Fractal and Multifractal Analysis in Cancer Diagnosis and Segmentation with MRI Data

Bengü Karaca[1] and Fatma Aslan-Tutak[2(✉)]

[1] Bilfen Çamlıca Schools, Istanbul, Turkey
[2] Faculty of Education, Mathematics and Science Education, Bogazici University, Istanbul, Turkey
fatma.tutak@bogazici.edu.tr

Abstract. Fractal and multifractal analysis methods are employed to find scale invariance, irregularity and self-similarity of computerized images and signals. Mathematical modeling solutions in cancer cases enable the comprehension of the biological complexity of cancer, optimization of treatment and development of approaches personalized for each patient. Diagnosis of cancer relies on the degree of accuracy, which depends on the development of parameter identifications against the measured data concerning observed phenomena as the parameters in biomedical models. Symmetry and symmetry-breaking notions of biology and physics may be applied to cancer whose features are examined in relation to symmetry breaking, among which tumor heterogeneity, fractal geometric and information structure, along with functional interaction networks can be addressed. To this end, this study has focused on the mathematical examination of the significant role of multifractal analysis methods in their application to medical images. 2D multifractal pumping method is applied to colon cancer MRI data. For these purposes, local regularity calculation (use of multifractal), Hölder exponent calculation, denoising and transformation with inverse wavelet have been performed analyzing all the pixels from the MRI images. All the pixels, namely matrices, have been calculated on the MRI images of colon cancer patients through computerized applications. The calculations and analyses have shown that computerized applications with technological foundation can facilitate diagnosis, treatment and management for patients with cancer and / or other complex dynamic diseases.

Keywords: Fractal analysis · Fractal dimension · Multifractal analysis · MRI Images · 2D Multifractal pumping method · Hölder regularity · Inverse wavelet transform · Denoising · Noise reduction · Colon cancer · Pattern recognition · Symmetry · Fractal patterns

1 Introduction

Multifractal systems, being common in nature, is known as a generalization of a fractal system where a single exponent or fractal dimension is not adequate to describe its dynamics [1]. Multifractality dates back to sequential or time series data ascribed to mathematical convergence effects and other geometric models [2]. On the other hand,

© The Author(s), under exclusive license to Springer Nature Switzerland AG 2026
O. Gervasi et al. (Eds.): ICCSA 2025 Workshops, LNCS 15886, pp. 165–181, 2026.
https://doi.org/10.1007/978-3-031-97576-9_11

multifractal analysis is employed for investigating large datasets, frequently along with other fractal methods, which requires distorting datasets obtained from patterns for the generation of multifractal spectra depicting the way scaling changes across the dataset. It has been utilized to interpret the generating rules and functionalities of complex networks [3]. With these attributes, multifractal analysis has been applied in practical instances such as the interpretation of images or prediction of earthquakes [4–6].

Multifractal methods can provide robust analytical means to understand and characterize the behaviors and patterns of complex dynamic systems in different fields of science. The multifaceted scaling properties can be captured by those systems with multifractal analysis being able to reveal the underlying irregularities as well as patterns which traditional methods could oversee. This means that the application of multifractal analysis and multifractal methods can enable researchers to understand the interconnection of complex dynamic systems' components, which can result in higher levels of accuracy in predictions, forecasting and risk models so that more robust and optimal models can be designed and implemented.

The advancements in biological discoveries have led to paradigm changes in modeling, which applies for the concept of cancerization field with mathematical propositions being infused with biological as well as genetic details and attributes [7]. Cancers, as complex adaptive diseases, are regulated by nonlinear feedback systems across environmental signals, instabilities, cellular protein flows, and so on. For these reasons, it is important to understand the cybernetics of cancer which necessities the incorporation of information dynamics along multidimensional spatial and temporal scales, which may include various networks from metabolic to epigenetic. Consideration of these multiple dynamics provides a perspective which cultivates the significance of computational systems oncology with regard to nonlinear dynamics, inverse problems, information theory, and complexity, to name some [8]. In view of these perspectives and approaches, it has become evident that mathematical modeling and computer simulations play a remarkable role in solving complex problems which emerge in different scientific disciplines including biomedical sciences by ensuring support to experimental and clinical practices within an interdisciplinary framework. While dealing with these sorts of applications, the generation and development of mathematical models and efficient numerical simulation means are of preliminary importance. Furthermore, accurate estimation techniques rely on the development of parameter identifications against the measured data concerning observed phenomena as the parameters in biomedical models show specific scientific interpretations. Amid all these developments, computational mathematics opens the way for validating mathematical models and investigating related problems [9]. When the relationship between fractal and medical progresses is in question, recent studies have demonstrated that fractal geometry, which entails the irregular shapes' terminology, is beneficial to describe tumors' pathological architecture, providing insights into the mechanisms of tumor growth and angiogenesis complementing the ones captured through modern molecular methods [10].

The importance of mathematical biology (or biomathematics) in cancer research in terms of mathematical modeling (with formulas and techniques) is increasingly recognized in the current times. Mathematical modeling has become an indispensable tool for understanding and controlling complex biological systems such as cancer, predicting the

dynamics of 3 these systems and developing treatment strategies. Mathematical modeling solutions in cancer research with the aid of mathematics enable one to comprehend the biological complexity of cancer, optimize treatment and develop approaches personalized and customized for each patient since cancer course varies from patient to patient. In addition, genetic diversity and environmental factors have effects on the success of treatments. Thus, patient-based mathematical models offer more effective and personalized treatment schemes, plans and strategies by simulating individual genetic profiles and treatment responses. All of these solutions have the potential to improve patients' life quality along with their survival rates by supporting advancements in cancer research and clinical practice [11].

