Algorithm of Determining Errors of Gimballed Inertial Navigation System

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Abstract. This paper deals with determining errors in gimballed inertial navigation systems. One of the aims of the study is to obtain expressions for transformations between different coordinate systems and expressions describing the system kinematics. The appropriate matrices of directing cosines have been obtained. The expressions for correction of reading of accelerometers taking into consideration rotation of the Earth, motion of the moving vehicle, and Coriolis acceleration have been obtained. Features of the considered system lie in the usage of the trajectory coordinate system as a basic navigation system contrary to the geographical reference frame and the usage of the two-axis gimballed platform. The structure chart and the necessary instrumentation of the researched system are represented. The equations for the determination of methodical errors for the inertial stabilization and inertial system have been obtained. The simulation of the basic errors of the studied system has been carried out. The obtained results of the simulation show the high accuracy of the researched system.

Keywords: Stabilization System · Moving Vehicle · Navigation Systems · Errors · Correction Expressions · Simulation

1 Introduction

The modern stage of the creation of moving vehicles requires improved techniques for the design of navigation systems, including course systems. For many vehicles, it is mandatory to use a course determination system based on gyroscopic devices [1–3]. If it is necessary to achieve high accuracy, systems with a gimballed platform should be

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used. Such a platform is mounted in a special suspension consisting of some frames and ensuring the possibility for angular motion in the inertial space. Dynamically adjusted gyroscopes and accelerometers could be used as sensing elements in such systems. One of the gyroscopes could be used as a vertical gyro reference, the other – as a directional gyroscope. The advantages of the chosen gyroscopes are the high stability of characteristics, low power consumption, low sensitivity to changing surrounding temperature, and small time of operation readiness. Accelerometers are used to correct gyroscopes to determine navigation parameters, namely speed and coordinates after appropriate double integration.

Requirements of high accuracy and mass and dimensions restrictions can be satisfied by the usage of a two-axis gyro-stabilized platform. Stabilization of the platform in the horizon plane is implemented by gyro vertical. In its turn, the gyro vertical is setting to the horizon plane by signals of accelerometers using the integral correction.

The application of the two-axis gyro-stabilized platform requires the usage of the trajectory reference frame. This is accompanied by complications algorithms of processing the primary navigation information.

The main objective of the paper is the assessment of errors in the gimballed platform stabilization. For this, the algorithm for determining errors and modelling stabilization processes have been carried out. The novelty of the approach lies in developing an algorithm for the determination of errors for gimballed inertial navigation systems operating in trajectory instead of the geographical reference frame.

The paper is organized as follows. In Sect. 2 we consider features of an inertial navigation system with a two-axis gimballed platform including basic kinematical relationships. The algorithm for determining errors of the studied system is in Sect. 3. The modelling results are given in Sect. 4.

2 Features of Inertial Navigation System with Two-Axis Gimballed Platform

Nowadays, gimballed inertial navigation systems are used in high-precision applications only. The main features of such systems are defined by the choice of the kinematic scheme. The choice of the kinematic scheme depends on many factors including accuracy and weight-dimension requirements. The usage of the stabilization and navigation system with a three-axis gimballed platform is the most widespread. However, such an approach requires significant mass and size costs. It is possible to improve these characteristics using a stabilization and navigation system based on a two-axis gyro-stabilized platform [4–6]. At the same time, additional hardware and algorithmic expenses arise. This is caused by the necessity to use an additional rotary device to ensure turns of the gyroscopic device. This gyroscopic unit performs the functions of the directional gyroscope [7–9]. The use of a two-axis platform also leads to the need to use a trajectory coordinate system. This leads to the complication of primary navigation information processing algorithms [10–12]. The position of the gyroscopic devices is controlled by the signals of the accelerometers. The stabilization motors ensure the coordination of the position of the platform with the horizon plane. This makes possible control of gyroscopes with dynamic adjustment using relatively small moments. Due to the use

of two-stage gimbals, the platform turns with the object in azimuth. Such peculiarity is connected with additional errors due to the change of object heading. For compensation, it is necessary to know the data about the linear velocity and course of the object. The choice of the kinematic scheme of this system depends on many factors, including accuracy and weight requirements [13].

