Application of Extended Bass Approach to Statistical Description of Some Cases of Distributed Finances on Example of Stablecoins

Victor Dostov[1,2(✉)], Pavel Shust[2], and Svetlana Krivoruchko[1]

[1] Department of Banking and Monetary Regulation, Financial University Under the Government of the Russian Federation, 49 Leningradsky Prospekt, Moscow 125993, Russia
dostov@npaed.ru

[2] Russian Electronic Money and Remittance Association, Novoslobodskaya 20, Moscow 107078, Russia

Abstract. Financial instruments based on blockchain technologies known as Distributed Finances (DeFi), including cryptocurrencies, smart-contract based on tokens, central bank digital currencies (CBDC), are now actively spreading around the world. They acquire a visible share in financial ecosystems, and this share is growing fast. It is important to have some instruments to predict at least some of parameters of their growth. In the article we propose an extension of Bass innovation diffusion model to DeFi on example of widespread stablecoins to describe dynamics of their extension. Specifically, we consider how a number of active users (active addresses) evolve over time, basing on equation of extended Bass innovation diffusion model. The model results describe well qualitative trend and average quantitative behavior of two widespread stablecoins. We also provide a modification of extended Bass equations which is more convenient for stablecoins dynamic approximation and interpretation. Beside usefulness of results for DeFi market analysis, we confirm applicability of innovation diffusion model to payment markets once again. We also discuss limitations of model and possible ways of its development, including introducing competition between different stablecoins.

Keywords: Distributed finances · DeFi dynamics · Bass innovation diffusion · USDT · USDC

1 Distributed Finances Scope

DeFi is an umbrella term for wide class of payment, investing and other financial instruments [1] based on blockchain technology. In turn, blockchain is distributed leger (DL) technology which allow to build extremely reliable and untampered databases which do not depend on single entity, such as database server but store a huge number of database copies in independent nodes under control of blockchain participants. DeFi, correspondingly, relates to blockchains containing financially related information [2].

© The Author(s), under exclusive license to Springer Nature Switzerland AG 2024
O. Gervasi et al. (Eds.): ICCSA 2024 Workshops, LNCS 14815, pp. 238–248, 2024.
https://doi.org/10.1007/978-3-031-65154-0_15

Some researchers [3] narrows this definition, limiting DeFi by smart-contract based products. Smart-contracts in this content are programs, which, using special technologies, are stored and executed in blockchain, processing financial information. However, we include in term DeFi all distributed financial services, specifically cryptocurrencies and CBDC.

First cryptocurrency, Bitcoin, was introduced in 2008 by Satoshi Nakamoto [4] and now is a biggest DeFi in the world in terms of capitalization [5]. A big number of clones and modification to follow, successful or not [2]. As a result, now we have almost ten thousand cryptocurrencies in circulation. For us, the following features of cryptocurrencies are important. First, cryptocurrencies are DL based, that means they are single level person to person (p2p) payment technologies without any centralized governance. All parameters such as emission, commissions, etc., are either defined algorithmically or defined by users. Second, in the lack of centralization, e.g. governmental control any person or company can start and run the own cryptocurrency, if he or she has enough competence and financial resources and related demand. In other worlds, cryptocurrencies are, opposite to traditional government-lead fiat money, the product which is determined in great extent by market demand [6]. To be exact, some government regulations of cryptocurrencies already exist [7], but their influence on market development is still relatively weak.

The basic property of bitcoin and other cryptocurrencies is sheer fluctuation of their price to fiat currencies. It may be considered as investment opportunity, but in terms of payments for goods and services often is not convenient as well as in other domains [2]. As a solution, stablecoin model was elaborates. Opposite to "classic" DL-based payment technologies, stablecoins are DL-based payment instruments under with (almost) fixed value to fiat currencies. They benefit from advances of DL technologies, from one side, from another allow to control exactly their monetary value.