Human body is made up of a multitude of very complex structures, and mathematical concepts of fractals which are marked by broken and irregular objects having details on all scales of observation control the structures at all levels of the human body owing to their organization and characteristics. On the other hand, Euclidean geometry has been proven to fail in analyzing irregularities in such complex structures. The use of fractal hypothesis was addressed in a study where statistical distributions of fractal dimensions were examined in the case of mammary cancer and mastopathy. The approach used made it possible to detect significant differences in the underlying distribution between the two groups besides conducting a multifractal analysis on the basis of a wavelet. The study provided directions regarding alternative cancer therapy and cancer prevention [12]. To give another example from literature, applications of fractals in medicine were examined in the diagnosis of diabetic retinopathy by calculating the fractal dimensions of normal and pathological samples of retinal images. Through the fractals' application, it was possible to make the distinction between normal retinas and pathological ones [13].

It can be stated that the interest of fractal geometry originates from its capability to describe fragmented or irregular patterns of natural features along with other complex objects. This feature is frequently conveyed by time-domains or spatial aspects in the behaviors of real-world physical systems, which ensures a geometrical interpretation that is simple and often encountered in different fields such as biology, fluid mechanics, geophysics, among other ones. Regarding the applicability of fractal geometry in medical image analysis is justified due to the fact that self-similarity can hardly be verified in biological objects that are imaged by a finite resolution. This means images are complex on spectral and spatial grounds, and what is more, objects generally display particular similarities at different spatial scales. At this point, it is observed that fractal geometry enables the description and characterization of the images' complexity. Fractal analysis evaluates the fractal dimension to have a global description of the images' inhomogeneities, and its efficiency is revealed in experiments regarding classification and segmentation, in which it was used as an additional texture parameter in addition to being used on its own characterizing healthy and pathological states [4].

With further strands, it has been noted that symmetry and symmetry breaking notions of biology and physics may be applied to cancer based on three categories, which are combinatorial, geometric and functional. Relevant cancer features are examined in relation to symmetry breaking, among which tumor heterogeneity, fractal geometric and

information structure besides functional interaction networks can be addressed. Symmetry concepts hold their importance regarding the origin, spread, treatment and resistance for generating solutions to cancer [14]. From this point onwards, the studies using combined methodologies with integrative approaches can be noted, too. For instance, Hidden Markov model (HMM) and multifractal method with predictive quantization complexity models were applied for the differential prognosis and differentiation of a complex disease Multiple Sclerosis (MS) [15]. This integrative proposed model revealed the critical and determining significance of predictive quantization in dynamic complexity while analyzing the dynamic and transient states in varying complex systems. In addition, the use of computational methods, namely Viterbi algorithm enabled optimal maximization and uncovering of the most probable hidden sequence states. Another study highlights the importance of magnetic resonance imaging (MRI) as a sensitive method to detect nervous systems' diseases, and Brownian motion Hölder regularity functions for 2D image along with multifractal methods were applied to brain images to identify the distressed regions in the brain more conveniently in the case of MS. The study provides the classification of MS disease based on multifractal method through the integration of Self-Organizing Map (SOM) algorithm and cluster analysis which included the identification of pixels from distressed regions by means of multifractal methods to diagnose MS subgroups based on artificial neural networks (ANNs) [16]. Many natural phenomena exhibit recurring self-similar patterns, and when a pattern appears to repeat itself, fractal is to be employed. The map put forth by Benoit B. Mandelbrot did not only have applications and impact on mathematics but also on various domains such as physical chemistry, medicine and biology, having a close relationship with a new geometrical system [17].

The applications of fractal and multifractal methods are extensive applications in neurosciences, for example, which require the examination of fractal properties like self-similarity in the brain, an organ characterized by complex structure. Since medical data analysis allows for analyses at various levels of observation, these abovementioned methods are foregrounded by capturing the subtle details which may be overlooked by physicians. In the case of stroke, which could be a life-threatening disease, one paper using a multifarious and integrated methodology identified the self-similar, significant and efficient attributes to attain high rates of classification accuracy for the subtypes of stroke implementing two approaches. The first one was the use of fractal and multifractal methods to identify the self-similar, regular and significant attributes from the dataset with box-counting dimension as well as Wavelet transform modulus maxima. The second approach was Feed Forward Back Propagation (FFBP) for stroke subtype classification. The comparative analyses yielded results concerning sensitivity and specificity for the three datasets in the study which provided contributions by identifying the significant, self-similar and regular attributes from the datasets, revealing the multifractional nature of complex dynamic systems and structures and showing a way to interpret them accurately and sensitively [18].

Concerning multifractal analysis and imaging, one study identified primary cancer in metastatic bone disease by classifying the shape of tissue cells from microscopis images. The study provided contributions through its results of computer-aided analyses [19]. Another work is concerned with the comparative multifractal analysis of dynamic

infrared thermograms and X-ray mammograms. The authors a 2D wavelet-based multifractal method for analyzing the spatial fluctuations of breast density in the X-ray mammograms [20]. One other work demonstrates the use of fractal/multifractal analysis for the detection of the melanoma patients' responses to treatment. The analysis proposed by the study showed its advantage of being operator independent, providing new diagnostic directions when compared to more laborious approaches of visual locating or manually contouring of lesions in images [21]. Consequently, advances in high-resolution biomedical imaging that focus on the properties of cells and issues including morphological, electrical and biochemical aspects and tissues have provided improvements in cancer diagnosis. In addition, multiscale imaging has revealed high complexity requiring further and advanced data processing methods of multifractal analysis [22]. Given these, multifractal analysis perceived as an extension of fractals proves to be an effective tool in identifying complex and irregular patterns and structures, so they are utilized for differentiating different tissues on the medical imaging processes. Multifractals can be, and a multi-fractal object is more complex as it is all the time invariant by translation.

