluctuations can be described as follows as shown by [10]:

- Higher values of q indicate larger fluctuations. When q is large, the partition function $Z(q)$ is dominated by contributions from regions with higher values of the measured quantity (such as the probability p_i). Thus, large q values emphasize the impact of rare, significant fluctuations.
- Lower values of q indicate smaller fluctuations. When q is small, the contributions to $Z(q)$ are more evenly distributed, capturing more typical, smaller fluctuations.

This relationship also shows that each moment q corresponds to a particular value of α, characterizing the nature of the fluctuations in the system. By analysing $Z(q)$ across a range of q values, one can understand how fluctuations of different magnitudes contribute to the overall system, with each q highlighting a specific aspect of these fluctuations and their corresponding α.

Computing $\tau(q)$ from $Z(q)$ for each q between $]-\infty, +\infty[$ provides the full spectrum.

2.5 Multifractal Detrended Cross-Correlation Analysis

The Multifractal Detrended Cross-Correlation Analysis (MFD-CCA) is a method that evaluates the relationship between two time series, $x(i)$ and $y(i)$, where i ranges from 1 to N. The steps involved in this method are as follows and are based upon the works of Podobnik and Stanley [14], which were further expanded by Zhou [15].

Step 1 - Construct the profiles: The profiles $X(i)$ and $Y(i)$ are calculated by summing the cumulative values of each time series after subtracting their respective average, as follows:

$$\begin{aligned} X(i) &= \sum_{t=1}^{i}(x(t) - \bar{x}) \\ Y(i) &= \sum_{t=1}^{i}(y(t) - \bar{y}) \end{aligned} \tag{10}$$

Notice that if the two time series are equal, then MFD-CCA is the same as Multifractal Detrended Fluctuation Analysis (MF-DFA) [6].

Step 2 - Partition of the profiles into windows: The profiles $X(i)$ and $Y(i)$ are partitioned into $2N_s$ non-overlapping windows of equal length s. This is achieved by dividing each profile into $[N/s]$ windows and then repeating the same procedure starting from the reverse end of each profile.

Step 3 - Evaluate local trends: The local trends $X_\nu(i)$ and $Y_\nu(i)$ for each segment ν, where ν ranges from 1 to $2N_s$, are evaluated by performing a least squares fit on the data for each segment. The detrended covariance is then determined using the following equation:

$$F^2(s,v) = \frac{1}{s}\sum_{i=1}^{s}|((\nu-1)s+i) - X_v(i)||Y((\nu-1)s+i) - Y_v(i)| \tag{11}$$

for each segment ν such that $v = 1, 2, ..., N_s$.

$$F^2(s,v) = \frac{1}{s}\sum_{i=1}^{s}|X(N-(v-N_s)s+i) - x_v(i)||Y(N-(v-N_s)s+i) - y_v(i)| \tag{12}$$

for each segment ν such that $v = N_{s+1}, N_{s+2}, ..., 2N_s$. The trends $x_v(i)$ and $y_v(i)$ are represented by a fitting polynomial with order m.

Step 4 - Obtain the order q fluctuation: The order q fluctuation function is calculated as follows:

$$F_q(s) = \left[\frac{1}{2N_s}\sum_{v=1}^{2N_s}[F^2(s,v)]^{q/2}\right]^{1/q} \tag{13}$$

and, if $q \neq 0$, the q-th order fluctuation function would be

$$F_0(s) = \exp\left[\frac{1}{4N_s}\sum_{v=1}^{2N_s}\ln[F^2(s,v)]\right] \tag{14}$$

Note that when $q = 2$, the MF-DCCA is equivalent to a traditional cross-correlation analysis (CCA) [17].

Step 5 - Analyze the results: The scaling behavior of the fluctuations is analyzed by plotting logarithmic graphs of $F_q(s)$ versus s for each value of q. If the two series are long-range cross-correlated, then $F_q(s)$ will increase for larger values of s, and the relationship can be represented as a power law:

$$F_q(s) \sim s^{H_{xy}(q)} \tag{15}$$

which can be simplified into

$$\log F_q(s) \sim H_{xy}(q) \log(s) \tag{16}$$

Here, $H_{xy}(q)$ is known as the generalized cross-correlation Hurst exponent, which describes the power-law relationship between the two series.

