Geometrical Proportioning of Masonry Arch Bridges

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Abstract. The design of masonry arches has been based for centuries on empirical rules, and many of the arch bridges currently in use were designed and built according to geometric proportions. All the elements of the bridge, thickness at the springer and at the crown, dimensions of the abutments, height of the abutments, were defined according to methods that were reported for the first time in Renaissance treatises. The structure of geometric relations took on increasingly complex forms in relation to the development of mathematical thought of the time. This article presents a reasoned catalogue of empirical formulas reported in treatises from the Renaissance to the first decades of the twentieth century, together with an evaluation of their reliability using modern calculations.

Keywords: Masonry arch · Collapse mechanism · Empirical rules

1 Introduction

Since the earliest civilizations masonry arches have been the solution to bridge a gap. Before the development of static methods, geometrical rules and empirical methods have been the only source of design methods for arch bridges [1]. Despite their design did not envisage actual heavy traffic loads, several of these bridges are, at this time, a "live" part of the railway and road network. Excellently designed bridges, nowadays in service, can be found everywhere in Europe. For local authorities in charge of masonry arch bridges still in service it is a key problem to understand the "rules" intended in the last centuries to estabilish the structural shape of the bridge [2-4]. The problem of stability has become a key issue in case of cultural heritage bridges located in seismic areas [5]. To address these needs, a historical-critical analysis taking into account the design methodology can be the first step toward a complete understanding of the construction and a correct methodological approach for planning minimum restoration interventions [6-7]. Thickness at the springing and at the crown, together with the dimensions of abutments, have been determined in the past centuries by means of empirical methods [8-12], based on simple geometrical relations. They were aimed at providing both the dimensions of several bridge components (i.e. span, rise and thickness of the arch, width and height of piers and abutments) and the safety of the structure based on the observation of existing bridges and past experience [13]. The empirical rules continued to be employed until the first half of the 20th century, when the developments of statics and continuum mechanics were consolidated [14] and already in use for framed structures.

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Essential parameters like weight and stiffness of the structure (due to arch, fill and spandrels), strength of the materials and problems related with the stability of the arch (snap-thorough) were not considered [1]. In every case, the geometrical approach was widely used and the results of these empirical formulae are in several cases masonry arch bridges actually in perfect conditions, playing until today their role in the traffic network. Recent researches have pointed out the strong link existing among modern formulation and pre-elastic theories, laying stress on the fact that *"the statics of the arch finally escaped from the labyrinth of empirical methods and theoretical conjectures*" [15]. On the contrary, it can be shown that several successful analysis methods, based on limt analysis for masonry arches, do not take into account the "essential parameters" named above.

The most popular and the first "mechanism method", proposed by Heyman in the second half of last century [16-19], is mainly based on the hypothesis of infinite compressive strength, null tensile strength and no-sliding among the voussoirs. Recent researches have in fact assessed that the material strength is not a key point in a stability analysis of an arch bridge [20], even in case of damage [21-22] since major cause of global failure can be envisaged as loss of equilibrium rather than the material failure. In many cases in fact the collapsed blocks are in perfect condition and restoration can be done merely by rebuilding, like in the Mostar bridge. The Heyman approach has recently been applied to the analysis of vaults, with successful and elegant formulations [23-26].

The search for the best shape of the arch that stimulated architects and engineers in the last centuries has produced as a result a lot of empirical formulae whose reliability could be evaluated taking into account the development of assessment methods [27-28]. The most usual method of verifying the adequacy of an arch proportioned according to these empirical methods has been made in the 19th century by means of the static procedures at that time consolidated and based on the line of thrust position. These methods have been successively extended to threedimensional surfaces, in order to analize masonry vaults [29-30]. In every case, a large part of empirical rules, although of doubtful mechanical orgin, has shown to be efficient [31].

In general the empirical approach calculates the crown thickness as a function of the span or the rise, according to the shape chosen for the intrados of the arch (circular, segmental, elliptical [31-34]. The analytical structure of the relations ranges from the simple linear relation proposed since the Renaissance [35] to the complex one given by Baker at the beginning of 20th century [3], according to the development of mathematics. A large part of these relations are reported in the treatises of the time, together with the examples effectively built by the author. Several of them are included in the major part of the tecnical manuals diffused among the engineers until the first half of last century [36-38], even in the code of practice by the Italian National Road Company (A.N.A.S.).