Studies on cancer have been on the rise for the identification and treatment of the disease besides the rigorous attempts for developing applicable approaches. Biology and geometry should be perfectly aligned while utilizing technological devices for the purpose of diagnosing cancer. The utilization of different imaging technologies is for the detection of cancerous tissues and structures located in the internal organs of patients. For colon cancer in particular, it is important the scientists and researchers from different disciplines cooperate due to accessibility challenges between internal organs and imaging limitations. Colon cancer is a type of cancer which does not display symptoms at the early stage, and yet if diagnosed at an early stage, it can often be treated through surgical intervention [23]. The accurate and proficient use of magnetic resonance imaging (MRI) does not require the cutting of the skin or installing a device into the body, so this technique with its high-resolution capabilities has become a vital one for the diagnosis of the tumors [24]. In view of these capabilities, this study involves the examination of images obtained by medical technologies and makes use of fractals for the diagnostic aspect. When further studies are examined, one inquires the use of multifractal analysis methods in cancer diagnosis [22]. Concerning colon cancer, another study used the fractal dimension analysis over optical microscopic images to detect the different stages of cancer. The method in question examines the fractal features of density distributions obtained by light transmittance. In this way, the heterogeneous and complexity changes in the tissues with the advancing of cancer can be evaluated quantitatively [2].

The extensive applications of fractal and multifractal methods stem from the nature of multifractal systems being common in nature and being a generalization of a fractal system in which a single exponent or fractal dimension is not adequate to describe its dynamics. The examination of the previous studies in literature shows that there are limited number of studies concerning colon cancer and use of multifractal approaches for the early detection and diagnosis of the disease. Therefore, the aim of this study is to conduct mathematical examinations of the multifractal analysis techniques for segmenting the tissues in medical images. 2D multifractal pumping method is applied to colon cancer MRI data. For early detection of colon cancer, two approaches are employed: i) numerical calculations are conducted for one image of colon cancer patient taken as a

sample, with the pixel value $(x, y) = (80, 90)$. . ii) 2D multifractal pumping method is applied for all the pixels in the entire MRI data. For these purposes, local regularity calculation (use of multifractal), Hölder exponent calculation, denoising and transformation with inverse wavelet are carried out analyzing all the pixels from the MRI images. It is the first time that numerical analysis concerning 2D multifractal pumping method has been performed on the MRI of a colon cancer patient for single pixel. Another novelty is that all the pixels, namely matrices, are calculated on the MRI images of colon cancer patients through computerized applications. All these calculations and analyses have demonstrated that computerized applications with technological foundation facilitate diagnosis, treatment and management concerning patients having cancer and / or other complex diseases. Moreover, such approaches enable prompt and accurate outcomes that can reveal subtleties and significant structures like tumors and lesions more distinctly. Thus, this paper is organized as follows: Sect. 2 addressing Methods and Methodology. Section 3 providing experimental results, while Sect. 4 presenting the application of noise reduction of a 2D signal with multifractal pumping method to the UW-Madison GI Tract Image Dataset. Section 5 concludes the study with conclusion, discussion and future directions.

2 Methods and Methodology

Noise Reduction of a 2D Signal with multifractal pumping method is a processing technique that employs a multifractal approach in order to eliminate noise while preserving significant features of the signal such as textures, edges as well as fine details. In this process, noise reduction is performed by using the multifractal pumping method (MFP).

2.1 Fractal Geometry and Multifractals

Fractal geometry and multifractals have a fundamental denominator with respect to their capability at describing and analyzing complex irregular structures as well as patterns that occur across multiple scales. Fractal geometry is focused on objects marked by self-similarity and fractional dimensions, while multifractals extend this notion through capturing the variability in scaling properties within these structures. With these features, it is possible that multifractals ensure a more nuanced comprehension of various systems that have varying regions with diverse scaling behaviors.

The examination of cancer cells with fractals has brought about the need to define a new dimension, referred to as the “fractal dimension” which is represented by rational numbers. The dimensions of fractals are usually not expressed in whole numbers but are measured using fractional values. Fractals are defined by structures repeating themselves at every scale, whereas multifractals are a generalization of fractals. Fractals exhibit similarity across all scales, while multifractals allow this similarity to change across different scales. This can also mean that multifractals may be identical at one scale but exhibit varying complexity at another. However, as multifractals display these properties only in certain regions, a multifractal can have different fractal dimensions in different regions [26]. The multifractal spectrum of a multifractal signal shows the distribution of fractal dimensions across different scales, allowing the modeling of complexity in

various areas of the signal at multiple levels. With these characteristics, multifractals have been employed in scientific research, specifically in cancer diagnosis studies.

The analysis of multifractals in studies concerning identification involves fractal dimensions. Yet, it is required to conduct computations in the diagnostic process so that decision making process can be initiated after data obtained geometrically are converted into numerical data. At this particular stage, multifractal spectrum is formed by using Hölder exponent (α) which shows how smooth or complex a signal is in a certain region. The distribution of exponents constitutes the multifractal spectrum of the signal [27]. Multifractal analysis constitutes the signals from which noise is eliminated, so it provides important advantages in terms of obtaining high accuracy rates owing to its capability of maintaining the complex structures of the signal [28]. Wavelet transforms are generally employed in multifractal analysis as wavelet transforms allow for the examination of the signal in different frequencies and resolutions. This upper hand is significant for identifying the signal's structural properties. The multiple resolution level of the signal's wavelet transform is used for differentiating the properties and structures of the signal at different scales. It is possible to differentiate the noisy (high-frequency) and core structure (low-frequency) components via wavelet transform. Subsequent paragraphs, however, are indented.