To obtain kinematic ratios, the following coordinate systems must be used:

- trajectory $O'\zeta\eta\xi$ (the $O'\eta$ axis is pointed along the object's velocity vector, the $O'\zeta$ axis is normal to the horizontal plane, the $O'\xi$ axis is pointed perpendicularly to the $O'\eta$ axis and is in the horizontal plane);
- body-axis $Ox_0y_0z_0$ (the Oy_0 axis is pointed along the longitudinal axis of the object, the Oz_0 axis is normal to the horizontal plane, the Ox_0 s is located perpendicularly to the Oy0 axis and is located in the horizon plane);
- the platform's $Ox_py_pz_p$ (the Oy_p axis is pointed along the axis of the outer frame of the platform gimbals, the Oz_p axis is normal to the horizontal plane, the Ox_p axis is located perpendicularly to the y_p axis and is located in the horizontal plane);
- $Ox_ry_rz_r$ connected with the Rezal's axes of a dynamically adjusted gyroscope, which performs the functions of a vertical gyro reference.

The vector radius of the platform installation place concerning the center of mass of the object r_p determines the position of the origin of the coordinate systems. The relative angular location of the entered coordinate systems is given in Figs. 1 and 2.

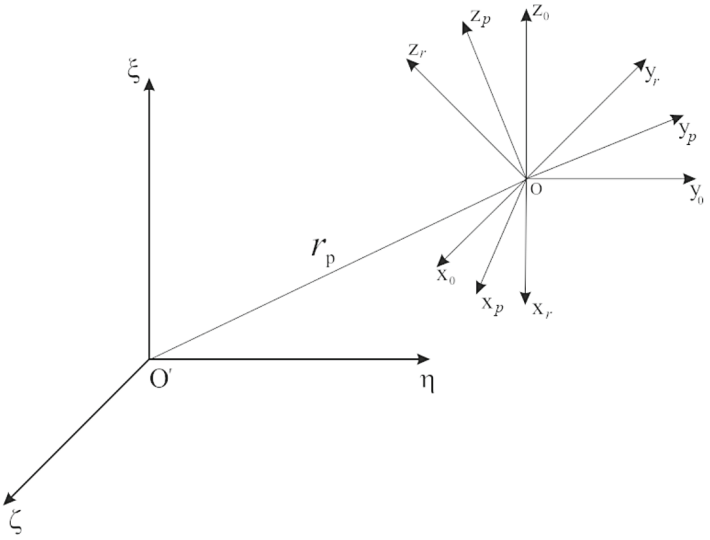

Fig. 1. The relative location of all the above mentioned systems of coordinates.

The scheme with a two-axis gimballed platform is presented in Fig. 3. Two gyroscopes with dynamic adjustment and two accelerometers are located on the platform.

The platform is stabilized in the horizontal plane by the data of the gyroscope, which performs the functions of a vertical gyro reference. The vertical gyro is brought

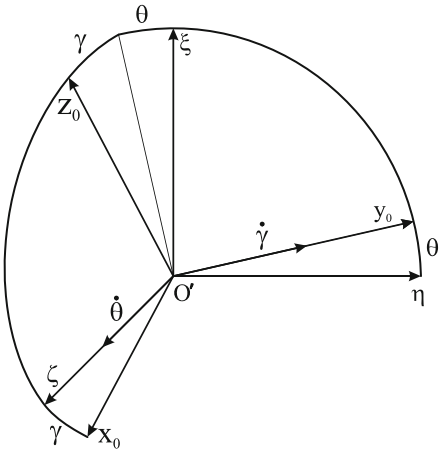

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Fig. 2. The relative location of body-axes system and the platform's system of coordinates: $\alpha_p \beta_p$ are angles that define a location of the platform).

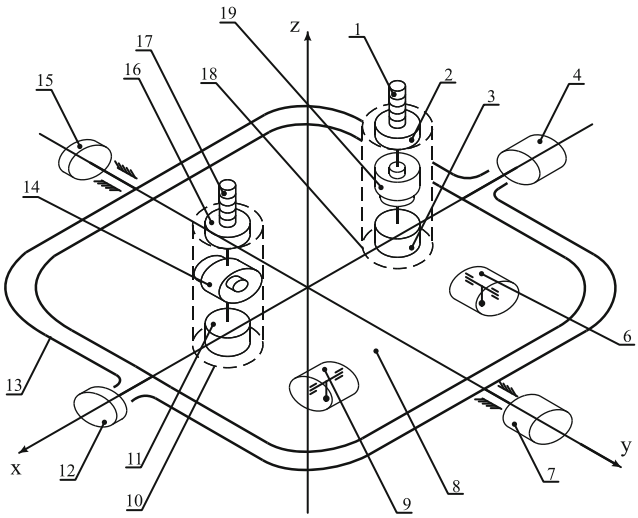

Fig. 3. The stabilization and navigation system using two-axes platform: x, y, z is the body-axes coordinate system; 15, 12, 16, 2 are angle-data sensors; 7, 4, 11, 3 are torque sensors; 13 is the outer gimbal; 1, 17 are shunts; 18, 10 are rotating units; 8 is the platform; 9, 6 are accelerometers; 14, 19 are dynamically adjusted gyroscopes

to the horizon plane by the signals of the corresponding accelerometers using integral correction. The directional gyroscope is installed on a separate rotary base. It should be noted that in the given scheme (Fig. 3), the vertical gyro reference is also installed on a rotating base. This is done to ensure checks of the directional gyroscope.

