The Effects of Localised Damage on the Structural Stability of Masonry Arches

Luciana Di Gennaro(✉), Mariateresa Gaudagnuolo, and Giorgio Frunzio

Department of Architecture and Industrial Design, University of Campania Luigi Vanvitelli, Aversa, CE, Italy
luciana.digennaro@unicampania.it

Abstract. Masonry arches represent a widely adopted structural solution in structure and infrastructure systems, including bridges, viaducts, and architectural constructions. They often exhibit signs of degradation due to ageing, insufficient maintenance, and exposure to natural actions, such as seismic events, soil settlements, and material deterioration. Such factors can significantly compromise structural stability, making the assessment and preservation process more complex. The present study investigates the stability of masonry arches subjected to horizontal loads, with particular attention to the influence of localised damage on the collapse multiplier across different geometric configurations. Arches with different geometry were analysed, ranging from elliptical arches with a quarter-span rise to semicircular (full-centre) arches. Damage was modelled by introducing localised reductions in thickness along the entire intrados to evaluate its impact on the structural response. Moreover, multiple thicknesses were considered for each geometry to assess slenderness's role in undamaged and damaged conditions. The findings identify the most critical damage locations regarding structural capacity loss under the applied loading configuration and provide useful insights for the preliminary assessment and conservation of historic masonry arch structures, particularly in contexts where rapid and simplified evaluations are required.

Keywords: Masonry arch · Collapse mechanism · Limit analysis

1 Introduction

Masonry arch bridges are a significant portion of the infrastructural heritage across Italy and Europe, bearing testimony to the evolution of civil engineering and architectural practice. Since antiquity, efforts have been made to understand and optimise arch geometries to enhance structural performance, developing numerous analytical theories. From the seminal treatises of De La Hire, Mascheroni, and Méry to the mechanical theories of Heyman and Benvenuto [1–5], significant progress has been made in modelling the behaviour of masonry arches. In particular, Heyman's application of limit analysis principles remains a cornerstone in the field [6, 7].

While modern numerical techniques such as the Finite Element and Discrete Element Methods offer accurate assessments [8–10], they require extensive data collection

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and considerable computational effort, making them less suitable for rapid assessments [11, 12]. Consequently, limit analysis methods—particularly those based on the kinematic theorem—remain valuable for preliminary evaluations, as they demand fewer input parameters while still providing meaningful insights into structural stability [13–19].

Recent experimental investigations have offered valuable insights into the structural behaviour of masonry arches, particularly for assessing their residual strength, identifying failure modes, and evaluating the influence of material degradation on collapse mechanisms [20–23].

A critical factor in such assessments is the presence of defects and damage, often caused by environmental or anthropogenic actions [24, 25]. These can lead to significant, yet difficult to predict, performance losses [26–28], particularly in seismic regions. Although modern surveying techniques enable accurate geometric documentation [29], detailed structural characterisation may be unfeasible in large-scale or urgent assessments [30–32]. In this context, modelling damage as a localised reduction in thickness has emerged as a feasible and effective approach within limit analysis frameworks [33].

This study investigates the structural capacity of masonry arches under horizontal loading through a two-dimensional limit analysis approach. The analysis focuses on elliptical arches with a horizontal major axis and variable geometries. A parametric study is conducted by varying both the rise and the thickness of the arch to explore the influence of these geometrical parameters on structural stability. Damage is modelled as a localised reduction in intradossal thickness, and collapse multipliers are computed to evaluate the loss of load-bearing capacity in each geometrical condition. The proposed methodology offers a rapid yet effective way of assessing damaged masonry arches' structural vulnerability, supporting conservation and maintenance strategies.