2.2 Multifractal Pumping Method (MFP)

Calculation of Local Regularity. Multifractal pumping method (MFP) is a special technique employed for noise reduction in the signal [29]. By benefiting from the multifractal nature of the signal, this method enhances the signal with the following approach:

Initially, with multifractal spectrum analysis, the signal is analyzed using multifractal scaling laws. The complexity of the signal at different scales is determined based on the fractal dimension and scaling properties. The next approach is noise identification. Noise with a less complex and more regular structure often has different scaling properties than the real signal it often has. Therefore, these kinds of regions are detected by multifractal analysis, and the noise is separated from the meaningful signal. Finally, signal pumping describes the process of amplifying signal components that comply with multifractal properties while weakening components that belong to noise. This method is known to enhance meaningful signal properties while filtering out components, namely noise that do not comply with multifractal properties.

Theoretical and Mathematical Foundations of Multifractal Denoising Method. This method aims to preserve the multifractal structure of the image while the image is denoised. Within the scope of this study, this method can be examined in the following three steps:

1. Local Regularity Calculation (use of Multifractal): A 2D signal, for example an image or time-series data, is differentiated by means of a method like multifractal wavelet transform. This process captures the scale-related properties of a signal at different resolutions.

The signal is modeled as a noisy original signal according to Eq. 1:

$$S(x, y) = F(x, y) + N(x, y) \tag{1}$$

In Eq. 1, S refers to signal, F denotes the multifractal structure and N means noise. As for the processes, the multifractal properties of the image (signal) are identified. These properties represent significant structures in the signal such as edges and textures, and they are determined by the changing of the fractal dimensions at different scales.

2. Hölder Exponent Calculation: Wavelet transform is used to calculate the regularity level (Hölder exponent) at each point of the signal. The Hölder exponent is used to measure the local regularity of the signal and is derived from the scaling properties of the wavelet transform coefficients.

Wavelet transform is employed to calculate the regularity level (Hölder exponent) at each point of the signal. Hölder exponent is used to measure the local regularity of the signal. The function estimates the Hölder exponent ($h(x, y)h(x, y)$) at each point:

Noise: Low Hölder exponent values ($h < 0.2$).

Multifractal structure: High and variable Hölder exponent values.

Denoising, on the other hand, is another step. After identifying the multifractal features, the noise signal is differentiated. This can be achieved by attenuating the components that have a more linear or flat structure and do not comply with the signal's multifractal structure weaker. The regions with low regularity (generally noise) are suppressed, while the areas where the clean signal's structure has been preserved are restructured.

This method is applied through the following processes outlined below:

i. Thresholding: In the areas where the Hölder exponent is low (noisy areas) wavelet coefficients are suppressed. The coefficients are preserved in the areas with high Hölder exponent.

ii. Redistribution of energy: The coefficients applicable to multifractal spectrum are rescaled, which reinforces the original multifractal features of the signal.

3. Transformation with inverse wavelet: Pumping mechanism: These related steps are repeated multiple times, and in each step, the multifractal structure of the signal becomes more pronounced, and the noise is further reduced. For the signal reconstruction, after the noise is filtered out, the remaining clean multifractal components are combined to reconstruct the signal. This step creates a noise-free signal while preserving important structures. Following the identifying of the multifractal features, the noise signal is distinguished. This can be done by attenuating the components that do not conform to the multifractal structure of the signal and have a flatter structure.

- The wavelet coefficients in regions with a low Hölder exponent (noisy) are suppressed.
- The coefficients in regions with a high Hölder exponent are preserved.

This method with three steps is implemented for each pixel, and it is repeated until the image (signal) reaches the desired level of regularity. This kind of an iterative step is referred to as pumping. At each step, the multifractal structure of the signal becomes more pronounced, and noise is reduced further. Finally, the remaining clean multifractal components are restructured by combining the multifractal components after the image has been filtered. This step is beneficial by creating a signal that has been denoised by protecting its important structures. Computer applications enable the pumping and signal reconstruction processes. This study focuses on the examination of the mathematical process made up of three stages as outlined above.

3 Experimental Results: Numerical Calculations and Computerized Applications

3.1 Numerical Calculation: Computation of the Noise Reduction Steps with Multifractal Pumping Method

The first process applied at this stage is to define the 2D visual with the use of fractals and multifractals. Subsequently, the data obtained geometrically are converted into numerical data and the analysis needs to be carried out. This process involves three steps, which are local regulatory calculation (multifractal use), Hölder exponent calculation and inverse wavelet transform.

For computerized applications, MathLab [30] and FracLab [31] software tools have been used for the analyses and figures in this study.

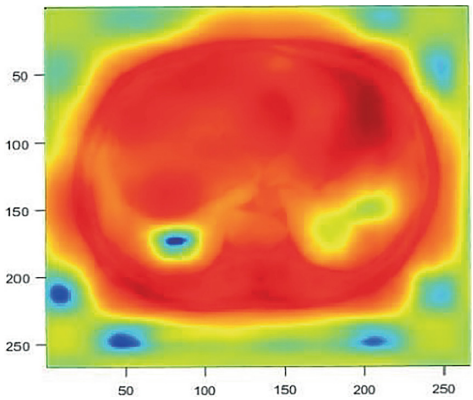


Fig. 1. Colon cancer image with jet coloring after the MFP method has been applied

Reducing noise of a 2D signal by means of MFP method increases regularity and denoising by subjecting each pixel to the mathematical operations presented below. This process allows the MRI image to become clearer and more suitable for segmentation. In the following calculations, the transformation of a pixel's value, for example, the value 5746 located at the 80 × 90 index, into the value of 4801.176 is realized according to the following mathematical operations.