A feature of the considered system is the distribution of control functions. At the same time, the position of the main axis of the dynamically adjusted gyroscope is controlled by the signals of accelerometers. Stabilizing motors ensure the perpendicular position of the main axis relative to the stabilized platform. This position coincides with the orientation of the dynamically adjusted gyroscope rotor. Such a distribution of functions ensures the formation of comparatively few control moments in the torque sensors of the dynamically adjusted gyroscopes [13].

Another feature of the considered system is the use of a two-stage gyro-stabilized platform. At the same time, the platform rotates with the object in the azimuthal plane. On contrary to the standard version with a platform using three-axe gimbals, this ensures an invariant position of the axis of the outer frame of the platform to the North. This feature makes the system susceptible to perturbations due to changes in the course of the moving object, as well as the Earth's elliptical shape. To compensate for such perturbations, there is a need to use data about the linear velocity, as well as the course of the object and the rate of change of course.

When compiling a mathematical description of the researched system, it is necessary to apply a trajectory coordinate system. This greatly complicates the algorithms of the studied system in terms of its functionality. The system is close to an attitude and heading reference system and requires the formation of appropriate corrections by accelerometer signals [14, 15].

3 Algorithm of Determining Errors

We will assume that the gyro-stabilized platform repeats the movement of the Rezal axes of the vertical gyro reference with accuracy to the stabilization errors. At the same time, platform stabilization errors are determined by angles α_r, β_r. The equations of motion of the vertical gyro reference can be obtained on the basis of the precession equations of motion of gyroscopes with dynamic adjustment. They have the form [4, 6] as presented in the system (1))

$$\begin{cases} I\ddot{\alpha}_{\mathrm{r}} - H\dot{\beta}_r + c\alpha_r + d\dot{\alpha}_r - \frac{H}{T}\beta_r = H_1\omega_x + I\dot{\omega}_{\mathrm{y}} - k_i\int\limits_0^t W_y + M_y^b \\ I\ddot{\beta}_{\mathrm{r}} + H\dot{\alpha}_r + c\beta_r + d\dot{\beta}_r - \frac{H}{T}\alpha_r = -H_1\omega_y + I\dot{\omega}_x - k_i\int\limits_0^t W_x + M_x^b \end{cases}; \qquad (1)$$

where H is the kinetical moment; c is the gimbals remanent inflexibility; d is the subsidence ratio; T is the constant time of the gyroscope precession; $H_1 = H(1+s)$; $s = 1 \cdot 10^{-3}$; I is the equatorial moment of the gimbals frames and the gyroscope rotor; ω_x, ω_y are projections of the platform's angle velocity; $\dot{\omega}_x$, $\dot{\omega}_y$ are projections of angular accelerations of the platform; k_i is the factor of the integrating correction; W_x, W_y are projections of seeming acceleration of the platform onto the sensitivity axes of accelerometers; M_x^b, M_y^b are moments caused by perturbation actions. To ensure the condition of system stability, the coefficient of integral correction should be determined as $k_i = \frac{H_\perp}{R_M}$, where R_M is the basic radius of bending the surface of the Earth's ellipsoid in the meridian plane.

To determine the projections of the angle velocity of the platform, it is necessary to project the angle velocities of the initial trajectory coordinate system on the axis associated with the platform. To evaluate the errors of a virtually vertical line, it is expedient to define projections as functions of the angles α_r, β_r. These angles determine the location of the vertical gyroscope reference concerning the platform and represent platform stabilization errors. The angles α, β determine the location of the vertical gyroscope reference concerning the horizontal plane. They represent errors of the virtual vertical line [16–18].