2 The Geometry of the Arch

The arches analysed in this study were idealised as two-dimensional structures with intrados profiles derived from a single geometrical family: elliptical curves with a horizontal major axis. This formulation allows for a continuous and systematic variation of the rise-to-span ratio while maintaining the typical structural and geometric characteristics of historical masonry arches. The intrados geometry was defined by the upper half of an ellipse governed by the equation:

$$\frac{x^2}{a^2}+\frac{y^2}{b^2} = 1, x \in [-a, a] \tag{1}$$

where $a = L/2$ is the semi-span, and $b = h$ represents the rise. The span L was kept constant in all analyses, while the rise h was parametrically varied to explore different arch shapes. A rise equal to half the span ($b = h = L/2$) yields a semicircular profile, which corresponds to the special case commonly referred to as a full-centered arch.

In all configurations, the arch was assumed to have a constant thickness t along the profile. This thickness was also treated as a variable parameter in the analysis, in order to investigate its influence on the structural response under horizontal loading.

The elliptical parametrisation here adopted ensures a smooth and symmetric geometry, suitable for discretisation into rigid blocks and fully compatible with the assumptions

of kinematic limit analysis. Moreover, it enables a unified treatment of both flatter and more curved profiles within a consistent mathematical framework.

3 Background and Theoretical Framework

The mechanical behaviour of masonry arches has long been studied through simplified models that capture their essential structural characteristics. Among these, Heyman's theory [3, 34] provides a foundational framework by idealising masonry as a material with no tensile strength, infinite compressive strength, and no sliding between blocks. Despite its simplicity, the model captures the essential features of masonry behaviour and has been widely employed and further developed in recent decades [35, 36]. These assumptions naturally lend themselves to the framework of limit analysis, which allows the evaluation of the ultimate load-bearing capacity without detailed stress computations. In particular, the kinematic theorem of limit analysis states that a structure is unstable if a compatible collapse mechanism exists for which the internal energy dissipation is lower than the external work [37–39]. This principle forms the basis of the kinematic approach, which is especially suited to the study of masonry as an assemblage of rigid blocks.

Through the kinematic method, possible failure mechanisms can be explicitly modelled, and upper-bound estimates of collapse loads can be obtained by evaluating virtual work equilibria.

Within the framework of limit analysis, damage may be introduced through geometric simplifications, such as a localised reduction in thickness, which has been adopted in several recent studies as a pragmatic proxy for crack patterns, voids, or material loss. This modelling strategy enables evaluation how damage influences structural capacity while maintaining computational efficiency. Despite its simplicity, the kinematic approach remains a valuable tool for preliminary assessment, particularly when limited geometric and material data are available. It provides a rational basis for early-stage engineering decisions and for prioritising further detailed investigations.

4 Methodology

The structural behaviour of the masonry arch is investigated under a system of increasing horizontal loads superimposed on the constant self-weight of the structure. The horizontal actions are expressed through a dimensionless load multiplier, denoted by λ, which governs the proportional increase in lateral loading.

According to established literature [40–42], this loading scenario typically induces the development of a four-hinge failure mechanism, which is characteristic of masonry arches subjected to dominant horizontal forces (Fig. 1). The arch is divided into four rigid blocks, separated by plastic hinges. The first block, adjacent to the springing on the left, remains constrained at the support, while the remaining three blocks rotate about the hinges as the mechanism evolves. Hinges typically form in a defined sequence: the first and third at the intrados and the second and fourth at the extrados.

Experimental and analytical studies have shown that the angular positions of the hinges—denoted as θ_1, θ_2, and θ_3—tend to fall within specific intervals, which depend

on the geometry and loading conditions of the arch [41]. These admissible ranges are defined as:

$$\Omega_{\theta 1} = [15°,\ 75°], \Omega_{\theta 2} = [30°,\ 90°], \Omega_{\theta 3} = [90°,\ 165°] \tag{2}$$

The kinematics of the collapse mechanism are fully described by a Lagrangian parameter, α, which governs the relative rotation of the rigid blocks. Each block rotates around a defined centre—C_1, C_2, and C_4 for the first, second, and third block, respectively—as illustrated in Fig. 1.