Application of Local Regularity Calculation (Multifractal use) and Wavelet Transform. Wavelet transform allows for the analysis of each pixel in an MRI image in terms of local details and general trends. Daubechies-10 wavelet was used in the application of this method. This wavelet decomposes the signal according to different scales (a) and positions (b) [18, 29]. This is expressed in Eq. 2.

$$\boldsymbol{T}(a,b) = \sum\nolimits_{\{x,y\}} \boldsymbol{S}(x,y) \cdot \boldsymbol{\psi}_{\boldsymbol{a},\boldsymbol{b}}(t) \tag{2}$$

Here,

$T(a, b)$ is the wavelet transform coefficient.

$S(x, y) = t$ denotes the pixel value in the input image (i.e. S(80,90) = 5746).

$\boldsymbol{\psi}_{a,b}(x)$: Wavelet function refers to the high-frequency components of the signal.

$$\boldsymbol{\psi}_{a,b}(x) = \frac{1}{\sqrt{a}}\boldsymbol{\psi}\left(\frac{t-b}{a}\right) \tag{3}$$

According to Eq. 3, the following points can be explained:

Scale parameter controls the width or frequency of the wavelet.

a>1: The wavelet widens, analyzing low-frequency components (general trends).

a<1: The wavelet compresses, analyzing high-frequency components (fine and subtle details)

Position parameter indicates where the wavelet is shifted along the signal.

$\frac{1}{\sqrt{a}}$**:** Normalization factor for energy conversation. It keeps the total energy of the wavelet independent from the scale.

$\boldsymbol{\psi}$: Core wavelet function. **Daubechies-10** includes a scaling function and has the prosperity of 10 vanishing moments.

$$\phi(t) = \sum_{k=0}^{N-1} \boldsymbol{h_k}.\phi(2t-k), N = 10 \tag{4}$$

As Eq. 4 shows, $\boldsymbol{g_k} = (-1)^k.h_{\{N-1-k\}}$: is the modified version of the scaling coefficients. ($g_0 = h_9, g_1 = -h_8, g_2 = h_7, g_3 = -h_6, g_4 = h_5, g_5 = -h_4, g_6 = h_3, g_7 = -h_2, g_8 = h_1, g_9 = -h_0$).

The multiplication operations implemented for each pixel to obtain the T(a,b) wavelet transform coefficient need to be summed up. Regularity and irregularity features around the pixels are analyzed by obtaining the $T(a,b)$ value at this stage. For example, wavelet coefficient corresponding to the value of 5746 is calculated based on a specific scale and location. For a pixel with (x, y) = (80, 90) indices, let input value be $t = 5746$, $a = 4$ and $b = 5000$. The continuing steps of application are presented in the following section:

By identifying the noise regions as regions with a low Hölder exponent ($h < 0.2$), these regions are suppressed, and regions with a high Hölder exponent ($h > 1.0$), which are more regular regions, are enhanced.

The regions with a low Hölder exponent ($h(x, y) < 0.2$) resulting from noise are suppressed, and more regular regions ($h(x, y) > 1.0$) are enhanced. The thresholding process is performed as follows:

The thresholding process is performed in the way described below:

$$\boldsymbol{T'}(a,b) = \begin{cases} T(a,b), & h(x,y) < 0.2 \textit{ if satisfies the threshold value} \\ T(a,b) \cdot f(\boldsymbol{h}(x,y)), & \textit{in other conditions} \end{cases}$$

In this expression, $\boldsymbol{f}(\boldsymbol{h}(x, y))$ is a weight function suitable for the multifractal spectrum. In this example, we have $T\prime(a, b) = T(a, b) \cdot (1 + \textit{RegularityIncrease})$, and the regularity increase is set to 0.5. This process enhances the coefficients in regular regions while suppressing the coefficients in irregular regions. In this particular step, the processed coefficient of the pixel corresponding to the value 5746 is obtained based on the regularity threshold value.

3.2 Hölder Exponent Calculation

By using the coefficient obtained from the wavelet transform, the Hölder exponent of ($\boldsymbol{h}(x, y)$) each pixel is calculated. This expresses the local regularity of the pixel [16, 29].

$$\begin{aligned} \boldsymbol{h}(x, y) &\sim \log\left(\frac{T(a, b)}{a}\right) \\ \boldsymbol{h}(80, 90) &\sim \log\left(\frac{T(a, b)}{a}\right) \\ &= \log\left(\frac{4160.104}{4}\right) = \log(1040.026) = 6.947 \end{aligned} \tag{5}$$

Equation 5 shows the calculation for the pixel with $(x, y) = (80, 90)$. The regularity of the pixel that the value **5746** corresponds to can be analyzed with the start level analysis.

3.3 Inverse Wavelet Transform

Inverse (Reconstruction). After the wavelet coefficients are rearranged, the signal is reconstructed by inverse wavelet transform:

$$\boldsymbol{S'}(x, y) = \sum_{\{a,b\}} \boldsymbol{T'}(a, b) \cdot \boldsymbol{\psi}_{\boldsymbol{a,b}}(x)$$

In this step, 5746 input pixels are transformed into the value of 0.2008 using adjusted coefficients and inverse transformation. Each pixel is denoised and its regularity is enhanced by making it subjected to the mathematical operations given above. This process enables the MRI image to become clearer and more appropriate for the purpose of segmentation. For example, the indices are $(x, y) = (80, 90)$ and the input value is $t = 5746$, $a = 4$ and $b = 5000$.

