

Vehicle Navigation with Hexagonal Hierarchical Spatial Index

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Abstract. Spatial index of geoinformation theory proposes a new possibility for development of navigation methods with global coverage and rich hierarchical addressing with different precision levels. Hexagonal Hierarchical Spatial Index (H3) and Open Location Codes (OLC) are the most frequently used spatial indexes. Global spatial indexes use indexes instead of geodetic coordinates of latitude and longitude to address any point or data associated with this place. Spatial indexes may use different shapes of cells and different projections could be applied for data transformation from WGS84 to cell space. In this paper, basic methods of vehicle navigation Time of Arrival and Angle of Arrival have been adopted for use in the global spatial index. We use a hexagonal hierarchical spatial index due to multiple advantages of cell shape for support navigation. Vehicle position is identified as a cell address based on available indexes of reference points and measured ranges between vehicle and reference points. Navigation in spatial index is grounded on set theory and logical operations only which makes positioning system cheap for computation in comparison to classical positioning methods. In numerical calculation we consider airplane positioning with Time of Arrival method in H3.

Keywords: Spatial Index · Hexagon · H3 · Air Navigation · Positioning · Vehicle · Coordinates

1 Introduction

1.1 Introduction to the Problem

Vehicle navigation is an important process of successful transportation. Modern transportation networks set a strong requirement for performance and efficiency of vehicle trajectory selection logic and precision of approved trajectory maintaining.

Navigation is a process of movement from one point to another in a defined space. Navigation includes two processes positioning and guidance. Positioning identifies vehicle current position and guidance chooses the way of application forces to move vehicle in the desired direction. There are four basic positioning methods commonly used in different applications: Time of Arrival (ToA) [1, 2], Angle or Arrival (AoA), AoA/ToA [3, 4], and hyperbolic [5, 6]. ToA method uses measured distances from vehicle position to navigational aids with known coordinates. AoA grounds on available angle data to

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navigational aids. Positioning in 2D with AoA or ToA requires at least two measured parameters. In the case of navigational aid supports both distance and angle measurement an AoA/ToA method could be used for positioning by only one navigational aid [7]. Hyperbolic method is grounded on the primary property of hyperbola: range difference between any point of hyperbola and its focuses is constant. Hyperbolic positioning system counts the time difference between two synchronized signals received at the point of vehicle position [8]. This difference is proportional to the distance difference and specifies a hyperboloidal curve of vehicle position.

Each positioning method could be used with different parameters which is a function of angle, distance, or difference in distances. Based on applied volume navigation could be local or global. Also, there are many methods of vehicle localization which is opposite to positioning and detecting vehicle coordinates by ground facility.

Accuracy of positioning methods is grounded on performance of data measured by a particular sensor type [9, 10]. In most cases, distance measuring methods are much more precise in comparison to angle measuring. It makes ToA more accurate than other types.

Spatial indexing systems have been developed rapidly for the last decade [11]. Hierarchical spatial indexes provide a global addressing system with multiple precision levels. Spatial index divides the surface of Earth's ellipsoidal model WGS84 at a set of indexed cells. Each cell has a particular code or index which is hierarchal structured to support a different level of precision. A cell of different shapes could be used to solve particular tasks. Commonly used spatial indexes include Open Location Codes (OLC), Hexagonal Hierarchical Spatial Index (H3), and Hierarchical Triangular Mesh [12]. OLC uses a square or rectangular cell. H3 operates with hexagons and pentagons. Unique address of each cell in spatial indexes makes it possible to easily access data associated with a particular place globally [13, 14]. Spatial indexing system provides efficient data storage [15, 16] and interrogation with database [17, 18].

1.2 Motivation

Global addresses of spatial indexing system could be used to replace geographic coordinates of latitude and longitude in a variety of navigation applications. A spatial indexing system creates a set of cells in which navigation methods (AoA and ToA) can be applied.

Practical implementation of positioning methods puts some additional requirements for computation performance of equipment to support "real-time" operation or guarantee adequate feedback from automatic control system. Thus, one of the important ways of navigation theory development is finding ways of increasing performance of computation methods used in navigation systems. This problem is critical for low-weight automatic vehicles, which are limited by size and cannot support a speedy computation process. As a solution, an external computational cluster could be used, however, its practical implementation meets perils of bandwidth of wireless communication data link.