This paper presents a reasoned catalogue of empirical formulas reported in the treatises from the Renaissance to the first decades of the 20th century, together with an evaluation of their reliability by means of modern calculations.

2 Empirical relations

The first step in designing a masonry bridge was the definition of the number of arches and the intrados curve, generally based on hydraulic requirements, for the passage of the water and/or the boats, if the bridge spanned a river or stream [39].

The aestethic and the stability requirements were strongly connected to these choices, considering that "...masonry bridges are more beautiful. They are more stable. They have durability. They are simple, both to design and to construct" [12].

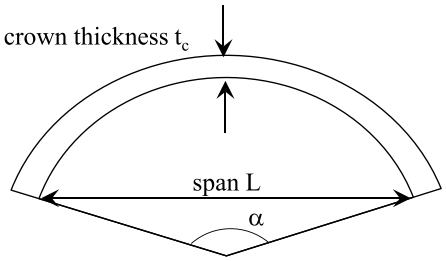

Fig. 1. Geometry of the masonry arch

The following step was the determination of the thickness at the crown, i.e. the depth of the keystone. Great part of the relations express the the crown depth (t_c) as a function of the span L or the radius R of the circle passing through the crown. Some of them are reported in Table 1 [9-11, 40-43].

As it will be shown in the following, the values of thickness obtained by the above relations 1 and 2 are too large for large span, unsafe for small span, while the relations 10 give values of smaller thickness for low-rise arches than for semicircular ones, as a further indication that the increase of flatness do not influence the bridge behaviour.

In this last ones in fact the influence of the rise is totally absent, so that buckling phaenomena for low-rise arches like the snap-through are not taken into account.

A picture of the variation of the crown thickness versus the span length is reported in Fig 2.

The decrease of the arch thickness over the years, especially for the large span arches, can be recognized. This is surely due to the development of masonry techniques and to the increasing trust in the materials strength, thanks to the diffusion of results of laboratory tests.

The red dots in Fig. 2 refer to real masonry bridges proportions, reported by Baker [3]. As it can be noted, several of them, actually in service, present crown thickness in the range under the Perronet formula.

The only particular case of a very low thickness is the footbridge at Pontypridd, Wales. It is not a coincidence that the minimum crown thickness is that of a bridge subjected to its own weight and limited traffic loads, the pedestrian only.

The dark blue dot, which seems in agreement with the Baker formula, refers to the Roman arch bridge in Pont Saint Martin, Italy, a shallow circular footbridge actually in perfect conditions (Fig. 3, left). The light blue dot refers to another Roman arch

Table 1. Empirical formulae for crown thickness.

	te	Author	Deep arch	Low-rise arch
1	XV c.	L.B. Alberti	$t_c = L/10$	
2	XVII c.	(before Perronet)	$t_c = L/15$	
3	1756	Perronet	$t_c = 0.325 + 0.035\ L$	$t_c = 0.325 + 0.0694$ R
4	1809	Gauthey (L<16m)	$t_c = 0.33 + 0.021\ L$	
5	1809	Gauthey (16m<L<32m)	$t_c = 0.042\ L$	
6	1809	Gauthey (L>32m)	$t_c = 0.67 + 0.021\ L$	
7	1826	Léveillér	$t_c = 0.333 + 0.033\ L$	
8	1828	Déjardin	$t_c = 0.3 + 0.045\ L$	
9	1865	Rankine	$t_c = 0,191\sqrt{R}$	
10	1855	Lesguiller	$t_c = 0.10 + 0.20\sqrt{L}$	
11	1870	Dupuit	$t_c = 0.20\sqrt{L}$	$t_c = 0,15\sqrt{L}$
12	1885	Croizette-Desnoyers	$t_c = 0.15 + 0.17\ R^2$	
13	1914	Séjourné	$t_c = 0.15 + 0.15\sqrt{2R}$	
14	1920	Baker	$t_c = 0.138\sqrt{R + \frac{L}{2}} + 0.061$	