2 Research Purpose

There are numerous attempts to model and predict DeFi behavior. Most efforts are put into modelling of behavior of price of cryptocurrencies. The prices are volatile and their knowledge would benefit far beyond academic purposes. A lot of approaches have been done using ARIMA [8], neural networks [9], Monte-Carlo simulations and other approaches [10, 11]. However, common believe is that DeFi markets generally fall under general stock market stochastic behavior [12] and there would be no chance for simple model academic advantages here.

Therefore, few years ago, the authors concentrated on prediction of user activity in cryptocurrencies basing on Bass innovation diffusion theory. Surprisingly, despite of exotic technologies underlining, growth of active users is described pretty well by classic approach, and shows similarity with model of "traditional" payment systems. Moreover, qualitative effects are explained even better, such as market saturation, differences of behavior of peer to peer (p2p) payment versus customer to business (c2b) payments and absence of correlation between early-stage success of payment schemes and long-term behavior [13].

In current paper we extend our consideration from "traditional" cryptocurrencies on stablecoin dynamics. We apply our modification of Bass model to dynamics of stablecoin

audience. Stablecoins are cryptocurrencies connected to fiat currencies and contrary to bitcoin and other "classic" cryptocurrencies their values to US dollar and other fiat currencies oscillate in very narrow corridor or are fixed one to one. Here we apply approach early described in [11, 14] with predictable reasonably good results.

3 Stablecoin Definition

As it was briefly aforementioned, stablecoins are modification of cryptocurrencies connected to fiat currencies and contrary to bitcoin and other "classic" cryptocurrencies their values to US dollar and other fiat currencies oscillate in very narrow corridor or are fixed one to one. Here we apply approach early described in [13] with predictable reasonably good results. As it was mentioned, stablecoins are cryptocurrencies which have fixed exchange rate to fiat currencies, mostly to dollars. This may be secured by

- Special, usually oversighted, centralized institution (issuer) which guarantee exchange rate to fiat currencies
- By distributed guarantee supported in "traditional" cryptocurrencies
- By algorithmic trade
- or by combination of these mechanisms [15].

In other aspects stablecoins are pure DeFi, completely private and market driven.

Stablecoins plays important rule in DeFi ecosystems, providing reliable payments and investments, so users activity for them is very high.

4 Case of USDT

First, we consider Tether USDT [16] which first stablecoin announced as early as in 2014. It runs on the platform by Tether Limited and is pegged to US dollar. It is typical DeFi which runs on different DLs. It is issued by Fig. 1 describes dynamics and exchange rate of Tether stablecoin versus time.

Beside fluctuation we can easily identify three typical stages of Bass-complying systems. 2018–2019 we see slow gaining on traction, 2019–2020 sharp growth, 2020–2024 – fluctuation around saturation level.

Following our previous investigation, to descried system behavior we use modified Bass equation [18]:

$$\frac{dx}{dt} = a_p(N - x)x + a_b(N - x) - bx \tag{1}$$

Here the following assumption, mostly following original Bass article [19], are made:

1. There is a limited number of potential users of the product, USDT in our case. Let call it maximal size of audience, N. Number x of them are already active users, N-x are not.
2. In every moment a non-user made a random decision to become a user with probability a_b. This is a decision, independent on other user behavior. Bass associated it with early adoption and/or mass promotion effects, like advertisement.

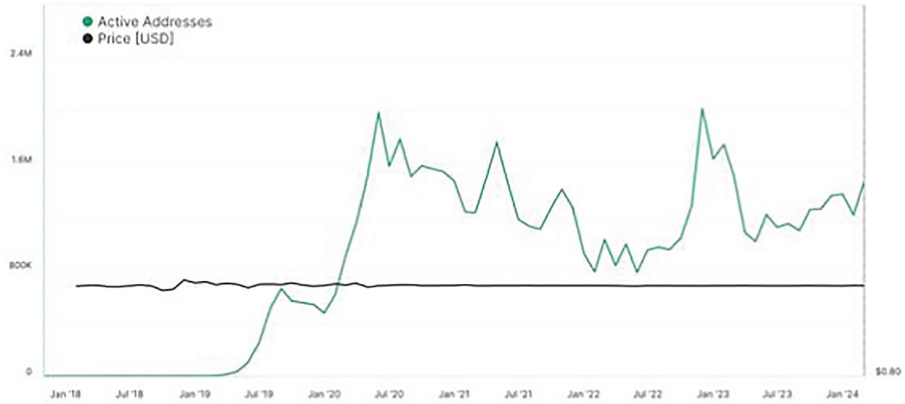

Fig. 1. Tether (USDT) price and active user number versus time [17].