The corresponding matrices of directing cosines between the trajectory coordinate system and the system of coordinates connected to the vertical gyro reference C_1 and between the coordinate system connected to the platform and the coordinate system connected to the vertical gyro reference C_2 have the forms, respectively, (2) and (3)

$$C_1 = \begin{bmatrix} \cos\alpha & 0 & -\sin\alpha \\ \sin\alpha \cdot \sin\beta & \cos\beta & \cos\alpha \cdot \sin\beta \\ \sin\alpha \cdot \cos\beta & -\sin\beta & \cos\alpha \cdot \cos\beta \end{bmatrix}; \tag{2}$$

$$C_2 = \begin{bmatrix} \cos\alpha_r & 0 & -\sin\alpha_r \\ \sin\alpha_r \cdot \sin\beta_r & \cos\beta_r & \cos\alpha_r \cdot \sin\beta_r \\ \sin\alpha_r \cdot \cos\beta_r & -\sin\beta_r & \cos\alpha_r \cdot \cos\beta_r \end{bmatrix}; \tag{3}$$

Considering the angles α_r, β_r, α, β to be small, after simplifications we obtain the matrix of directing cosines of the trajectory coordinate system and the coordinate system connected to the platform:

$$C_2C_1 = \begin{bmatrix} 1+\alpha\alpha_r\beta\beta_r+\alpha\alpha_r & \alpha_r\beta_r-\beta\alpha_r & -\alpha+\alpha_r\beta\beta_r+\alpha_r \\ \alpha\beta-\alpha\beta_r & 1+\beta\beta_r & \beta-\beta_r \\ -\alpha_r+\alpha\beta\beta_r+\alpha & \beta_r-\beta & \alpha\alpha_r+\beta\beta_r+1 \end{bmatrix} \approx$$

$$\approx \begin{bmatrix} 1 & 0 & -\alpha+\alpha_r \\ 0 & 1 & \beta-\beta_r \\ \alpha-\alpha_r & -\beta+\beta_r & 1 \end{bmatrix}. \tag{4}$$

Conformably, the projections of the angle velocity of the platform onto its axes based on expression (1) and matrices C_1, C_2 described by expressions (2)–(4) are determined as follows:

$$\begin{aligned} \omega_x &= \dot{\beta} - \dot{\beta}_r + \omega_\zeta - \omega_\xi(\alpha - \alpha_r); \\ \omega_y &= \dot{\alpha} - \dot{\alpha}_r + \omega_\eta + \omega_\xi(\beta - \beta_r); \end{aligned} \tag{5}$$

In Eqs. (5), ω_ζ, ω_η, ω_ξ represent the projections of the angler velocity of the trajectory coordinate system onto its axes, which concerning [7] have the form, respectively, (6) and (7)

$$\begin{cases} \omega_\zeta = -\frac{V_\eta \sin K + V_\zeta \cos K}{R_1}\sin K - \frac{V_\eta \cos K - V_\zeta \sin K}{R_M}\cos K - \Omega\cos\phi\sin K; \\ \omega_\eta = \frac{V_\eta \sin K + V_\zeta \cos K}{R_1}\cos K - \frac{V_\eta \cos K - V_\zeta \sin K}{R_M}\sin K + \Omega\cos\phi\cos K; \\ \omega_\xi = \frac{V_\eta \sin K + V_\zeta \cos K}{R_1} tg\phi - \dot{K} + \Omega\sin\phi; \end{cases} \tag{6}$$

where V_ζ, V_η, V_ξ are lateral, longitudinal, and vertical projections of the moving vehicle's velocity; K is the course of the moving vehicle; R_1 is the radius of bending of the terrestrial ellipsoid surface; Ω is the angle velocity of the daily rotation of the Earth; $\dot{K}$ is the angle velocity of changes of the course [19, 20].

Projections of the acceleration vector of the platform onto the axis of the coordinate system connected to the moving object are determined as follows [7, 8]

$$\begin{bmatrix} W_{x_0} \\ W_{y_0} \\ W_{z_0} \end{bmatrix} = \mathrm{C}_1 \begin{bmatrix} W_\zeta \\ W_\eta \\ W_\xi \end{bmatrix} + \begin{bmatrix} \dot{\omega}_{x_0} z_\mathrm{p} - \dot{\omega}_{z_0} y_\mathrm{p} \\ \dot{\omega}_{z_0} x_\mathrm{p} - \dot{\omega}_{x_0} z_\mathrm{p} \\ \dot{\omega}_{x_0} y_\mathrm{p} - \dot{\omega}_{y_0} x_\mathrm{p} \end{bmatrix} + \\ + \begin{bmatrix} \omega_{y_0}(\omega_{x_0} y_\mathrm{p} - \omega_{y_0} x_\mathrm{p}) - \omega_{z_0}(\omega_{z_0} x_\mathrm{p} - \omega_{x_0} z_\mathrm{p}) \\ \omega_{z_0}(\omega_{y_0} z_\mathrm{p} - \omega_{z_0} y_\mathrm{p}) - \omega_{x_0}(\omega_{x_0} y_\mathrm{p} - \omega_{y_0} x_\mathrm{p}) \\ \omega_{x_0}(\omega_{z_0} x_\mathrm{p} - \omega_{x_0} z_\mathrm{p}) - \omega_{y_0}(\omega_{y_0} z_\mathrm{p} - \omega_{z_0} y_\mathrm{p}) \end{bmatrix}; \tag{7}$$