Wavelet Transform

$$\boldsymbol{T}(4, 5000) = \boldsymbol{S}(80, 90) \cdot \boldsymbol{\psi}_{(4,5000)}(5746)$$

The calculation processes of the wavelet function ($\boldsymbol{\psi}$) is an iterative process which is extremely hard to conduct manually. The following result is achieved as obtained regarding the function:

$$\begin{aligned} \boldsymbol{T}(4, 5000) &= 5746\frac{\boldsymbol{\psi}(1436.5)}{2} \\ &= 5746 \times 0.7240 \\ &= 4160.104 \end{aligned}$$

The final step in the noise reduction of a 2D Signal with MFP method is the suppression of regions according to the Hölder exponent value by using inverse wavelet transform for the analyzed pixel. This process enables imaging to become usable and applicable by utilizing medical technology. Regions with low Hölder exponents ($h(x, y) < 0.2$) due to noise are suppressed, and more regular regions ($h(x, y) > 1.0$) are enhanced. The thresholding process is performed as follows [29].

- *Noisecondition*$(h(x, y) < 0.2)$

$$\boldsymbol{U}(a, b) = \boldsymbol{T}(a, b)$$

- *Inotherconditions*

$$\boldsymbol{U}(a, b) = \boldsymbol{T}(a, b) \cdot \boldsymbol{f}(\boldsymbol{h}(x, y))$$

In this expression, $\boldsymbol{f}(\boldsymbol{h}(x, y))$ is the weight function suitable for multifractal spectrum. The example is identified as $T\prime(a, b) = T(a, b)\cdot(1 + \mathit{RegularityIncrease})$, and regularity increase is identified as 0.5.

This process enhances coefficients in regular regions while suppressing coefficients in irregular regions. Returning to the example in which calculations above were done, the processed coefficient of the pixel corresponding to the value 5746 is obtained based on the regularity threshold value.

Pumping

Since $h > 1.0$, regularity increase coefficient is used.

$$\boldsymbol{T'}(a, b) = \boldsymbol{T}(a, b) \cdot \boldsymbol{f}(\boldsymbol{h}(x, y)) = 4160.104*(1 + 0.5)$$

$$\boldsymbol{T'}(a, b) = \boldsymbol{T}(a, b) \cdot \boldsymbol{f}(\boldsymbol{h}(x, y)) = 6240.156$$

After the wavelet coefficients are rearranged, the signal is reconstructed using the inverse wavelet transform.

Inverse Wavelet Transform

$$\begin{aligned}
\boldsymbol{S'}(80, 90) &= \sum_{\{4,5000\}} \boldsymbol{T'}(4, 5000) \cdot \boldsymbol{\psi}_{4,5000}(6240.156) \\
\boldsymbol{S'}(80, 90) &= \sum_{\{4,5000\}} \boldsymbol{T'}(4, 5000) \cdot \boldsymbol{\psi}_{4,5000}(6240.156) \\
&= 6240.156 \times 0.7694 \\
&= 4801.176
\end{aligned}$$

Hölder exponent calculation and corrected value after pumping is:

$$4.801176 \times 10^3 \simeq \mathbf{4801.176}$$

Noise reduction of a 2D Signal with MFP method is explained with a numerical example. While diagnosing cancer, it is not possible to manually perform this process repetitively for a cross-sectional image made up of 266x266 pixels. One area in medicine where technology is used is data analysis using programming languages. To demonstrate the way noise reduction of a 2D Signal with MFP method is used, an example which shows how the image presented in Fig. 1 (including the pixel analyzed manually above) is analyzed with the relevant computer application [30, 31].

4 Application of Noise Reduction of a 2D Signal with Multifractal Pumping Method to the UW-Madison GI Tract Image Dataset

4.1 UW-Madison GI Tract Image Dataset

Applying the multifractal image processing method to the UW-Madison GI Tract Image dataset can prove to be useful, particularly in terms of better identification of the intestinal tissue boundaries, detailed analysis of tissue structures as well as increased segmentation accuracy. This application enables segmentation operations to produce clearer and more reliable results without being impacted by noise.

The UW-Madison Carbone Cancer Center is a leading center in MR-Linac (Integrated magnetic resonance imaging (MRI) and linear accelerator systems (MR-Linac)) based radiotherapy. The center has been applying MRI-guided radiotherapy based on the daily anatomies of patients since 2015. The UW-Madison GI Tract Image dataset consists of anonymized MRIs of patients receiving treatment at the UW-Madison Carbone Cancer Center. Mask files in PNG format are provided for each image, and these masks contain pixel values corresponding to different tissues in the image. The masks were prepared by expert radiologists, which makes the dataset clinically reliable.

The dataset includes the segmentation images of organ cells, and it is in the form of masks encoded as **RLE (Run-Length Encoding)**. The images are in 16-bit grayscale PNG format. The names of the image files contain 4 numbers (e.g. 276_276_1.63_1.63.png). These four numbers express the following:

- The first two numbers (e.g., 276_276) indicate the slice's width/height (integers in pixels).
- The last two numbers (e.g. 1.63_1.63) record the pixel interval spacing (width/height) as physical dimensions (decimal numbers in millimeters).

The first two numbers define the resolution of the slice. The last two numbers indicate the physical size of each pixel [32].

4.2 The Application Steps of 2D MFP Method on the MRI Data

The initial image of the application of the noise reduction of a 2D Signal with MFP method using MathLab [30] and FracLab [31] is provided in the upper left side of Fig. 2. As can be observed in the initial image, the dark regions may be cancerous areas, but it is nearly impossible to make a decision based merely on this image due to the low clarity of the image. The images resulting from the application of the three steps mentioned in the study to this colon image analyzed using FracLab [31] (Example 1: slice_0065_266_266_1.50_1.50.png) are presented in Fig. 2.

Figure 2 shows the initial image (original image) located in the upper left side The image in the lower right side is the visual obtained as a result of the steps applied as the noise Reduction of a 2D signal with MFP which have been performed iteratively. The cancerous tissue can be seen in a clearer way in this last visual depiction.