Focusing now on $q = 2$, the covariance of the fluctuations of the multifractal behavior [18], which will be important during the analyses of the data. If $H_{xy}(q = 2) > 0.5$, the cross-correlation between the return fluctuations of the two series related to q are long-range persistent. If $H_{xy}(q = 2) < 0.5$, the cross-correlations between the return fluctuations of the two series are anti-persistent. If $H_{xy}(q = 2) = 0.5$, there is no cross-correlation between the two series.

The Renyi function $\tau_{xy}(q)$ is used to characterize the multifractal nature of the time series, which was previously defined as (4) now takes the shape of

$$\tau_{xy}(q) = H_{xy}(q)q - 1 \tag{17}$$

If the Renyi function $\tau_{xy}(q)$ is linear, we can conclude that the cross-correlation between the two series is monofractal, otherwise, it is multifractal.

As proposed by Yuan *et al.* [16], the strength of the multifractality, meaning the degree of multifractality present and it can be estimated with ΔH and it is given by

$$\Delta H = H_{max}(q) - H_{min}(q) \tag{18}$$

The multifractal spectra $f_{xy}(\alpha)$ could be derived through the Legendre transform [10,17]:

$$\alpha_{xy}(q) = H_{xy}(q) + qH'_{xy}(q) \tag{19}$$

$$f_{xy}(\alpha) = q(\alpha_{xy} - H_{xy}(q)) + 1 \tag{20}$$

where H'_{xy} denotes the first derivative of H_{xy} with respect to q.

Using the multifractal spectrum, one can measure its width to examine the multifractal level [19], given by:

$$\Delta\alpha = \max(\alpha_{xy}) - \min(\alpha_{xy}) \tag{21}$$

Using this parameters will be key to further understand the nature of the multifractal behavior, with a larger $\Delta\alpha$ indicating a more complex, varied structure, while a smaller $\Delta\alpha$ suggests a more uniform structure.

3 Empirical Results

3.1 Dataset

The dataset analyzed in this study consists of the spot price of Gold in dollars[1], the S&P500 index[2], the DAX40[3], the dollar index (DXY)[4], the Nikkei225[5] and the crude Oil commodity (WTI)[6]. For each of these assets, we calculate the logarithmic daily returns using the formula $r_t = ln(P_t/P_{t-1})$, where P_t is the closing price at time t and P_{t-1} is the closing price at time $t-1$. The choice of logarithmic returns over simple returns or raw prices is motivated by several factors. First, log returns are more likely to be additive over time and tend to be more normally distributed than simple returns, which makes them more suitable for statistical analysis. Second, unlike raw prices or cumulative returns which can exhibit strong trends and non-stationarity, daily log returns are more likely to be stationary, a crucial property that as we discussed before, is a necessary condition for the application of multifractal analysis.

All the price data comes from investing.com, and consists of daily prices from January 1st, 2006 to December 31st, 2022. The selection of daily frequency, as opposed to weekly or monthly data is that it captures more granular market movements and immediate responses to geopolitical events. This is particularly important as geopolitical events can have rapid, sometimes intraday impacts on financial markets. Since we are analyzing commodities, forex and indices from different countries, we encounter the problem where some days some assets may be tradable and others are not, due to national holidays or other issues. To mitigate this problem, we have normalised the data so we only analyze the assets on the same days. This is also important since we are going to be analyzing these assets against the Geopolitical Risk Index[7]. For the Geopolitical Risk Index, we also considered growth rates to ensure consistency with the return-based approach used for financial assets and to maintain stationarity in the data series, which is essential for our multifractal analysis framework.