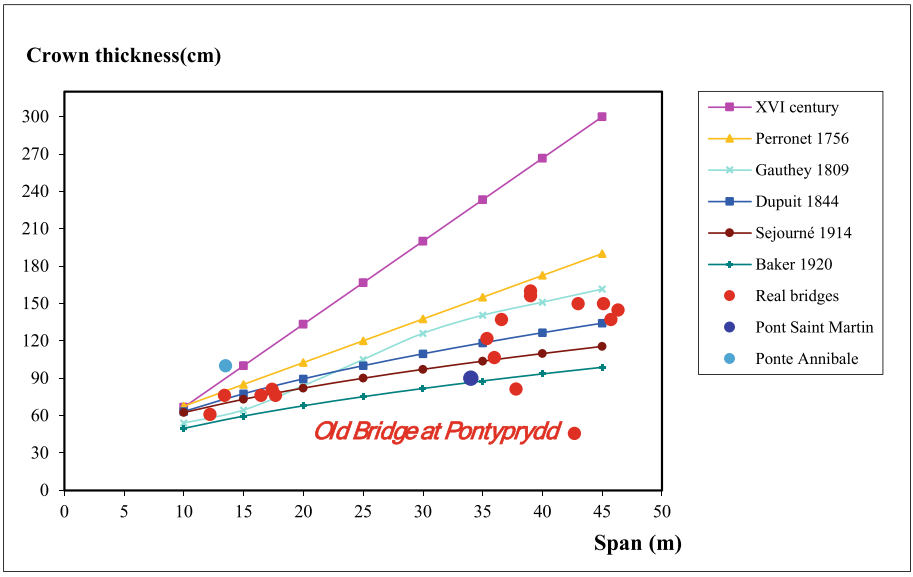

Fig. 2. Variation of the crown thickness with the span

bridge, in San Mango sul Calore, Italy (Fig. 3, right), whose proportions are far above the Renaissance relation.

A small non-scientific note: Roman engineers designed arch bridges according the local workmanship and material availability, so that it is impossible to force their proportions into relations that do not take into account the set of aspects Roman engineers considered. On the contrary, the bridges designed from the Middle Ages onwards were proportioned according to the geometrical relations reported in the treatises of the time.



Fig. 3. The Roman arch bridges in Pont Saint Martin (left) and San Mango sul Calore (right)

In general, bridges with low span show a constant thickness of the arch, while large span bridges present depth increment at springing, and empirical rules for the design of the bridge complete geometry can be found in the same treatises.

According to the Sejourné treatise [12], depth at springing in low-rise arches (angle $\alpha \leq 120°$ in Fig. 1) can be expressed in function of the crown thickness as:

$$t_a = t_c\,(1 + 12\,n^2)$$

where t_a is the springing depth, t_c is the crown thickness, n is the height at springing with respect to the intrados curve center, so that:

$$n \leq \frac{1}{2\sqrt{3}}\ (\alpha \leq 120°)$$

while the thickness at springing for deep arches can be designed according the following relation [12]:

$$t_a = t_c\,(1 + 2\,n)$$

It should be underlined that the thickness calculated according to the previous relation can be given, according to Sejourné, to the sections corresponding to α=120°, since, for Sejourné and other French authors, the remaining part of the bridge does not behave like an arch, due in general to the strong backing of the abutments. In every case, with the development of structural mechanics the geometrical approach was not abandoned, but used as predimensioning for design basis [31], until the first half of last century.

3 Kinematic analysis

In the following, a kinematic sensitive analysis has been conducted on semicircular masonry arches designed according the empirical formulae above reported.

The analysis of the arch has been perfomed according classical limit analysis, considering masonry a no-tension material with infinite compressive strength and infinite friction.

The collapse occurs for loss of equilibrium, due to the formation of hinges in the sections where the thrust line touches extrados and intrados curve.

The arch has been considered subjected to the selfweight (supposed constant in all the analyses) and a single point force whose position is defined by the ratio a/L (see Fig. 4).