1.3 Contribution

In this paper, we study the application of general positioning methods in the theory of global spatial indexing systems. The paper contributes with a theoretical background of

implementation positioning methods in the set of addresses of global spatial indexing. The aim of paper is to implement AoA and ToA positioning methods to use with Hexagonal Hierarchical Spatial Index. Proposed methods are grounded on logical operations only that are computationally more efficient than solution of navigation equations with trigonometric functions.

1.4 Organization of the Paper

The paper consists of six sections. The first section includes introduction to the problem with a detailed literature review, motivation, and contribution. The second section describes the basics of Hexagonal Hierarchical Spatial Index. The third section considers application of ToA positioning method in a hexagonal hierarchical spatial indexing system. The fourth section gives a theoretical statement of AoA positioning method application in spatial indexing systems. The fifth section provides a numerical demonstration of proposed positioning method for airplane navigation based on trajectory data of DLH 1010 flight.

2 Hexagonal Hierarchical Spatial Index

Global spatial index uses a set of cells to be associated with a particular map projection. Which represents a surface of reference ellipsoid with a set of cells. Hierarchic structure makes unique addressing for each cell of different precision levels.

There are four basic types of cells used in global spatial indexes: triangular, rectangular, pentagon, and hexagon (see Fig. 1). The cell shape with sides of equal length is more useful, due to simple recalculation of distances and computational cheap cell identification.

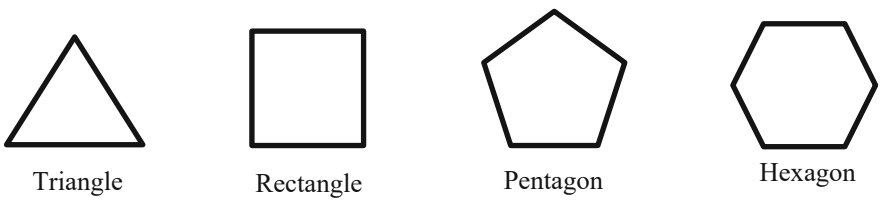


Fig. 1. Geometrical shapes of cells used in spatial index.

The number of neighbor cells and length of connection border with other cells depend on the shape configuration that is critical for navigation purposes. Thus, a triangular shape has 12 neighbor cells, rectangular – 8, hexagonal – 6, pentagonal – 5. Another important criterion for navigation is the equality of ranges between centers of neighbor cells and reference cell. Unfortunately, triangular and rectangular shapes do not meet this requirement. Complementary area of cell at a circle could be used to choose an efficient shape for cell identification. Hexagon gives the smallest complementary area for the circles of identical radius which is important for algorithms of nearest neighbors search.

All of these advantages make hexagonal cells more useful for developing navigation applications based on spatial index.

Regular hexagon are formed by regular triangles with sides equal to circumcircle radius. Hexagon of regular shape has six rotations and lines of symmetry. Also, all internal angles are equal to 120º. Let's the side length of regular hexagon H be equal to a. Then the radius of inscribed circle could be calculated as follows:

$$r = \frac{\sqrt{3}}{2}a. \tag{1}$$

Geometry of hexagonal shape of the cell is presented in Fig. 2.

Representation of spherical coordinates with spatial index is performed with the help of map projections. Map projection transforms spherical data to 2D cartesian plane. Orientation of hexagonal cell in the spatial index could be different based on general system configuration [19]. If we use a cartesian reference frame with an X-axis directed to the North and a Y-axis pointed to the East, then coordinates of vertexes in the hexagon could be calculated as follows:

$$V_{xi} = \left[x_i + \frac{\sqrt{3}}{2}a; x_i; x_i - \frac{\sqrt{3}}{2}a; \;\; x_i - \frac{\sqrt{3}}{2}a; x_i; x_i + \frac{\sqrt{3}}{2}a \right], \tag{2}$$

$$V_{yi} = \left[y_i + \frac{a}{2}; y_i + a; y_i + \frac{a}{2}; \;\; y_i - \frac{a}{2}; y_i - a; y_i - \frac{a}{2} \right], \tag{3}$$

where x_i, y_i is coordinate of hexagon center; a is regular hexagon side length; V_x, V_y are matrixes of vertexes coordinates in NE reference frame.