This code enhances an MRI image by using multifractal denoising method (as you can see Table 1). The output generated shows the original image and mask on the top side and the input image as well as the denoised output image on the bottom side. Figure 3

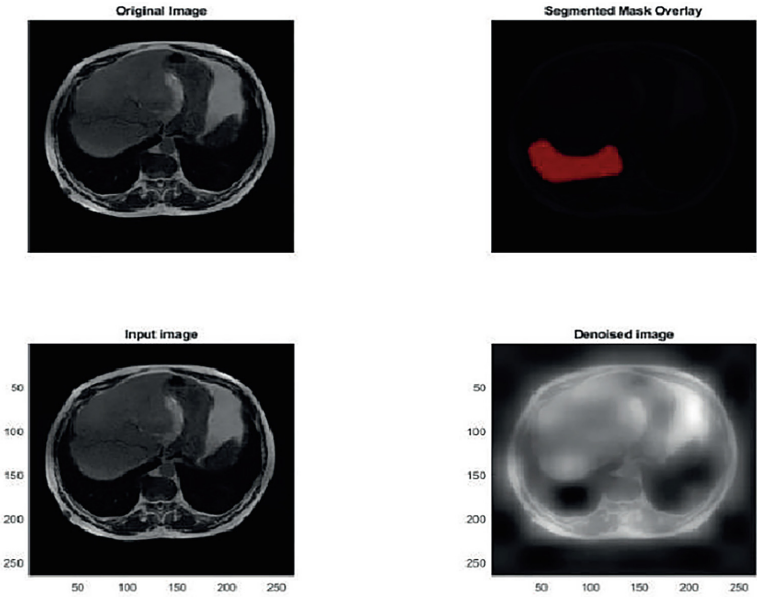


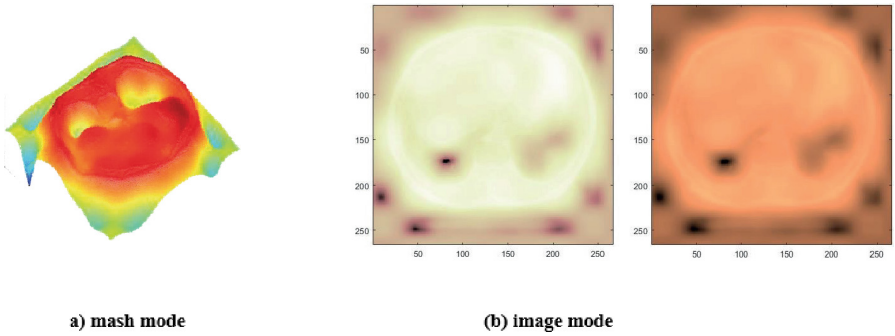
Fig. 2. Noise reduction steps with multifractal pumping method (a) original MRI data (raw data), (b) segmented mask overlay, (c) input image and (d) denoised image

Table 1. Application of the Code: Noise Reduction of a 2D Signal with Multifractal Pumping Method.

```
x = imread('slice_0082_266_266_150_150.png');
(the parts for printing the mask by decoding have been removed.)
QMF = MakeQMF('daubechies',6);
y = mfdpumping2d(x,QMF);
figure; subplot(1,2,1); imagesc(x); title('Input image'); axis image;
subplot(1,2,2); imagesc(y); title('Denoised image'); axis image;
colormap(gray);
```

depicts the colon cancer MRI image with the image mode of the MRI data with jet coloring and the mash mode of the MRI data with pink and copper coloring.

Overall, this method is an effective tool for denoising images in complex structures like MRI images. The preservation of the multifractal structure ensures that the essential information of the image is not lost, and thus this increases its applicability in further analyses.



a) mash mode (b) image mode

Fig. 3. Colon cancer MRI image (a) image mode of the MRI data with jet coloring and (b) Mash mode of the MRI data with pink and copper coloring

5 Conclusion, Discussion and Future Directions

Fractal and multifractal geometries offer application benefits in various medical signal analyses including but are not limited to texture analysis, pattern recognition and segmentation. The application of this geometry depends on estimation of the fractal features, and fractal analysis is applied in many fields of science from computer graphics to image compression through medicine. Furthermore, the multifractal properties of tumor tissue in MRI images can be used for automatic segmentation. To this end, this study has focused on the mathematical examination of the significant role of multifractal analysis methods in their application to medical images. Specifically, the application of the Multifractal Pumping Method (MFP) for noise reduction in a signal as has been the case in this study can provide a remarkable transformation by encouraging the use of innovative and advanced methods in medical imaging. This study has been the first one in which numerical analysis concerning 2D multifractal pumping method has been carried out on the MRI of a colon cancer patient for single pixel. Another novel point is that all the pixels, namely matrices, are calculated on the MRI images of colon cancer patients through computerized applications. All these calculations and analyses demonstrate that computerized applications with technological foundation facilitate diagnosis, treatment and management concerning patients having cancer and / or other complex diseases. Such approaches with computerized applications can facilitate the processes for achieving prompt and accurate outcomes.

High-quality images free of noise with preserved multifractal properties have significant effects on both diagnostic and treatment processes. It supports a better understanding of complex diseases such as cancer and the development of early diagnosis tools. Through these means, medical innovations can contribute to a higher quality of life worldwide and upgrade the general level of healthcare services. Last but not least, these kinds of advanced image processing methods developed using multifractals optimize existing diagnostic and treatment processes while bringing a new perspective to medical research. It has been observed how mathematics is addressed in different disciplines with such a method that could not be achieved without mathematical modeling. The use of multifractal analysis can support a better understanding of complex diseases such as cancer and the development of early diagnosis tools. In these ways, medical

innovations can contribute to a higher quality of life worldwide and elevate the general level of healthcare services.