Each asset in our dataset represents a different aspect of the global financial market:

- Gold (XAU/USD): Traditionally viewed as a safe-haven asset and hedge against geopolitical uncertainty
- S&P500: The benchmark U.S. equity index, representing the performance of the largest U.S. companies
- DAX40: The primary German stock index, representing Europe's largest economy

[1] https://www.investing.com/currencies/xau-usd.
[2] https://www.investing.com/indices/us-spx-500.
[3] https://www.investing.com/indices/germany-30.
[4] https://www.investing.com/indices/usdollar.
[5] https://www.investing.com/indices/japan-ni225.
[6] https://www.investing.com/commodities/crude-Oil.
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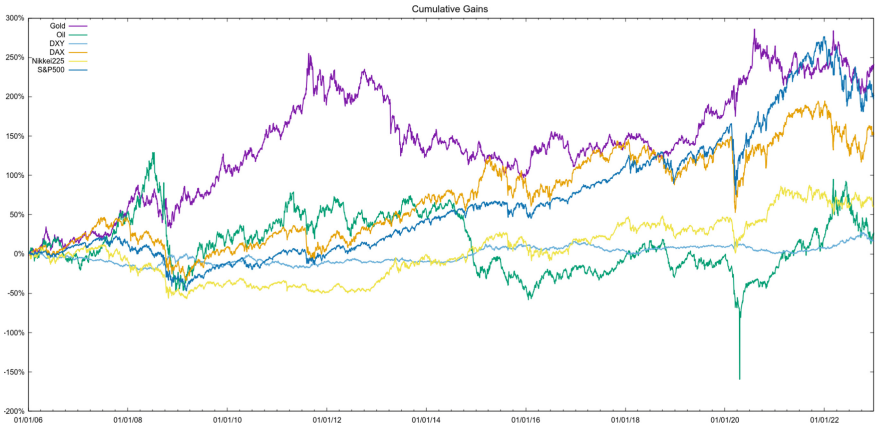

Fig. 1. Cumulative Gains of Assets

- Dollar Index (DXY): A measure of the U.S. dollar's strength against a basket of major currencies
- Nikkei225: Japan's primary stock index, representing the Asian market perspective
- Crude Oil (WTI): A key commodity highly sensitive to geopolitical events, particularly in oil-producing regions

The data from geopolitical risk is based on computerized text searches from the archives of ten newspapers (Chicago Tribune, The Daily Telegraph, Financial Times, The Globe and Mail, The Guardian, The Los Angeles Times, The New York Times, USA Today, The Wall Street Journal, and The Washington Post) [20]. It tracks the number of times per day that adverse geopolitical events took place and takes into consideration: war threats, peace threats, military buildups, nuclear threats, terror threats, beginning of war, escalation of war, and terror acts.

One can see from Fig. 1 that during the timeline being analyzed, Gold was indeed one of the best performing assets, with a return of 241.90%, beating the S&P500 and the DAX40 (formerly DAX30), which had a return of 202.61% and 154.98% respectively. Meanwhile, Oil and the DXY end up with the a return of 26.88% and 16.88% respectively while the Nikkei225 had a return of 59.44%

From a superficial level, by analyzing the data, one can also see that Gold had a significantly better performance during some periods of crisis, namely a very quick recovery during the 2008 financial crisis and the COVID19 pandemic, where, due to logistical reasons and lack of demand, Oil futures went into negative territories. One can also observe a stagnation between those 2 time periods while the DAX40 and S&P500 experienced an upward trend which, on the surface level, means that Gold was indeed outperforming the other indices during periods of crisis. However, one can also verify that Gold, along side the other

Table 1. Descriptive Statistic Analysis of prices from 2006 to 2022

	Mean	SD	Median	Min	Max	Skewness	Kurtosis	Jarque Bera
Gold	1292.979	364.5634	1281.52	525.05	2063.81	−0.18270	−0.69140	2204.459
DXY	88.22696	8.970505	87.555	71.33	114.11	0.187923	−0.91754	2481.376
S&P500	2125.139	1002.838	1894.47	676.53	4793.54	0.910085	−0.14018	2110.54
Nikkei225	17129.69	6102.594	16764.36	7054.98	30670.1	0.295492	−0.84842	2428.692
DAX40	9490.028	3159.364	9452.875	3666.41	16271.75	0.222918	−1.15141	2792.91
Oil	72.27448	22.77575	70.335	-37.63	145.29	0.233772	−0.42753	1917.147