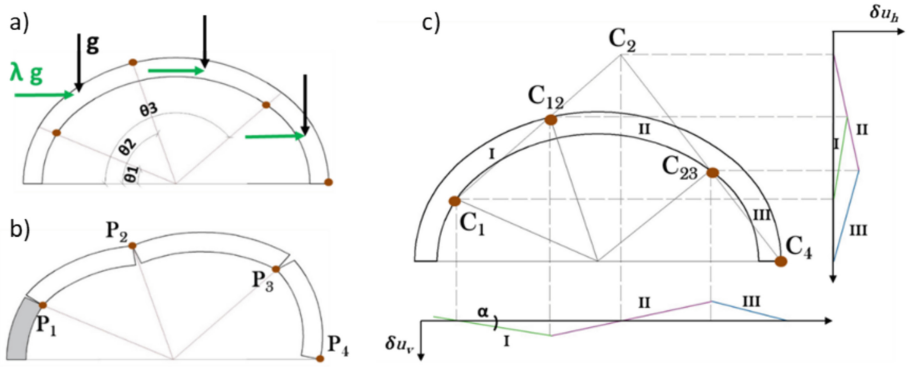

Fig. 1. a) Load condition and angular hinge position; b) Schematic representation of the four-hinge mechanism; c) Collapse mechanism and displacement profiles.

The structure remains in equilibrium under the load system $\mathbf{g} + \lambda\mathbf{g}$, where g denotes the self-weight and $\lambda\mathbf{g}$ the increasing horizontal forces, provided the Principle of Virtual Work is satisfied. This condition is expressed as:

$$\mathrm{g},\ \delta u_{\mathrm{v}} + \lambda g,\ \delta u_{\mathrm{h}} \leq 0;\ \forall\, \delta u \in \mathrm{M} \tag{3}$$

At the onset of collapse, the system reaches a limit state where the virtual work of external actions vanishes along the kinematically admissible mechanism **M**, leading to the equality:

$$\mathrm{g},\ \delta u_{\mathrm{v}} + \lambda g,\ \delta u_{\mathrm{h}} = 0;\ \forall\, \delta u \in \mathrm{M} \tag{4}$$

From this, the collapse multiplier λ can be computed explicitly as:

$$\lambda = -\frac{\mathrm{g},\ \delta u_{\mathrm{v}}}{\mathrm{g},\ \delta u_{\mathrm{h}}} \tag{5}$$

The multiplier λ is a function of the hinge angles (θ1, θ2, θ3). Each block's geometry, mass and centre of gravity are calculated for each admissible triplet, allowing for the corresponding value of λ to be evaluated. According to the kinematic (upper bound) theorem of limit analysis, the critical multiplier corresponds to the minimum value of λ across all admissible configurations, beyond which equilibrium cannot be maintained.

A constrained optimisation problem is formulated to identify this minimum. The domain of admissible configurations Ω is defined as the Cartesian product of the angular intervals:

$$\Omega = \Omega_{\theta 1} \times \Omega_{\theta 2} \times \Omega_{\theta 3} \tag{6}$$

Further constraints are imposed to ensure the physical plausibility of the mechanism, notably:

$$\theta 2 - \theta 1 \ \geq \ 15^{\circ}, \ \ \theta 3 - \theta 2 \ \geq \ 15^{\circ} \tag{7}$$

The resulting admissible set $\overline{\Omega}$ is thus defined as:

$$\overline{\Omega} = \begin{cases} \Omega = \Omega_{\theta 1} \times \Omega_{\theta 2} \times \Omega_{\theta 3} \\ \theta 2 - \theta 1 \geq 15^{\circ} \\ \theta 3 - \theta 2 \geq 15^{\circ} \end{cases} \tag{8}$$

The collapse multiplier is then obtained by solving:

$$\lambda \ = \min_{(\theta 1, \theta 2, \theta 3) \forall \overline{\Omega}} \lambda \, (\theta 1, \ \ \theta 2, \ \ \theta 3). \tag{9}$$

This procedure allows for systematically identifying the most unfavourable hinge configuration, providing an upper-bound estimate of the load capacity under horizontal actions.