3. In every moment a non-user also made a decision to become a user which is influenced by active users around. Bass called this following. The following probability is a_p. In payments it is especially important, as most of payments are peer to peer interaction.
4. In every moment a user makes a random decision to stop using the product with probability b, due to non-satisfaction, lack of need or any other reason.
5. Starting audience size at $t = 0$ is x_0. This parameter is introduced to describe systems, data for which is not available from very beginning. Actually, we can safely suppose x_0 negatively small for a lot of cases, including stablecoins.

Equation (1) is, actually, Ricatti equation [20]:

$$\frac{dx}{dt} = -a_p x^2 + lx + a_b N \tag{2}$$

where

$$l = a_p N - a_b - b \tag{3}$$

The solution of this equation gives

$$x = \frac{1}{2a_p}\left(D \tanh\left(\frac{1}{2}tD - C\right) + l\right) \tag{4}$$

where

$$D^2 = l^2 + 4a_b a_p N \tag{5}$$

which defines the reverse typical time of audience change. For $t \to \infty$ (11) transforms into

$$x_\infty = \frac{1}{2a_p}(D + l) \tag{6}$$

giving the maximum active audience size that the system aims for a long time.

The constant C can be found from the initial condition x(0) = x0

$$C = \operatorname{arctanh}\left(\left(l - 2a_p x_0\right)/D\right)) \tag{7}$$

As it was shown earlier [18] we can formally also rewrite Eq. (1) in a more general dimensionless form redefining the values to dimensionless

$$\chi \to \left(2a_p x - l\right)/D \tag{8}$$

$$\tau \to \frac{Dt}{2} - C \tag{9}$$

and getting the most general form of the solution (11) as

$$\chi = \tanh \tau \tag{10}$$

This means that evolution of all payment systems, DeFi or "classic" falling under extended Bass conditions 1–5 comes along singe dimensionless trajectory. In Fig. 2 it is shown for different payment systems.

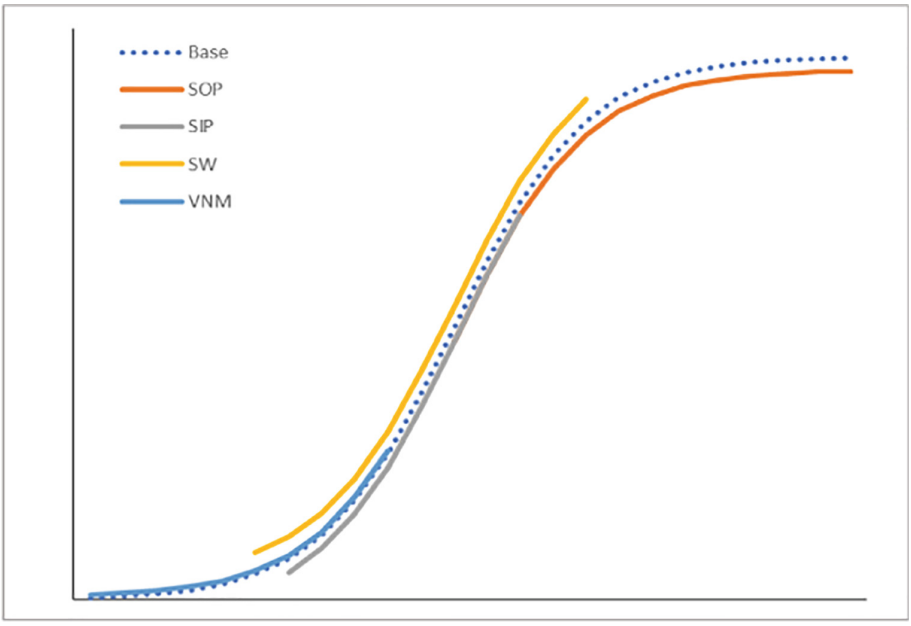

Fig. 2. Base trajectory (10) and different case of placement of payment system trajectories on it. Taken from [18].