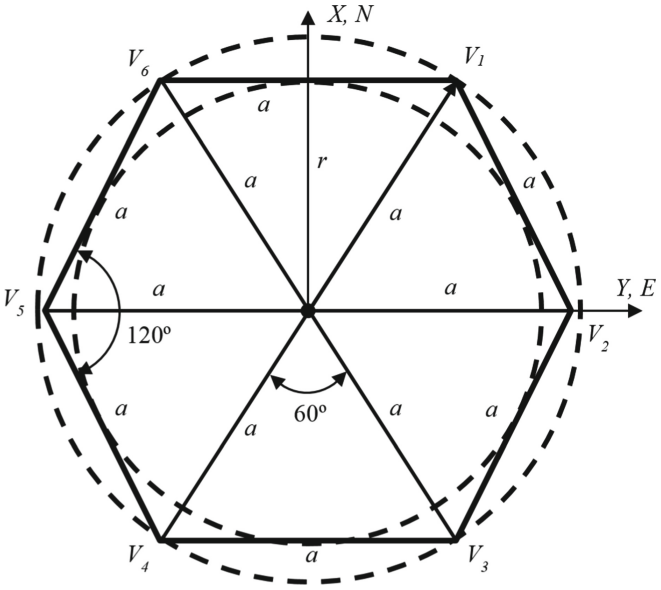


Fig. 2. Geometry of regular hexagon.

Regular hexagon configuration of the cell gives a possibility for easy operations with neighbor cells. Centers of all neighbor cells are located in vertexes of another bigger hexagon rotated at 30º (see Fig. 3).

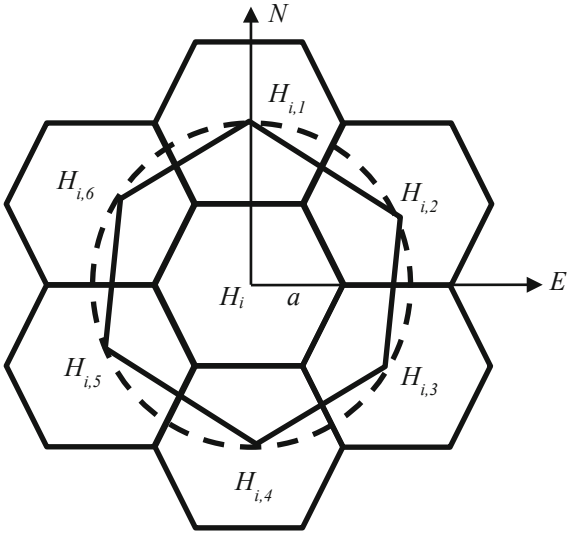


Fig. 3. Geometry of neighbor hexagons.

A set of neighbor hexagons $H_i = \{H_{i,1}, H_{i,2}, H_{i,3}, H_{i,4}, H_{i,5}, H_{i,6}\}$ could be calculated by hexagon centers:

$$n_{xi} = \left[x_i + \sqrt{3}a;\ x_i + \frac{\sqrt{3}}{2}a;\ x_i - \frac{\sqrt{3}}{2}a;\quad x_i - \sqrt{3}a;\ x_i - \frac{\sqrt{3}}{2}a;\ x_i + \frac{\sqrt{3}}{2}a \right], \quad (4)$$

$$n_{yi} = \left[y_i;\ y_i + \frac{3a}{2};\ y_i + \frac{3a}{2};\ y_i;\ y_i - \frac{3a}{2};\ y_i - \frac{3a}{2} \right]. \quad (5)$$

Based on centers of neighbor hexagons to the reference one, coordinates of vertexes of each neighbor hexagon could be obtained by (2) and (3).

Hexagonal symmetry helps to identify hexagons placed on the next level of grid distance. If we denote k as ordered number of hexagonal rings around a particular reference hexagon, then each side of hexagonal ring will include k + 1hexagons (see Fig. 4). Coordinates of hexagons center on a particular side are obtained after division of side length into lines of 2r width. The side length of the hexagonal circle could be calculated as follows:

$$L(k) = 2kr = ka\sqrt{3}. \quad (6)$$

Cells located in hexagonal circles are very important for navigation purposes. Each hexagonal circle includes 6 × k hexagons.