4.1 Analysis Parameters

A parametric analysis was performed by varying the rise (h) and the thickness (t) of a masonry arch with a fixed span ($L = 3$ m) to evaluate the influence of geometry on the collapse multiplier λ. The applied load condition corresponds to the increasing horizontal forces system described in Eq. 9.

The rise h, defined as the vertical distance between the crown and the springing line, was varied between a quarter of L and half of L, discretised into five equally spaced values, each of them representative of typical historical configurations:

$$h_1 = \frac{1}{4}L; \ h_2 = \frac{5}{16}L; \ h_3 = \frac{3}{8}L; \ h_4 = \frac{7}{16}L; \ h_5 = \frac{L}{2} \tag{10}$$

The arch thickness t was defined according to the empirical formula suitable for small-span bridges [41]:

$$t = A \left(1 + \sqrt{L}\right) \tag{11}$$

where the coefficient A ranges from 0.15 to 0.18. Five discrete thickness values were derived accordingly.

For each (h, t) combination, the collapse multiplier was computed through constrained minimisation over the hinge angle domain $(\theta 1, \theta 2, \theta 3)$, discretised with 5° increments and subjected to geometric constraints. The procedure was implemented in Wolfram Mathematica© [43], allowing for systematic evaluation of λ across all admissible configurations.

4.2 Damage Modelling

Localised damage was introduced into the model using three key parameters:

- Thickness Reduction (I): the defect was modelled as a uniform local decrease in thickness, set to one-third of the original value.
- Damage Extension (E): the angular extent of the damaged region was kept constant along the intrados, with a fixed radial spread equal to 15°.
- Damage Position (P): the position of the defect was defined by its initial angular coordinate φi, which varied from 15° to 165° in 5° increments, resulting in 31 discrete locations.

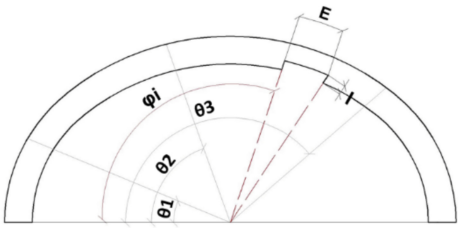

Fig. 2. Representation of damage

This parameterisation enabled the systematic assessment of how defect placement affects the collapse multiplier λ in arches subjected to horizontal loading (Fig. 2).

In this context, the reduction in thickness is intended to represent a localised loss of material at the intrados, such as that caused by erosion, detachment, or mechanical degradation [44, 45]. The assumption enables a simplified yet physically meaningful representation of damage effects within the analytical framework.

5 Results

5.1 Undamaged State

The collapse behaviour of the undamaged masonry arch was examined by evaluating the collapse multiplier λ across a range of geometric configurations. The analysis was conducted with a constant span $L = 3$ m and considered systematic variations of the rise (h) and thickness (t), as previously defined. The masonry was assumed to have a unit weight $\gamma = 16$ kN/m3.

The objective was to assess the sensitivity of the structural capacity to these parameters and to investigate the potential presence of nonlinear trends in the collapse response. Table 1 reports the computed values of λ for each (h, t) combination.

The results reveal that increasing the thickness t leads to a systematic rise in the collapse multiplier λ across all configurations, thereby confirming that thicker arches are more resistant to horizontal forces. In contrast, an increase in the rise h is associated with a reduction in λ, indicating that flatter arches perform better in terms of structural capacity under lateral actions. Although the effect of increasing thickness appears

Table 1. Collapse multiplier for each (h, t) combination – undamaged state.

	t_1	t_2	t_3	t_4	t_5
h_1	0.482	0.539	0.593	0.644	0.692
h_2	0.334	0.375	0.414	0.450	0.486
h_3	0.253	0.285	0.315	0.342	0.370
h_4	0.204	0.231	0.253	0.276	0.299
h_5	0.171	0.193	0.214	0.233	0.251

approximately linear, its influence is more pronounced in arches with lower rise. As the rise increases, the benefit of additional thickness becomes less significant, suggesting a nonlinear interaction between the two parameters (Fig. 3).