where W_{x_0}, W_{y_0}, W_{z_0} represent projections of the susceptible acceleration of the platform onto the axes of the body-axis coordinate system; ω_{x_0}, ω_{y_0}, ω_{z_0} are projections of moving vehicle absolute angle velocity; x_p, y_p, z_p are coordinates of the platform, which define the place of location of the platform concerning the center of a mass of the moving vehicle.

The matrices of directing cosines between the coordinate system associated with the moving vehicle and the trajectory and platform coordinate systems C_3, C_4 have the form, respectively, (8), (9)

$$C_3 = \begin{bmatrix} \cos\gamma & \sin\theta \cdot \sin\gamma & -\cos\theta \cdot \sin\gamma \\ 0 & \cos\theta & \sin\theta \\ \sin\gamma & -\sin\theta \cdot \cos\gamma & \cos\theta \cdot \cos\gamma \end{bmatrix}; \tag{8}$$

$$C_4 = \begin{bmatrix} \cos\alpha_p & 0 & -\sin\alpha_p \\ \sin\alpha_p \cdot \sin\beta_p & \cos\beta_p & \cos\alpha_p \cdot \sin\beta_p \\ \sin\alpha_p \cdot \cos\beta_p & -\sin\beta_\pi & \cos\alpha_p \cdot \cos\beta_p \end{bmatrix}. \tag{9}$$

The projections of the object's absolute angular velocity on its axes are as follows

$$\begin{cases} \omega_{x_0} = (\omega_\zeta + \dot{\theta})\cos\gamma + \omega_\eta \sin\gamma \sin\theta - \omega_\xi \sin\gamma \cos\theta; \\ \omega_{y_0} = \dot{\gamma} + \omega_\eta \cos\theta + \omega_\xi \sin\theta; \\ \omega_{z_0} = (\omega_\zeta + \dot{\theta})\sin\gamma - \omega_\eta \cos\gamma \sin\theta + \omega_\xi \cos\gamma \cos\theta. \end{cases} \tag{10}$$

One of the most important points for determining errors in the inertial stabilization and navigation system is the necessity to determine corrections for calculating accelerations. These corrections are caused by the daily rotation of the Earth, the motion of a moving vehicle, and also Coriolis acceleration.

Taking into account the formulas (5)–(10) including matrices C_3, C_4, the expressions for determining the projections of the susceptible acceleration on the axes of the platform