In all the diagrams the minimum value of the force F that corresponds to incipient collapse (four-hinge mechanism) is reported, in function of the ratio a/L.

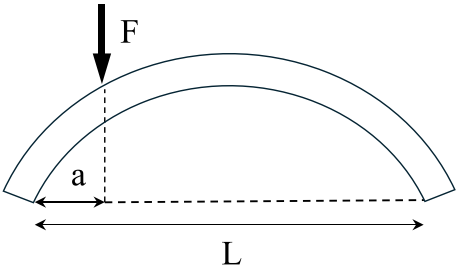

Fig. 4. Position of the point load on the arch

The analysis examines bridges of different spans, proportioned with the empirical relations reported in Table 1, starting from the Renaissance-type formula (Fig. 5). Figures 6-8 make reference to the remaining relations.

Since the Séjourné formula gives a reasonable factor of safety, while the other relations give an overestimation of the crown thickness, especially for large span, all the analyses are related to the upper limit of the Séjourné formula, reported in Figs. 5-8 with red line, in order to have a direct comparison with the famous XIX century formula.

The diffused trust in the geometrical approach should be probably found in this oversizing of the masonry structure.

The safety factor evaluable from the Séjourné formula is somewhat independent on the arch span, as it can be deduced by Fig. 8, while significant differences can be observed in the previous relations.

A summary of the analyses is reported in Fig. 9, in which an analysis of one of the two Roman masonry arch bridges considered, the one with the lower crown/span thickness ratio, is compared with the results of the sensitive analysis. As it can be easily seen, the safety degree obtained in this case is comparable with those obtained by means of the more ancient formulae.

This confirms that the geometrical approach, based on the observation of the realized structures, gives smaller dimensions as time passes by [44].

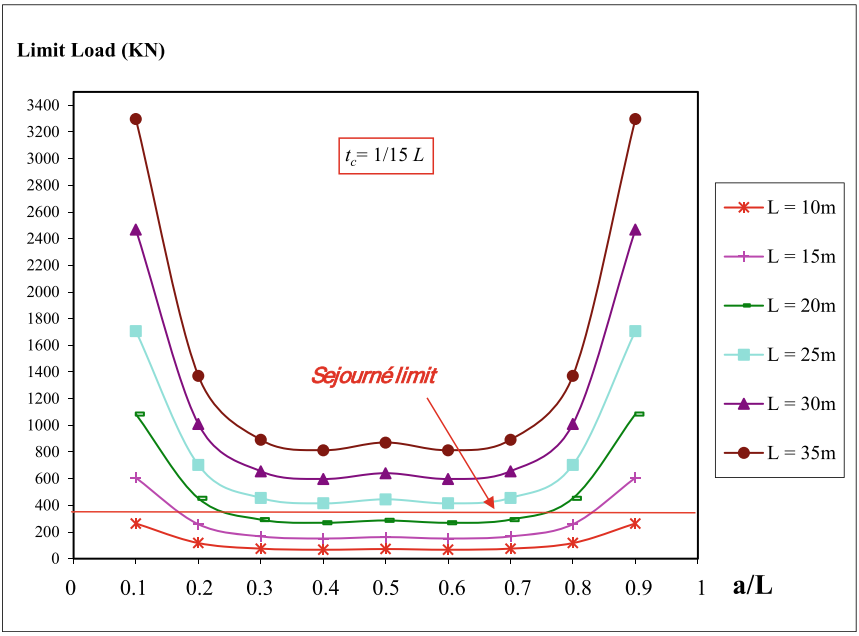

Fig. 5. Limit load vs loading position for different crown thickness according the XVI century formula