As directions for future research, fractal and multifractal analyses can be extended with the combination of other advanced methods based on mathematical foundations as well as computerized applications to reveal their potential as objective measure of seemingly random structures and as means for examining the origins of pathological, cancerous and / or damaged forms. The advantages of the protection of edges, adaptive filtering and processing of complex structures can be employed for tissues or structures resembling fractals.

Acknowledgments. The authors would sincerely like to express their thanks to Professor Yeliz Karaca, from University of Massachusetts for her orientation, guidance and validation throughout the stages and processes of the study.

References

1. Harte, D.: Multifractals: Theory and Applications. Chapman and Hall/CRC (2001)
2. Jørgensen, B., Kokonendji, C.C.: Dispersion models for geometric sums (2011)
3. Xiao, X., Chen, H., Bogdan, P.: Deciphering the generating rules and functionalities of complex networks. Sci. Rep. **11**(1), 22964 (2021)
4. Lopes, R., Betrouni, N.: Fractal and multifractal analysis: a review. Med. Image Anal. **13**(4), 634–649 (2009)
5. Moreno, P.A., et al.: The human genome: a multifractal analysis. BMC Genomics **12**, 506 (2011)
6. Atupelage, C., Nagahashi, H., Yamaguchi, M., Sakamoto, M., Hashiguchi, A.: Multifractal feature descriptor for histopathology. Anal. Cell. Pathol. **35**(2), 123–126 (2012)
7. Kimmel, M.: Evolution and cancer: a mathematical biology approach. Biol. Direct **5**, 1–3 (2010)
8. Uthamacumaran, A., Zenil, H.: A review of mathematical and computational methods in cancer dynamics. Front. Oncol. **12**, 850731 (2022)
9. Bretti, G., Natalini, R., Palumbo, P., Preziosi, L. (Eds.): Mathematical Models and Computer Simulations for Biomedical Applications, vol. 33. Springer Nature (2023)
10. Baish, J.W., Jain, R.K.: Fractals and cancer. Can. Res. **60**(14), 3683–3688 (2000)
11. Murray, J.D.: Mathematical biology: I. An introduction (Vol. 17). Springer Science & Business Media (2007)
12. Stehlik, M., Hermann, P.: Fractal based cancer modelling. REVSTAT-Stat. J. **14**(2), 139–155 (2016)
13. Uahabi, K.L., Atounti, M.: Applications of fractals in medicine. Ann. Univ. Craiova-Math. Comput. Sci. Ser. **42**(1), 167–174 (2015)
14. Frost, J.J., Pienta, K.J., Coffey, D.S.: Symmetry and symmetry breaking in cancer: a foundational approach to the cancer problem. Oncotarget **9**(14), 11429 (2017)
15. Karaca, Y., Baleanu, D., Karabudak, R.: Hidden markov model and multifractal method-based predictive quantization complexity models vis-á-vis the differential prognosis and differentiation of multiple sclerosis' subgroups. Knowl.-Based Syst. **246**, 108694 (2022)
16. Karaca, Y., Cattani, C.: Clustering multiple sclerosis subgroups with multifractal methods and self-organizing map algorithm. Fractals **25**(04), 1740001 (2017)
17. Mandelbrot, B.B.: The fractal geometry of nature/Revised and enlarged edition. New York (1983)

18. Karaca, Y., Moonis, M., Baleanu, D.: Fractal and multifractional-based predictive optimization model for stroke subtypes' classification. Chaos Solitons Fractals **136**, 109820 (2020)
19. Vasiljevic, J., Reljin, B., Sopta, J., Mijucic, V., Tulic, G., Reljin, I.: Application of multifractal analysis on microscopic images in the classification of metastatic bone disease. Biomed. Microdevice **14**, 541–548 (2012)
20. Gerasimova-Chechkina, E., et al.: Comparative multifractal analysis of dynamic infrared thermograms and X-ray mammograms enlightens changes in the environment of malignant tumors. Front. Physiol. **7**, 336 (2016)
21. Breki, C.M., et al.: Fractal and multifractal analysis of PET/CT images of metastatic melanoma before and after treatment with ipilimumab. EJNMMI Res. **6**, 1–16 (2016)
22. Huynh, P.K., Nguyen, D., Binder, G., Ambardar, S., Le, T.Q., Voronine, D.V.: Multifractality in surface potential for cancer diagnosis. J. Phys. Chem. B **127**(31), 6867–6877 (2023)
23. Rodrigo, L. (Ed.): Colorectal Cancer: From Pathogenesis to Treatment. BoD–Books on Demand (2016)
24. Chaudry, M.A., Nicholls, R.J.: Handbook of Colorectal Cancer. Springer (2016)
25. Elkington, L., Adhikari, P., Pradhan, P.: Fractal dimension analysis to detect the progress of cancer using transmission optical microscopy. Biophysica **2**(1), 59–69 (2022)
26. Olsen, L.: Multifractal geometry. In: Fractal Geometry and Stochastics II, pp. 3–37. Birkhäuser Basel (2000)
27. Feder, J.: Fractals. Springer Science+ Business Media, LLC (1988)
28. Legrand, P., Lutton, E., Olague, G.: Evolutionary denoising based on an estimation of Hölder exponents with oscillations. In: Workshops on Applications of Evolutionary Computation, pp. 520–524. Springer, Berlin, Heidelberg (2006)
29. Jaffard, S., Lashermes, B., Abry, P.: Wavelet leaders in multifractal analysis. In Wavelet analysis and applications, pp. 201–246. Birkhäuser Basel (2007)
30. The MathWorks, Inc., MATLAB 2023 version 9.12.0(R2023a), Software, Natick, MA (2023)
31. Vehel J.L.: Fraclab (2025), project.inria.fr/fraclab, 5 November 2024
32. https://www.kaggle.com/competitions/uw-madison-gi-tract-image-segmentation/data, 2 Dec 2024