Table 2. Descriptive Statistic Analysis of the daily returns from 2006 to 2022

2006–2022								
	Mean	SD	Median	Min	Max	Skewness	Kurtosis	Jarque Bera
Gold	0.000211	0.0112	0.000416	−0.08459	0.112227	−0.13669	6.629835	2122.831
DXY	0.0000033	0.004626	0	−0.02996	0.024038	−0.074	2.264713	90.12574
S&P500	0.000156	0.011181	0.000599	−0.08723	0.10789	−0.40375	9.706802	7310.828
Nikkei225	−0.00017	0.010833	−0.0001	-0.10025	0.123649	−0.37202	14.94745	22957.1
DAX40	−0,0000060	0.0111877	0.000408	−0.07125	0.117856	0.008130	6.616755	2095.7148
Oil	−0.00134	0.062373	0.00063	−3.12239	0.204604	−38.0117	1778.752	5.06E+08

assets, had a short crash immediately following said crisis, which puts in question the stability of Gold as a short term stable asset.

By doing a descriptive statistical analysis, Table 1, one can see that Gold is the only asset with a negative skewness, which indicates that the distribution of Gold prices is skewed to the left. This means that the tail of distribution is heavier on the left side. This means that Gold has a tendency for larger downward movements, however, since the value of skewness is not highly pronounced, the asymmetry is relatively small. This is the opposite for all other assets being analyzed. Furthermore, by performing the Jarque-Bera test, which is a statistical test that assesses whether the data has skewness and kurtosis matching a normal distribution, one can find that the Jarque-Bera values for the assets prices are all above the usual significance levels, we can see in this case testing for $\alpha = 0.05$, we can reject the null hypothesis that none of the pricing distribution follows a normal distribution.

By focusing now on the daily returns of the assets, Table 2, one can see that all assets but the DAX40 have negative skewness, meaning that they have more negative daily returns than positive returns. This indicates that the COVID19 crisis and the flash crash of Oil futures prices into negative territory had a significant impact in Oil returns. One can also observe that all assets' returns present positive kurtosis and that with the exception of DXY. The daily returns of each asset present leptokurtic distribution, with Gold and the DAX40 daily returns having the smallest kurtosis from the leptokurtic assets. This means that, although their distribution present heavier tails, compared to the other assets, they are less likely to present heavier fluctuations in daily returns. Once again, considering the value of the Jarque-Bera test, none of the daily returns of the assets follow a normal distribution.

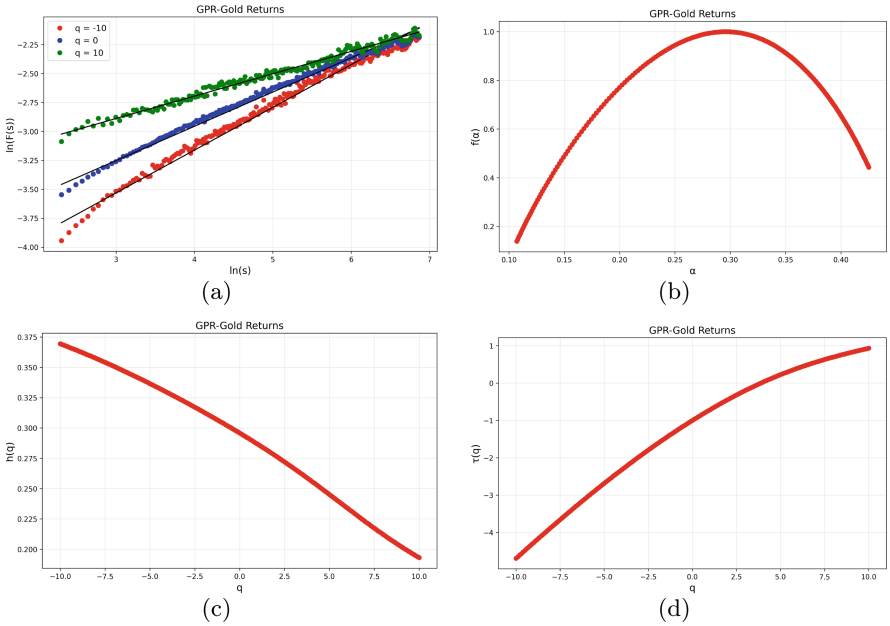

Fig. 2. MF-DCCA for geopolitical risk index and Gold

3.2 Multifractal Dentrended Cross Correlation Analysis

By employing now the MF-DCCA between each asset and the Geopolitical Risk Index, one can calculate the scaling exponents $h_{XY}(q)$ of q ranging from -10 to 10. Note that using those ranges, one can provide enough coverage to observe both the local and global scaling behavior. The main hypothesis being tested are:

H_1: Gold loses its safe haven attributes during times of increased Geopolitical risk

H_2: Its ability to offset downside risk is lower than the other assets

H_3: The cross-correlation between the different assets and the geopolitical risk index present multifractal properties.