Now, we can apply these equations to initial data of Fig. 1. For more evident interpretation we will use intermediate form between (4) and (10). We also can safely suppose $x_0 = 0$, thus obtaining

$$x = \frac{x_\infty}{2}(\tanh\left(\frac{Dt}{2} - C\right) + 1) \tag{11}$$

Here, evidently, C corresponds to time in which fastest growth occurs, t = 2s/D.

Sometimes, it is more convenient, to exclude high frequency oscillations due to high market volatility by analyzing quarterly values and/or taking average for long periods. Averages are given in Table 1.

Table 1. Active USDT addresses number, averaged by years.

Year	Average active USDT addresses number
2018	1044,667
2019	269669,6
2020	1352683
2021	1323774
2022	1047334
2023	1288761
2024	1349838

Here saturation effect is especially evident. To obtain parameters in (11) we can apply least square method both to initial data as well as to averaged data obtaining Table 2.

Table 2. Parameters for USDT.

x_∞	D	C
1380000	0,4 1/month	4,8

Correspondent curve is shown on Fig. 3.

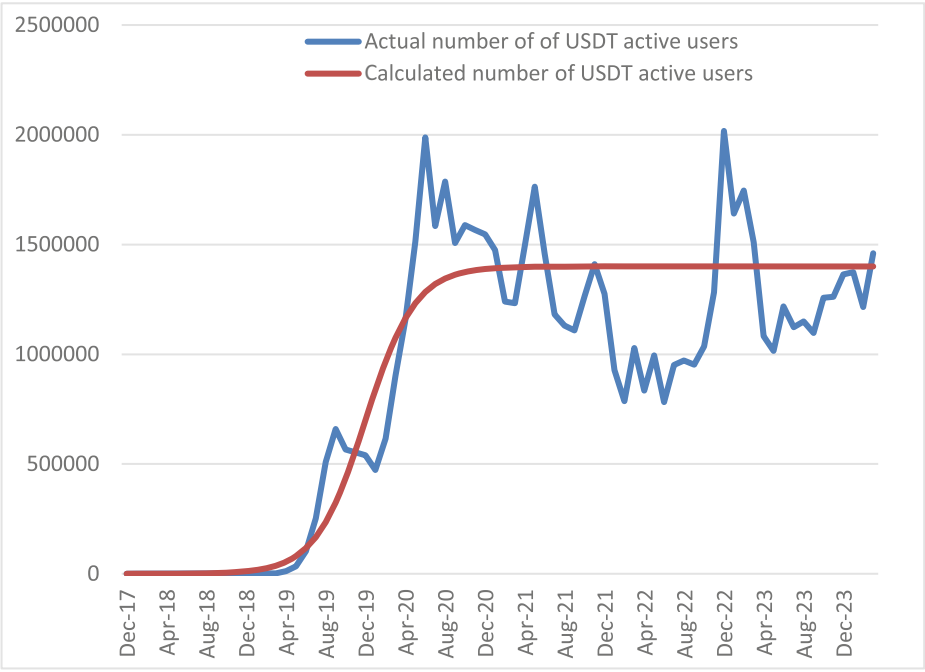

Fig. 3. Actual and calculated number of USDT active users.

Concerning empirical nature of Bass approach that parameters must be considered as qualitative, so fine fitting does not make sense. However, we see quite reasonable coincidence of model with real data and trend.

5 USD Coin

USD Coin is another example of popular stablecoins. It is run by consortium Circle [21] and is also pegged to US dollar. It is typical DeFi as well and as USDT it runs on different DLs. The dynamics demonstrated is similar to USDT. Applying the same approach as to USDT, we obtain similar results in Fig. 4, Tables 3 and 4.

As we see, parameters for USDC are close to USDT ones, but USDC has more slow penetration and less users. Qualitative but general in behavior is generally the same. Both curves are positioned on the same part of general trajectory on Fig. 5. Surprisingly, both real and calculated data supports the fact of saturation of size of audience, even if DeFi market is growing in total volume. This mean that specific sectors using these stablecoins lags from general market development.