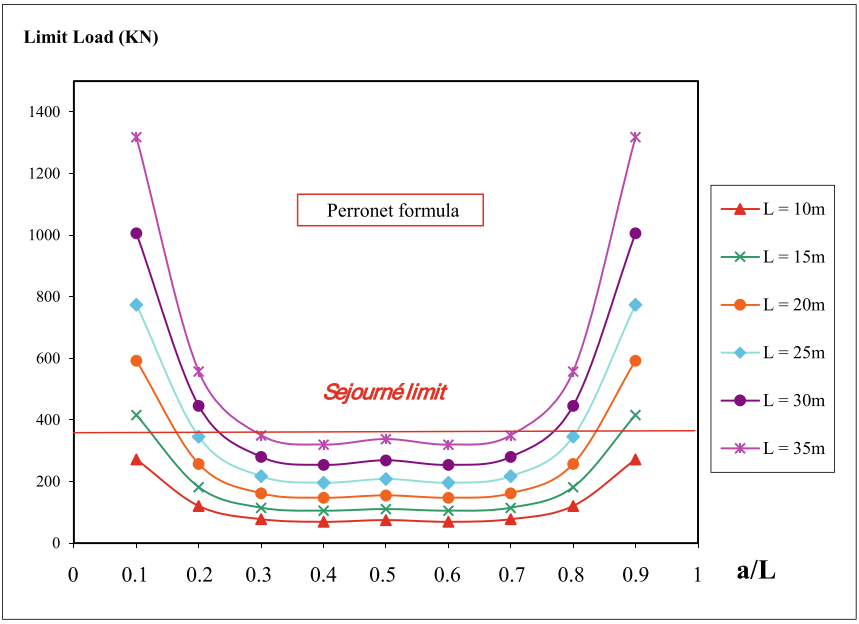

Fig 6. Limit load vs loading position for different crown thickness according Perronet formula

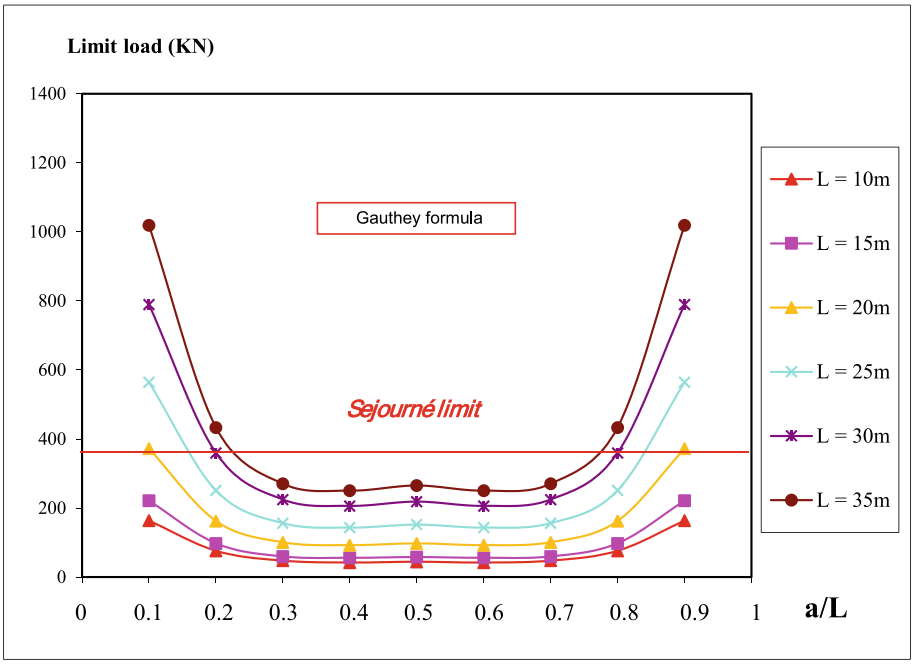

Fig. 7. Limit load vs loading position for different crown thickness according Gauthey formula