Figure 2 represent a MC-DCCA analysis between the daily returns of gold and geopolitical risk. The figure shows: (a) the log-log of the fluctuation $F_q(s)$ versus s, (b) the multifractal spectrum, $f(\alpha)$ with dependencies for $q \in [-10, 10]$, (c) the generalized Hurst exponent H_{xy} and (d) the Reyni function τ, respectively. The code implemented in python was based on the Fathon library [21], a detrendend fluctuation analysis library that was modified to be used for the dataset we have. The same analysis was performed for the remaining assets.

From the results, the log-log plots demonstrate clear power-law scaling behavior between the $F_q(s)$ and the scale s for different values of q. Notably, the $q = -10$ line (red, representing smaller fluctuations) consistently shows a steeper slope than the $q = 0$ line (blue), which in turn has a steeper slope than the

$q = 10$ line (green, representing larger fluctuations). This pattern indicates that the scaling properties depend on the magnitude of the fluctuations, with smaller market movements exhibiting stronger scaling behavior.

The Hurst exponent, $H_{xy}(q)$, for all pairs is not constant and exhibits a systematic decrease as q ranges from -10 to 10. This declining trend indicates that the cross-correlation between geopolitical risk and financial assets shows stronger persistence for smaller fluctuations (negative q values) and weaker persistence for larger fluctuations (positive q values).

Since the Hurst exponent depends on q and is not constant, it confirms that the cross-correlation between the different assets and the geopolitical risk index presents multifractal properties, validating the third hypothesis. This conclusion is further supported by the multifractal spectrum $f(\alpha)$ plots, which show a clear inverted parabolic shape, and by the Renyi function $\tau(q)$, which exhibits nonlinear behavior, both characteristic signatures of multifractality in the cross-correlation structure.

In our MF-DCCA framework, the generalized Hurst exponent at $q = 2$, denoted as $H_{xy}(2)$, is particularly significant as it quantifies the degree of persistence in the cross-correlations. According to the formalism described before, values of $H_{xy}(2)$ below 0.5 indicate anti-persistent relationships, while values above 0.5 indicate persistent relationships. Examining Table 3 we observe that all $H_{xy}(2)$ values fall below 0.5, confirming anti-persistent cross-correlations. There is a tendency to invert the correlation between asset returns and geopolitical growth rates. Notably, Gold demonstrates more pronounced anti-persistence (0.277) compared to most other assets except S&P500 (0.274), with DXY

Table 3. Cross correlation generalized Hurst exponent of order q between Geopolitical Risk and different assets

q	GPR GOLD	GPR DXY	GPR SP500	GPR Nikkei225	GPR DAX40	GPR Oil
−10	0.36938978	0.38388519	0.35755061	0.37977183	0.38702615	0.38714654
−-9	0.36358157	0.37738052	0.35262529	0.37478466	0.38086403	0.38078975
−7	0.35079828	0.36309373	0.34150839	0.36368922	0.36692152	0.36718170
−6	0.34386611	0.35564659	0.33532631	0.35759318	0.35930896	0.36036595
−5	0.33660801	0.34829446	0.32876212	0.35115432	0.35148181	0.35389229
−4	0.32904682	0.34121358	0.32186352	0.34439859	0.34362745	0.34795635
−3	0.32119721	0.33447774	0.31469023	0.33735525	0.33588802	0.34262257
−2	0.31306135	0.32806651	0.30729518	0.33005053	0.32831591	0.33781699
−1	0.30462457	0.32189258	0.29968962	0.32249688	0.32085423	0.33335355
0	0.29585169	0.31582122	0.29180174	0.31467663	0.31334217	0.32896083
1	0.28668687	0.30968295	0.28346188	0.30652169	0.30554588	0.32430153
2	**0.27706266**	**0.30328708**	**0.27445828**	**0.29789942**	**0.29721907**	**0.31900112**
3	0.26692468	0.29644343	0.26467580	0.28862834	0.28819528	0.31271061
4	0.25627378	0.28900087	0.25424628	0.27855442	0.27849031	0.30521953
5	0.24521316	0.28090584	0.24357511	0.26768523	0.26835130	0.29658919
6	0.23396919	0.27225802	0.23319014	0.25629441	0.25819280	0.28719896
7	0.22285653	0.26330976	0.22353407	0.24487355	0.24844690	0.27761391
8	0.21219723	0.25438766	0.21485374	0.23394280	0.23943148	0.26836134
9	0.20224269	0.24579140	0.20721114	0.22387693	0.23130551	0.25978838
10	0.19313828	0.23773290	0.20054979	0.21485242	0.22409616	0.25205037