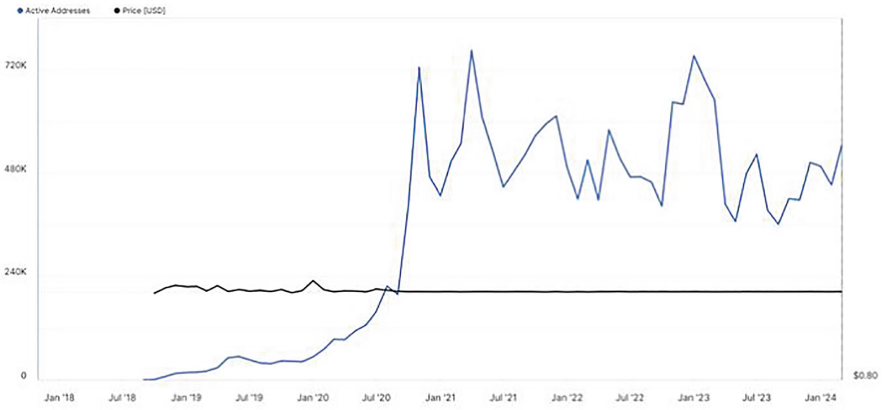

Fig. 4. USD Coin (USDC) price (gray line) and active user number (blue line) versus time [22].

Table 3. Active USDC addresses number, averaged by years.

Year	Average active USDC addresses number
2018	5881
2019	36602,83
2020	227520,4
2021	552035
2022	502833,5
2023	498207
2024	498078

Table 4. Parameters for USDC.

x_∞	D	C
500000	0,25 1/month	6

6 Some Common Outcomes

Another useful interpretation, common for both USDT and USDC arises from meaning of parameters of Eq. (1). As it was noted in [18] that relation between a_p which is responsible for peer-to-peer interaction and a_b, which is responsible for independent single person adoption is shifted to a_p in cryptocurrencies targeted to peer-to-peer payments. Value of $C > 0$ obtained from real data indicates clear peer to peer dominance in stablecoins which fit reality.

We may also note that deriving extended Bass equation, we assume that in average, the behavior of the masses, described by a huge set of parameters, is equivalent to

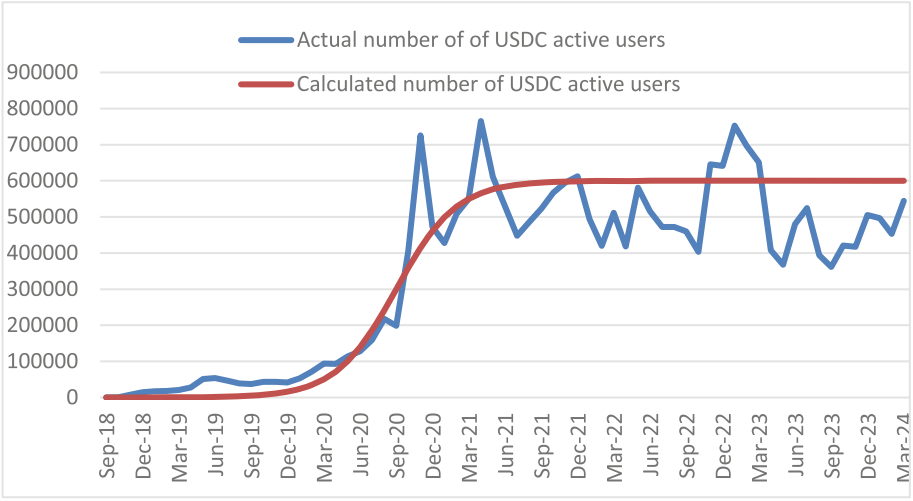

Fig. 5. Actual and calculated number of USDC active users.

the behavior of a single user, described by a small number of statistically measurable parameters. This may be called ergodic hypothesis [23] for payments. Note that the currently accepted approach to describing Bass-like diffusion of innovations explicitly or implicitly relies on a similar assumption about the possibility to empirically describe the behavior of complex systems by a small number of parameters. This paper introduces one more case which confirms basic Bass approach.