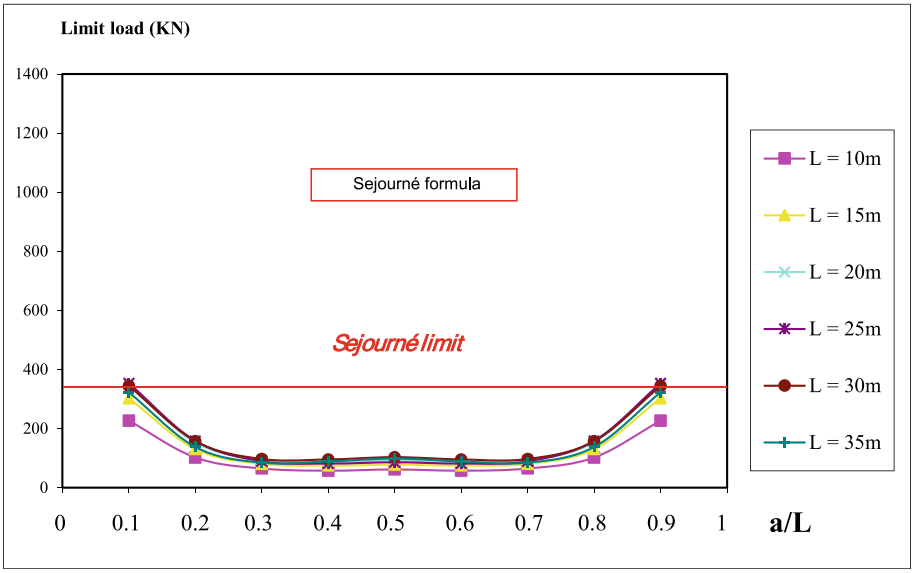

Fig. 8. Limit load vs loading position for different crown thickness according Sejourné formula

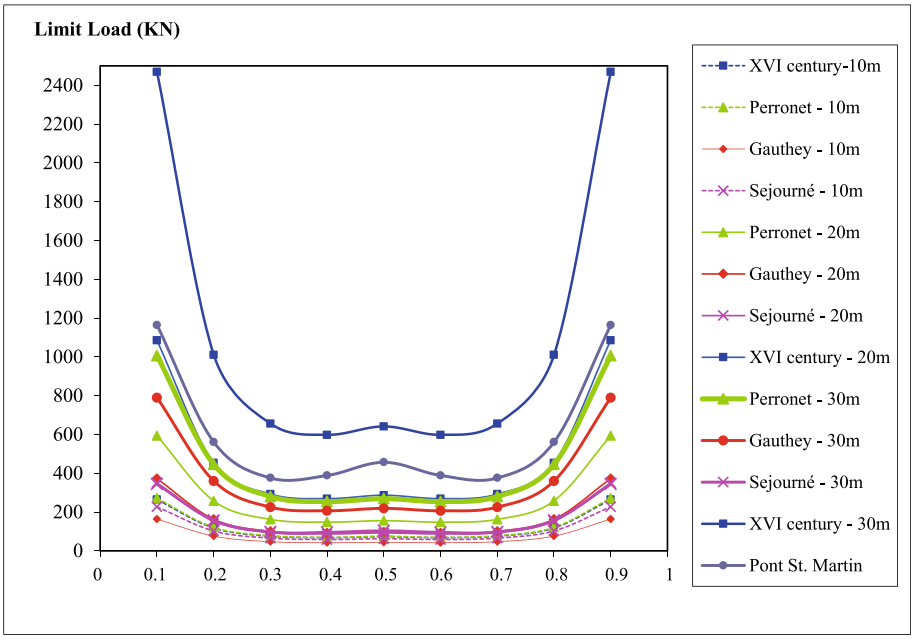

Fig. 9 Limit load vs loading position for different crown thickness according empirical formulae

The experience and tests performed on existing structures [45-46] confirm that good load carrying capacity of the structures can be obtained with a more limited volume of material, although the limits of these formulas have been recognized since the first half of last century [31].

4 Discussion and conclusion

The structural shape of a large part of the masonry bridges actually in service and part of the traffic network has been obtained over the centuries by means of geometrical rules. These empirical relations seem nonconservative in several cases (shallow arches) but they do not take into account the contribution given by spandrels and fill to the good structural performance of the bridge. Nevertheless, maintenance and repair of these bridges should take into account the principles on which the design process was based. This paper reports and discusses a catalogue of historical empirical formulas reported in the treatises from the Renaissance to the first decades of the 20th century, together with an evaluation of the reliability of the geometrical approach after a sensitive analysis.

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