Table 4. Generalized Hurst exponents $H_{xy}(2)$ and multifractal strength ΔH [in brackets] for the cross-correlation between GPR and financial assets across different market regimes

Asset	$H_{xy}(2)$ [Multifractal Strength ΔH]				
	Pre-Crisis	Financial Crisis	European Debt Crisis	Post-Crisis	COVID Crisis
Gold	0.46104 [0.458]	0.40057 [0.305]	0.40467 [0.263]	0.35643 [0.223]	0.28488 [0.321]
DXY	0.42126 [0.779]	0.39836 [0.619]	0.42132 [0.137]	0.32571 [0.136]	0.32077 [0.353]
S&P500	0.24057 [0.550]	0.30776 [0.717]	0.36503 [0.245]	0.29020 [0.229]	0.46002 [0.286]
Nikkei225	0.44996 [0.638]	0.27941 [0.710]	0.43966 [0.295]	0.30576 [0.229]	0.35212 [0.111]
DAX40	0.39145 [0.468]	0.28287 [0.932]	0.46470 [0.115]	0.32400 [0.162]	0.41143 [0.352]
Oil	0.37580 [0.448]	0.36001 [0.653]	0.41165 [0.288]	0.37213 [0.092]	0.42463 [0.361]

(0.303), Nikkei225 (0.298), DAX40 (0.297), and Oil (0.319) all showing less anti-persistence. This stronger anti-persistence suggests that Gold exhibits more rapid mean-reverting behavior in response to geopolitical shocks, making it particularly effective as a short-term hedge where investors rush to Gold during crises but sell once the situation begins to stabilize. This rapid mean-reversion characteristic offers a different kind of protection than assets with less pronounced anti-persistence, which tend to show less anti-persistent directional movements in response to geopolitical events.

However, these broad patterns mask significant variations across different market regimes. To investigate this further, a sub-period analysis covering distinct economic phases: Pre-Crisis (2006–2007), Financial Crisis (2008–2009), European Debt Crisis (2010–2012), Post-Crisis (2013–2019), and COVID Crisis (2020–2021), was conducted. The sub-period analysis reveals significant temporal variations in the relationship between Gold and geopolitical risk across different market regimes. Table 4 presents the generalized Hurst exponents at $q = 2$ and the multifractal strength parameters across these periods.

The temporal analysis demonstrates that Gold's relationship with geopolitical risk is not constant but varies substantially across different market regimes. During the 2008–2009 financial crisis, Gold showed relatively stable multifractal properties compared to equity markets, supporting its safe haven reputation during financial turbulence. However, during the COVID crisis, Gold displayed heightened negative sensitivity to geopolitical risks, likely because unprecedented monetary interventions and the unique nature of the pandemic created market conditions where traditional relationships were distorted. Our findings align with Bentes, who applied MF-DCCA to CIVETS emerging markets during 2018–2023 and discovered that Gold's cross-correlations with financial markets shifted from negative before the pandemic to positive during the COVID-19 and Russia-Ukraine war crises, demonstrating that Gold lost its safe haven attributes during these extreme market conditions [19].

4 Conclusion

Gold is widely regarded as a safe-haven asset during geopolitical uncertainty [1]. Its intrinsic value, scarcity, and historical role as a store of wealth make it attractive during crises, offering stability and wealth preservation in turbulent times. Our research reveals this relationship is more nuanced and context-dependent than traditionally understood.