could be calculated in the following way

$$\begin{gathered}
W_x = \cos\alpha_\pi\cos\gamma W_\zeta + \cos\alpha_\pi\sin\theta\sin\gamma W_\eta + \cos\alpha_\pi\sin\gamma W_\xi + \\
+\cos\alpha_\pi\dot{\omega}_{y_0}z_\mathrm{p} - \cos\alpha_\pi\dot{\omega}_{z_0}y_\mathrm{p} + \\
+\cos\alpha_\pi[\omega_{y_0}(\omega_{x_0}y_\mathrm{p} - \omega_{y_0}x_\mathrm{p}) - \omega_{z_0}(\omega_{z_0}x_\mathrm{p} - \omega_{x_0}z_\mathrm{p})] - \\
-\sin\alpha_\pi\sin\gamma W_\zeta + \sin\alpha_\pi\sin\theta\cos\gamma W_\eta - \sin\alpha_\pi\cos\theta\cos\gamma W_\xi - \\
-\sin\alpha_\pi(\dot{\omega}_{x_0}y_\mathrm{p} - \dot{\omega}_{y_0}x_\mathrm{p}) - \\
-\sin\alpha_\pi[\omega_{x_0}(\omega_{z_0}x_\mathrm{p} - \omega_{x_0}z_\mathrm{p}) - \omega_{y_0}(\omega_{y_0}z_\mathrm{p} - \omega_{z_0}y_\mathrm{p})]; \\
W_y = \sin\alpha_\pi\sin\beta_\pi\cos\gamma W_\zeta + \sin\alpha_\pi\sin\beta_\pi\sin\theta\sin\gamma W_\eta + \sin\alpha_\pi\sin\beta_\pi\sin\gamma W_\xi + \\
+\sin\alpha_\pi\sin\beta_\pi(\dot{\omega}_{y_0}z_\mathrm{p} - \dot{\omega}_{z_0}y_\mathrm{p}) + \\
+\sin\alpha_\pi\sin\beta_\pi[\omega_{y_0}(\omega_{x_0}y_\mathrm{p} - \omega_{y_0}x_\mathrm{p}) - \omega_{z_0}(\omega_{z_0}x_\mathrm{p} - \omega_{x_0}z_\mathrm{p})] + \\
+\cos\beta_\pi\cos\theta W_\eta + \cos\beta_\pi\sin\theta W_\xi + \\
+\cos\beta_\pi(\dot{\omega}_{z_0}x_\mathrm{p} - \dot{\omega}_{x_0}z_\mathrm{p}) + \\
+\cos\beta_\pi[\omega_{z_0}(\omega_{y_0}z_\mathrm{p} - \omega_{z_0}y_\mathrm{p}) - \omega_{x_0}(\omega_{x_0}y_\mathrm{p} - \omega_{y_0}x_\mathrm{p})] + \\
+\cos\alpha_\pi\sin\beta_\pi\sin\gamma W_\zeta - \cos\alpha_\pi\sin\beta_\pi\sin\theta\cos\gamma W_\eta + \cos\alpha_\pi\sin\beta_\pi\cos\theta\cos\gamma W_\xi + \\
+\cos\alpha_\pi\sin\beta_\pi(\dot{\omega}_{x_0}y_\mathrm{p} - \dot{\omega}_{y_0}x_\mathrm{p}) + \\
+\cos\alpha_\pi\sin\beta_\pi[\omega_{x_0}(\omega_{z_0}x_\mathrm{p} - \omega_{x_0}z_\mathrm{p}) - \omega_{y_0}(\omega_{y_0}z_\mathrm{p} - \omega_{z_0}y_\mathrm{p})]; \\
W_z = \sin\alpha_\pi\cos\beta_\pi\cos\gamma W_\zeta + \sin\alpha_\pi\cos\beta_\pi\sin\theta\sin\gamma W_\eta + \sin\alpha_\pi\cos\beta_\pi\sin\gamma W_\xi + \\
+\sin\alpha_\pi\cos\beta_\pi(\dot{\omega}_{y_0}z_\mathrm{p} - \dot{\omega}_{z_0}y_\mathrm{p}) + \\
+\sin\alpha_\pi\cos\beta_\pi[\omega_{y_0}(\omega_{x_0}y_\mathrm{p} - \omega_{y_0}x_\mathrm{p}) - \omega_{z_0}(\omega_{z_0}x_\mathrm{p} - \omega_{x_0}z_\mathrm{p})] - \\
-\sin\beta_\pi\cos\theta W_\eta - \sin\beta_\pi\sin\theta W_\xi - \\
-\sin\beta_\pi(\dot{\omega}_{z_0}x_\mathrm{p} - \dot{\omega}_{x_0}z_\mathrm{p}) - \\
-\sin\beta_\pi[\omega_{z_0}(\omega_{y_0}z_\mathrm{p} - \omega_{z_0}y_\mathrm{p}) - \omega_{x_0}(\omega_{x_0}y_\mathrm{p} - \omega_{y_0}x_\mathrm{p})] + \\
+\cos\alpha_\pi\cos\beta_\pi\sin\gamma W_\zeta - \cos\alpha_\pi\cos\beta_\pi\sin\theta\cos\gamma W_\eta + \cos\alpha_\pi\cos\beta_\pi\cos\theta\cos\gamma W_\xi + \\
+\cos\alpha_\pi\cos\beta_\pi(\dot{\omega}_{x_0}y_\mathrm{p} - \dot{\omega}_{y_0}x_\mathrm{p}) + \\
+\cos\alpha_\pi\cos\beta_\pi[\omega_{x_0}(\omega_{z_0}x_\mathrm{p} - \omega_{x_0}z_\mathrm{p}) - \omega_{y_0}(\omega_{y_0}z_\mathrm{p} - \omega_{z_0}y_\mathrm{p})];
\end{gathered} \tag{11}$$

In expressions (11), the projections of the acceleration of the object's centre of mass on the axis of the trajectory coordinate system have the form

$$\begin{gathered}
W_\zeta = \dot{V}_\zeta + V_\xi\omega_\eta - V_\eta\omega_\xi + V_\xi\Omega\cos\phi\cos K - V_\eta\Omega\sin\phi; \\
W_\eta = \dot{V}_\eta + V_\zeta\omega_\xi - V_\xi\omega_\zeta + V_\zeta\Omega\sin\phi + V_\xi\Omega\cos\phi\sin K; \\
W_\xi = \dot{V}_\xi + V_\eta\omega_\zeta - V_\zeta\omega_\eta - V_\eta\Omega\cos\phi\sin K - V_\zeta\Omega\cos\phi\cos K - g.
\end{gathered} \tag{12}$$

Equations (1), supplemented by formulas (5)–(12), describe a model of vertical gyroscope errors.