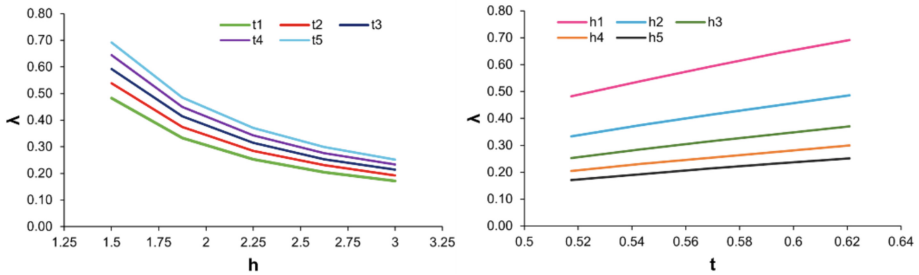

Fig. 3. Undamaged condition. a) h versus λ graph, b) t versus λ graph

5.2 Damaged State

The influence of localised damage on the collapse behaviour of masonry arches was assessed by evaluating the collapse multiplier λ as a function of the damage position φi for each combination of rise h and thickness t. Figure 4 presents the results for the low-rise cases, h_1 and h_2, with each curve corresponding to a different thickness (t_1–t_5).

The trends highlight a clear dependence on both geometry and damage location. λ remains relatively high and exhibits moderate variation across the angular domain, indicating a limited sensitivity to damage position. The worst conditions appear near the springs (φi equal to 15° and 165°) and in the right part of the arch (φi equal to 135° for h_1 and 130° for h_2).

As the rise increases (h_3 to h_5), the influence of damage becomes more pronounced and spatially localised (Fig. 5). In particular, a significant reduction in λ is often observed around $\varphi \approx 115°$–$125°$, depending on the geometry. These angular positions appear to be especially critical in triggering collapse mechanisms, as indicated by the sharp drop in load-carrying capacity.