7 Conclusion

Here we see, that our model represents stablecoins trend pretty well. Evidently, it cannot predict external-driven fluctuation which are connected to general trends on cryptomarkets, but for average values we have good predictions. The generalized Bass model describes adequately the initial stage of getting traction, sharp growth stage and, eventually, slow move to saturation. Also, qualitative description is reasonably accurate. Previously, authors proposed a combination of Bass model and an ARMA statistical approach [24], which can also be used to develop the model for stablecoins. The model will also allow to describe some of the usage fluctuations.

Of course, the model has some limitation. Some of the parameters introduced into the model are constant. This is always true for the short periods of time and might be a good hypothesis for middle-term behavior. In longer-term horizons, technology modifications or regulatory changes may change usability and attractiveness of the product. Then, parameters will need to be changed accordingly. Also, competition between different stablecoins is not taken into account in the research: usually, stablecoins with big audience cannibalize those with smaller user base. More exact model must include a system of Bass equation with cross-terms which describe mutual influence. This may be subject of further studies.

Another interesting application of this approach is dynamics of central bank digital currencies, CBDC, which are being introduced now in different countries. In a sense, CBDC are also stablecoins, but issued by a government [25]. After accumulation of enough amount of statistic we can apply the model to analyze CBDC dynamics and its deviations from dynamics of private stablecoins.

References

1. Manda, V.K., et al.: Revolutionizing Finance with Decentralized Finance (DeFi). In: Decentralizing the Online Experience with Web3 Technologies, edited by Dina Darwish, pp. 127–150. Hershey, PA: IGI Global (2024). https://doi.org/10.4018/979-8-3693-1532-3.ch006
2. Birch, D.: The Currency Cold War: Cash and Cryptography Hash Rates and Hegemony. Publishing Partnership, London (2020)
3. Aquilina, M., Frost, J., Schrimpf, A.: Decentralized finance (DeFi): a functional approach. J. Fin. Regulation (2024). https://doi.org/10.1093/jfr/fjad013
4. Satoshi Nakamoto: Bitcoin: A Peer-to-Peer Electronic Cash System. https://www.bitgo.com/content-library/bitcoin.pdf
5. Today's Cryptocurrency Prices by Market Cap. https://coinmarketcap.com/
6. Dostov, V., Shust, P.: Cryptocurrencies: an unconventional challenge to the AML/CFT regulators? J Fin. Crime **21**(3), 249–263 (2014). https://doi.org/10.1108/JFC-06-2013-0043
7. Alekseenko, A.P.: Model framework for consumer protection and crypto-exchanges regulation. J. Risk Fin. Manage. **16**(7), 305 (2023). https://doi.org/10.3390/jrfm16070305
8. Gunawan, D., Febrianti, I.: Ethereum value forecasting model using autoregressive integrated moving average (ARIMA). Int. J. Adv. Soc. Sci. Hum. **2**(1), 29–35 (2023). https://doi.org/10.56225/ijassh.v2i1.151
9. Dempere, J.M., Il-Agure, Z., Allam, H., Ramasamy, L.K., Alqasemi, S.S.: The Influence of AI-identified consumer sentiment metrics on crypto-asset prices. In: 9th International Conference on Information Technology Trends (ITT), pp. 90–95. Dubai, United Arab Emirates, (2023). https://doi.org/10.1109/ITT59889.2023.10184246
10. Likitratcharoen, D., Chudasring, P., Pinmanee, C., Wiwattanalamphong, K.: The efficiency of value-at-risk models during extreme market stress in cryptocurrencies. Sustainability **15**(5), 4395 (2023). https://doi.org/10.3390/su15054395
11. Dostov, V., Pimenov, P., Shoust, P.: Comparison of deterministic, stochastic, and mixed approaches to cryptocurrency dynamics analysis. In: Gervasi, O., et al. Computational Science and Its Applications – ICCSA 2021. ICCSA 2021. Lecture Notes in Computer Science, vol. 12952. Springer, Cham (2021). https://doi.org/10.1007/978-3-030-86973-1_2
12. Tassopoulos, P., Protonotarios, Y.: Brownian motion & the stochastic behavior of stocks. J. Math. Finance **12**, 138–149 (2022). https://doi.org/10.4236/jmf.2022.121009
13. Dostov, V., Shoust, P., Popova, E.: Using mathematical models to describe the dynamics of the spread of traditional and cryptocurrency payment systems. In: Misra, S., et al. Computational Science and Its Applications – ICCSA 2019. ICCSA 2019. Lecture Notes in Computer Science, vol. 11620. Springer, Cham (2019). https://doi.org/10.1007/978-3-030-24296-1_36
14. Dostov, V., Pimenov, P., Shoust, P., Titov, V.: The influence of the effect of marketing incentives on the dynamics of the development of classic and cryptocurrency payment systems. In: Proceedings of the 3rd International Symposium on Automation, Information and Computing - ISAIC; ISBN 978–989–758–622–4; ISSN 2975–9463, SciTePress, pp. 580–585 (2023). https://doi.org/10.5220/0011961700003612