The equations of motion of the gyroscope, which performs the functions of a course-measuring instruments, are as follows:

$$\begin{cases}
I\ddot{\alpha}_K + H\dot{\beta}_K + c\alpha_K + d\dot{\alpha}_K + \frac{H}{T}\beta_K = -H_1\omega_{x_1} + M_{z_1}^b + M_{z_1}^{\text{y}\pi\text{p}}; \\
I\ddot{\beta}_K + H\dot{\alpha}_K - c\beta_K - d\dot{\beta}_K + \frac{H}{T}\alpha_K = H_1\omega_{z_1} - M_{x_1}^b - M_{x_1}^{\text{y}\pi\text{p}}.
\end{cases} \tag{13}$$

The expressions for determining the angular velocity projections of the rotary device of the heading gyroscope have the form as presented in (14)

$$\begin{cases}
\omega_{x_1} = \omega_x\cos(K_1 + \Delta K) + \omega_\text{y}\sin(K_1 + \Delta K); \\
\omega_{z_1} = \omega_z + \dot{K} + \Delta\dot{K}.
\end{cases} \tag{14}$$

Accordingly, the control moments can be defined as follows

$$\begin{cases} M_{z_1}^{\text{упр}} = \text{H}_1 k_z \cdot W_y / g + \text{H}_1 \omega_{x_1}^{\text{кор}}; \\ M_{x_1}^{\text{упр}} = \text{H}_1 k_x \cdot W_x / g + \text{H}_1 \omega_{z_1}^{\text{кор}}. \end{cases} \tag{15}$$

where k_x, k_z are transfer coefficients of control loops; W_x, W_y are accelerations; $\omega_{x_1}^{cor}$, $\omega_{z_1}^{cor}$ are correction speeds. In its turn, these speeds are defined by expressions

$$\begin{gathered} \omega_{x_1}^{cor} = \omega_\zeta \cos(K - K_1) + \omega_\eta \sin(K - K_1) = \\ = \left(-\frac{V_\eta \sin K + V_\zeta \cos K}{R_1} \sin K - \frac{V_\eta \cos K - V_\zeta \sin K}{R_M} \cos K - \Omega \cos\phi \sin K\right) \cos(K - K_1) + \\ + \left(\frac{V_\eta \sin K + V_\zeta \cos K}{R_1} \cos K - \frac{V_\eta \cos K - V_\zeta \sin K}{R_M} \sin K + \Omega \cos\phi \cos K\right) \sin(K - K_1); \\ \omega_{z_1}^{cor} = \frac{V_\eta \sin K + V_\zeta \cos K}{R_1} tg\phi - \dot{K} + \Omega \sin\phi. \end{gathered} \tag{16}$$

The equations of motion of the heading gyroscope (13), supplemented by expressions (14)–(16) represent the model of errors of the course measuring instrument.

4 Modelling Results

The main objective of modelling is obtaining assessments of the accuracy of stabilization and navigation processes. To achieve this goal, models of stabilization and navigation errors in the software MATLAB have been developed. More precisely, these models have been created using Simulink Toolbox. The results of determining errors of the inertial stabilization and navigation system in the form of transient processes for regular and irregular perturbations are presented in Figs. 4, 5, 6 and 7.

Simulation has been carried out for the gimballed inertial navigation system assigned for a marine vehicle. Simulation results are given in the form of transient processes with the aim of taking into consideration the dynamics of stabilization. Figures 4, and 5 show transient processes of stabilization by angles of the pitch and roll for regular perturbations.

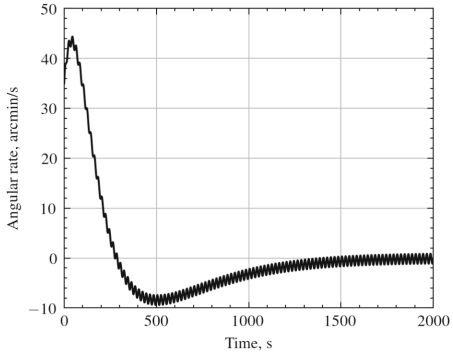

Fig. 4. The transient process of the error on the angle α for the regular perturbations.