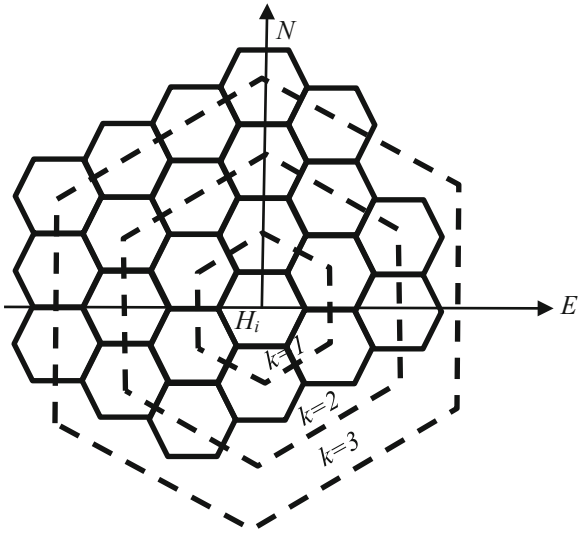


Fig. 4. Configuration of hexagons in k^{th} ring.

Indexing of hexagons could be organized differently. As an example, H3 uses a specific planar grid system with three axes of coordinates *i-j-k* applied at 120º. Coordinates of each hexagon in *i-j-k* integer frame are used to generate the address of each cell in the spatial index.

3 Time of Arrival Method

ToA is the most useful positioning method in navigation. ToA grounds on measuring distances from vehicle location to at least two navigational aids with given coordinates. Navigational aid is a specific reference point usually equipped with navigation equipment to perform measurements of distances to them [20, 21]. Navigation equation of ToA is based on equation of distances between two points in the local cartesian frame which could be applied for any number of measured distances to reference points. Navigation equation for the case of 2D positioning is the following:

$$D^2 = (x - \mathrm{X})^2 + (y - \mathrm{Y})^2,$$
$$D = [D_1, D_2, D_3, \ldots, D_m],\ X = [X_1, X_2, X_3, \ldots, X_m],\ Y = [Y_1, Y_2, Y_3, \ldots, Y_m], \tag{7}$$

where D is a matrix of measured distances; X and Y are matrixes of navigational aids coordinates; m is a number of available distances; x and y are coordinates of vehicle position.

If there are only two navigational aids ($m = 2$) solution of (7) is analytical for 2D. In the case of $m > 2$ solution of (7) could be obtained as an iterative process of non-linear equation solution.

Let's consider application of ToA in the space of spatial indexes. Navigational aids or reference points could be associated with particular cells and specified as a set of indexes at the same level of precision:

$$NAV = \{nav_1, nav_2, nav_3, \ldots nav_m\}, \tag{8}$$

where m is the number of navigational aids used.

Vehicle is equipped with a distance measuring sensor which measures distances D to navigational aids NAV. Each distance D_i forms a circle of hexagons with a radius of k. Values D could be easily transformed to the metric of k:

$$\begin{gathered} K = \frac{D}{2r} = \frac{D}{\sqrt{3}a}, \\ K = [k_1, k_2, k_3, \ldots, k_m], \end{gathered} \tag{9}$$

where K is a matrix of transformed distances to k space.

A set of hexagon identification codes H_k which are located at the circle of a particular level k_i could be obtained as follows:

$$H_k = \{h_1, h_2, \ldots, h_j\}. \tag{10}$$

where h is a hexagon placed on a circular line; j is the number of hexagons placed on a hexagonal line.

The power of the set for each hexagonal circle is different based on k:

$$j_k = 6k. \tag{11}$$

Sets of identification codes (10) are subsets of a navigation set:

$$H_{NAV} = \{H_1, H_2, H_3, \ldots, H_m\}. \tag{12}$$

Result of navigation set (12) solution gives a unique hexagon index which is present in each subset of H_{NAV}. Hexagonal lines of vehicle position for the case of $m = 3$ are presented in Fig. 5.