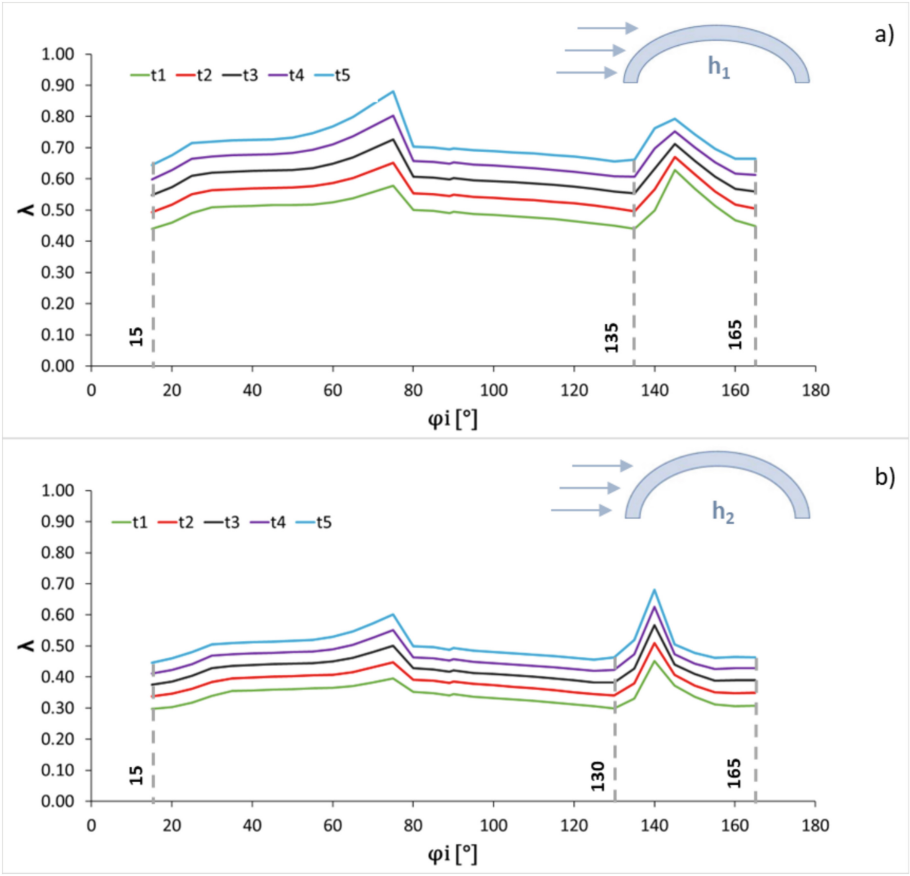

Fig. 4. Collapse multiplier λ as a function of the damage starting angle φi: a) h_1; b) h_2

Interestingly, for φi values immediately following this range (e.g. 135°–145°), λ tends to increase again, suggesting that the arch is less sensitive to damage located in those sectors. The graphical results confirm that even small shifts in the position of the damaged zone can lead to notable variations in structural response. The influence of thickness t is present across all cases, with higher thickness values generally associated with increased collapse multipliers. However, this effect becomes less marked as the rise increases: the curves for different t values tend to converge, suggesting that geometry becomes the dominant factor in structural performance in high-rise configurations.