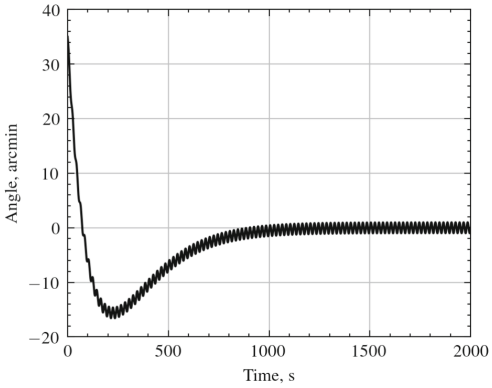

Fig. 5. The transient process of the error on the angle β for the regular perturbations.

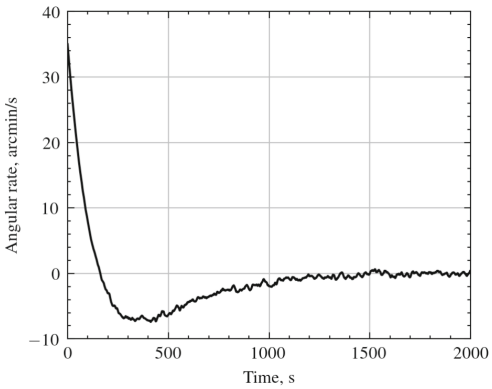

Fig. 6. The transient process of the error on the angle α for irregular perturbations.

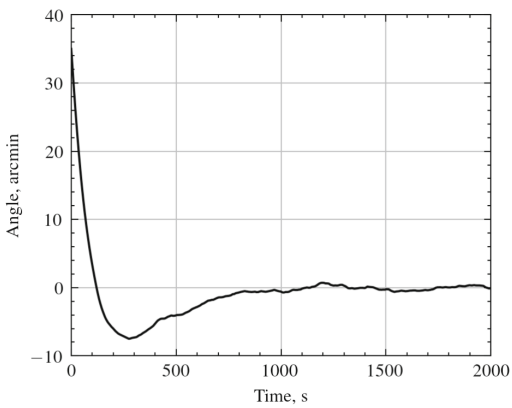

Fig. 7. The transient process of the error on the angle β for irregular perturbations.

Figures 6, and 7 present transient processes of stabilization by angles of the pitch and roll for irregular perturbations. Regular perturbations are given as harmonic signals with constant magnitude and frequency. Irregular perturbations are simulated based on known spectral densities of sea irregularities. Hence, both deterministic and stochastic factors are taken into consideration.

The represented results of the simulation show the high accuracy of the studied stabilization and navigation system. Magnitudes of stabilization errors do not exceed 1.5–2 arcmins and 3 arcmins for regular and irregular perturbations respectively.

5 Conclusions

The expressions for determining errors of the gimballed stabilization and navigation system are derived.

The important feature of the research is obtaining of expressions for determining phantom accelerations using basic concepts of the theoretical mechanics and directional cosines between the trajectory and body axes systems of coordinates. The obtained expressions ensure improving the accuracy of both stabilization and navigation processes. The complex conversions between different coordinate systems are defined in the form of directing cosines.

The novelty of the research lies in obtaining expressions based on using the trajectory coordinate system contrary to the well-known geographical coordinate system.

The expressions for correction of methodical errors for readings of accelerometers have been derived.

The obtained expressions for correction take into consideration the daily rotation of the Earth, the motion of the moving vehicle, and Coriolis acceleration.

The simulation of errors in the stabilization and navigation systems has been carried out. The simulation results prove the high accuracy of the studied system.

In future work, it is proposed to study errors of similar systems with a three-axis gimballed platform.

6 Discussion

The aim of the paper was to the research accuracy characteristics of the inertial navigation system with a two-axis gimballed platform. The research is based on the creation of models of stabilization and navigation system errors. The main result of the research is the creation of a model for determining errors in the precision navigation system with a two-axis gyro-stabilized platform. The developed model takes into consideration kinematical relationships and features of control in modes of gyroscopic vertical and course determination. The results of the research prove the possibility of the usage of the gimballed inertial navigation system in applications that require high precision of navigation and stabilization processes.

The obtained results can be useful for gimballed inertial navigation systems operated on moving vehicles of the wide class. In other words, obtained results could be spread on aerial and land moving vehicles. It should be noted that the accuracy of stabilization

leads to comfortable conditions of operation of inertial measuring instruments including gyroscopic devices and accelerometers.

In future work, it is proposed to apply this approach for systems, which combine navigation of moving objects and stabilization of payload mounted on these objects such as observation and measuring devices. Also, the creation of a full model including control features and the dynamics of moving objects is planned.

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