In order to understand these connections, we applied MF-DCCA to the daily returns of gold and from other assets, from the 1st of January 2006 to the 31st of December 2022. However, considering that the assets are from different countries and different sectors, the data had to be normalized for a fair comparison. Our research was guided by three central hypotheses: whether Gold loses its safe haven attributes during times of increased geopolitical risk (H1), whether its ability to offset downside risk is lower than other assets (H2), and whether the cross-correlations between assets and the geopolitical risk index present multifractal properties (H3).

This study found that all the assets we examined, when compared to geopolitical risk index, present long-range correlations, indicating the presence of multifractality, which supports our third hypothesis (H3). The non-linear dependency of the Hurst exponent on q, the inverted parabolic shape of the multifractal spectrum $f(\alpha)$, and the non-linear Renyi function $\tau(q)$ all confirm this multifractality. However, the strength of multifractality in the cross-correlation varies across all pairs. The complexity parameters (ΔH and $\Delta\alpha$) quantify this multifractality across all assets and time periods. All the assets showed anti-persistent cross-correlation with GPR, with Hurst exponent values below 0.5.

Regarding our first hypothesis (H1), our findings indicate it is partially true and significantly dependent on the nature of the crisis. During the 2008–2009 financial crisis, Gold maintained relatively stable multifractal properties compared to equity markets, supporting its traditional safe haven reputation. However, during the COVID crisis, Gold exhibited heightened negative sensitivity to geopolitical risks, with its Hurst exponent dropping to 0.28488. This aligns with recent research by Bentes, who found that Gold's cross-correlations with financial markets shifted from negative before the pandemic to positive during the COVID-19 and Russia-Ukraine war crises [19].

For our second hypothesis (H2), our analysis largely rejects the notion that Gold's ability to offset downside risk is lower than other assets, but for more nuanced reasons than initially hypothesized. In the full period analysis (2006–2022), Gold actually demonstrates more pronounced anti-persistence compared to most other assets (except S&P500), indicating more rapid mean-reverting behavior in response to geopolitical shocks. This stronger anti-persistence suggests that Gold functions as an effective short-term hedge where investors quickly move to Gold during crises but sell once the situation begins to stabilize. This rapid reaction characteristic differs from traditional views of Gold's stability but nonetheless confirms its special role during periods of geopolitical instability [1]. The pattern varies across different market regimes, however. During the financial crisis, Gold maintained a higher Hurst exponent (0.40057) than major

equity indices, suggesting more stable behavior during this specific type of market turbulence. Only during the unprecedented conditions of the COVID crisis did Gold's hedging dynamics change dramatically, with the S&P500 showing a remarkable reversal in behavior.

Our study found that Hurst values for all assets exhibited a systematic decrease as q ranges from -10 to 10. This declining trend indicates that the cross-correlation between geopolitical risk and financial assets shows weaker anti-persistence for smaller fluctuations and stronger anti-persistence for larger fluctuations. This pattern suggests complex market behavior where the scaling properties depend on the magnitude of the fluctuations.

A particularly important finding of our research is the significant temporal variation in the relationship between Gold and geopolitical risk across different market regimes. During the pre-crisis period (2006–2007), Gold exhibited a Hurst exponent of 0.46104, very close to 0.5, indicating an almost random walk relationship with geopolitical risk. This relationship evolved through various market regimes, with each period showing distinct patterns in the cross-correlations [18].

Overall, our research shows that gold is indeed generally one of the better hedges against geopolitical risk compared to other financial assets, except during the COVID crisis when traditional relationships were distorted [22]. Although this does not mean that it presented positive returns during times of uncertainty, but rather that its relative performance in relation to Geopolitical Risk was typically better than other assets.

In addition to the findings presented, it is essential to recognize that Gold price movements are influenced by factors beyond those captured in the GPR index. Interest rate expectations, central bank balance sheet expansions, inflation concerns, and currency debasement fears significantly impact Gold prices but are not explicitly measured in geopolitical risk metrics. During 2020-2021, Gold reached all-time highs despite showing strong anti-persistent behavior with geopolitical risk, suggesting dominant influence from monetary policy expectations and inflation concerns.

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