15. Nejadmalayeri, A., Molchanovsky, L., Paleo, B.W., Prescott, R.W.: Stablecoins: past, present, and future. In: Essex, A., et al. Financial Cryptography and Data Security. FC 2023 International Workshops. FC 2023. Lecture Notes in Computer Science, vol. 13953. Springer, Cham (2024). https://doi.org/10.1007/978-3-031-48806-1_13
16. Tether. https://tether.to/
17. Tether: Number of Active Addresses. Glassnode. https://studio.glassnode.com/metrics?a=USDT&m=addresses.ActiveCount&resolution=1month&s=1509494400&u=1709251200&zoom=
18. Dostov, V., Shust, P., Krivoruchko, S.: Fast payment systems dynamics: lessons from diffusion of innovation models. In: Abramowicz, W., Auer, S., Stróżyna, M. (eds) Business Information Systems Workshops. BIS 2021. Lecture Notes in Business Information Processing, vol. 444. Springer, Cham (2022). https://doi.org/10.1007/978-3-031-04216-4_31
19. Bass, F.: A new product growth for model consumer durables. Manage. Sci. **15**(5), 215–227 (1969). https://doi.org/10.1287/mnsc.15.5.215
20. Simmons, G., Robertson, J.: Differential Equations with Applications and Historical Notes, 2nd Edition. In: International Series in Pure and Applied Mathematics, McGraw-Hill (1991)
21. Jeremy Allaire: Ushering in the next chapter for USDC. https://www.circle.com/blog/ushering-in-the-next-chapter-for-usdc
22. USD Coin: Number of Active Addresses. Glassnode. https://studio.glassnode.com/metrics?a=USDC&ema=0&m=addresses.ActiveCount&mAvg=0&mMedian=0&resolution=1month&s=1509494400&u=1709251200&zoom
23. Birkhoff, G.D.: What is the ergodic theorem? Amer. Math. Monthly **49**(4), 222–226 (1942). https://doi.org/10.2307/2303229, JSTOR2303229
24. Dostov, V., Pimenov, P., Shoust, P., Fedorova, R.: Using an economically justified trend for the stationarity of time series in ARMA models. In: Gervasi, O., Murgante, B., Hendrix, E.M.T., Taniar, D., Apduhan, B.O. (eds.) Computational Science and Its Applications – ICCSA 2022: 22nd International Conference, Malaga, Spain, July 4–7, 2022, Proceedings, Part II, pp. 404–415. Springer International Publishing, Cham (2022). https://doi.org/10.1007/978-3-031-10450-3_35
25. Bindseil, U., Pantelopoulos, G.: Unbacked Crypto-Assets, Stablecoins and CBDC. In: Bindseil, U., Pantelopoulos, G. (eds.) Introduction to Payments and Financial Market Infrastructures, pp. 125–152. Springer Nature Switzerland, Cham (2023). https://doi.org/10.1007/978-3-031-39520-8_9