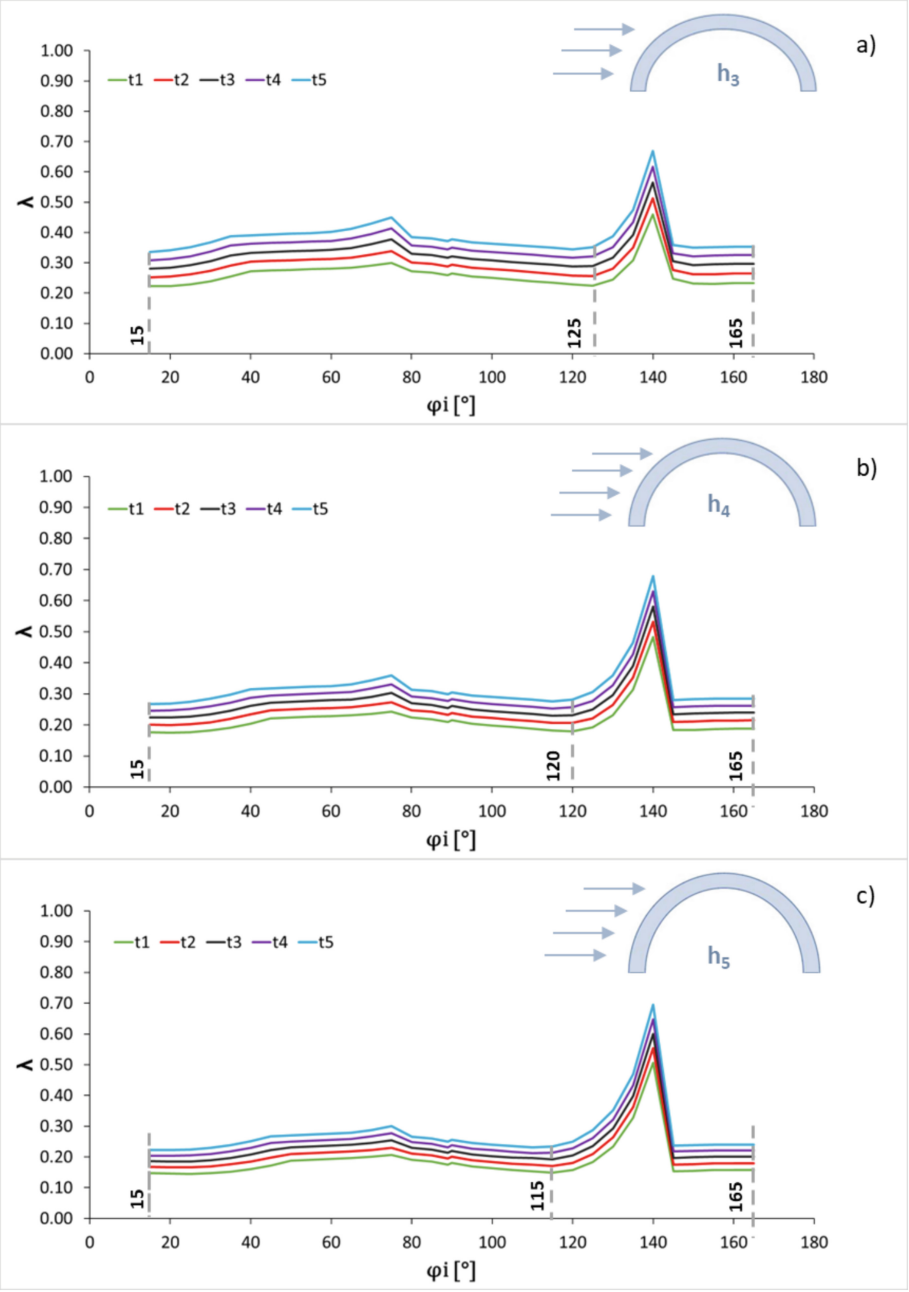

Fig. 5. Collapse multiplier λ as a function of the damage starting angle φi: a)h_3; b) h_4; c) h_5

6 Discussion and Conclusion

This study investigated the stability of masonry arches subjected to horizontal loads, with a specific focus on the effects of localised damage. A parametric approach was adopted to examine how variations in arch geometry, i.e. the rise h, the thickness t, and angular position of damage φi, influence the collapse multiplier λ. Both undamaged and damaged configurations were analysed, allowing for a direct comparison of their structural responses.

In the undamaged state, the analysis confirmed that increasing the arch thickness improves structural capacity, while higher rise values reduce the collapse multiplier. This inverse trend reflects the superior efficiency of flatter arches in resisting horizontal actions. The interaction between rise and thickness is nonlinear: the beneficial effect of thickness becomes less pronounced as the geometry becomes more slender. In the damaged condition, the sensitivity of λ to the angular position of the defect was systematically assessed. The results show that the structural response is strongly dependent on the exact location of the damaged zone. For lower-rise configurations, the variation of λ across the angular domain is relatively smooth, suggesting a more distributed structural response. In contrast, in higher-rise arches, a sharp drop in ultimate capacity is observed when the damage begins within specific angular intervals—particularly between $\varphi \approx 115°$ and 130°, depending on the geometry. Interestingly, neighbouring angular positions, such as $\varphi \approx 135°–145°$, result in significantly lower reduction of λ, highlighting the importance of precise localisation in damage modelling.

Thickness continues to influence the structural behaviour across all cases; however, its relative impact diminishes with increasing rise as the ratio h/L becomes the dominant factor. These findings suggest that while increasing cross-sectional dimensions may be beneficial, its effectiveness is conditional on both the global shape of the arch and the location of potential damage.

From a practical perspective, the outcomes of this study underscore the importance of assessment strategies that consider not only the extent but also the precise angular position of damage. In particular, for arches with greater rise, localised reductions in thickness—when occurring in structurally sensitive zones—may result in substantial losses in load-bearing capacity. Based on limit analysis and parametric investigation, the methodology adopted provides a rational and computationally efficient tool for the preliminary evaluation of structural vulnerability in masonry arch systems.

Disclosure of Interests. The authors have no competing interests to declare that are relevant to the content of this article.

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