Hexagon index which includes vehicle position could be obtained with logical operation of intersection of subsets in H_{NAV}:

$$POS = \cap_{i=1}^{m} H_i, \tag{13}$$

where POS is an identification code of a hexagon.

Identification code of hexagon obtained by (13) is a solution of navigation set (12). Address POS could be associated with vehicle position. Hexagonal shape of vehicle position line may give multiple solutions for the case of $m = 2$. Thus, all hexagons at the circular hexagon side could be marked a solution. This problem could be solved by using bigger values of m. Also, as a simplification for the case if a method returns more than one solution the closest cell could be selected.

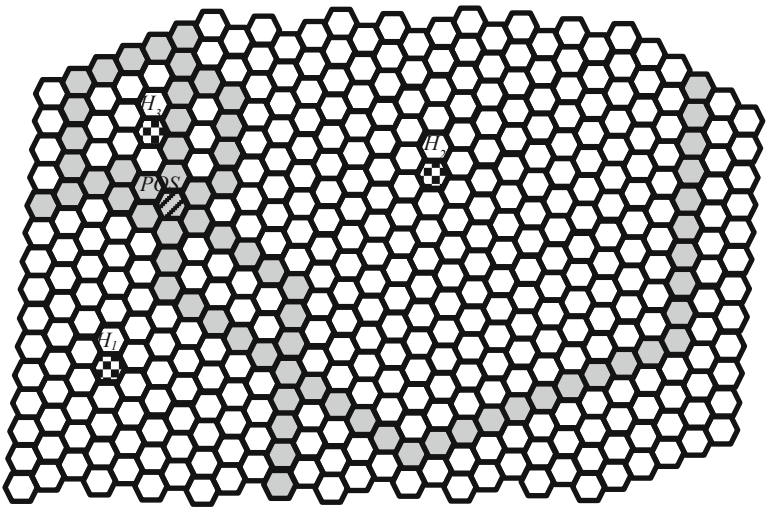


Fig. 5. Positioning by hexagonal rings.

4 Angle of Arrival Method

Method AoA uses angles as input data for positioning. AoA could use different reference directions to count angles [22, 23]. Angle of direction to navigational aid could be counted from some axis of body reference frame associated with vehicle construction (for example longitudinal axis could be used) or azimuth angle counted clockwise from vehicle center to the North. Also, angles counted from navigational aids to vehicle could be used [24]. As an example, an angle in navigational aid position from North to the vehicle is widely used in aviation for air navigation purposes. Navigation equation for AoA is based on trigonometric relations:

$$\begin{gathered} AP = B, \\ A = [tg(\alpha), -1];\ P = [x, y]^T;\ B = [Xtg(\alpha) - Y], \end{gathered} \tag{14}$$

where α is a matrix of measured angles from navigational aids; P is a matrix of vehicle position; X and Y are matrixes of navigational aid coordinates.

In the case of two navigational aids, a solution of (14) could be obtained analytically. For more numbers of input data, a Least Squares Method is helpful:

$$P = \left(\left(A^T A\right)^{-1} A^T B\right)^T. \tag{15}$$

In the space of spatial indexes, AoA could be realized based on an identified set of hexagons which are placed on a line by the given angle. If α is the angle counted from navigational aid to vehicle in the North direction clockwise then hexagons placed on line:

$$f(z) = tg(\alpha)z. \tag{16}$$

where z coincides with x axis of North-East plane; $f(z)$ corresponds to y.

Also, constraints to the geometrical quarter are used with reference to navigational aid location. Finally a set of hexagon identification codes H_k which placed on the line of particular angle from navigational aid could be obtained as follows:

$$S_i = \{h_1, h_2, \ldots, h_l\}. \tag{17}$$

Size l of set H_k is selected based on AoA maximum operational performance. A hexagon associated with vehicle position could be identified as an intersection of sets H identical to (13):

$$POS = \bigcap_{i=1}^{m} S_i, \tag{18}$$

where POS is an identification code of hexagon.

Geometrical representation of hexagons for the case of $m = 3$ is presented in Fig. 6.

Method AoA in hexagons is heavier than ToA based on computation time due to identification of sets by (16). Also, AoA is required to set up a size l. Bigger values l require more computation time for (18), lower l may lead to missing vehicle position point. Thus, the best l could be selected based on sensor capability level.

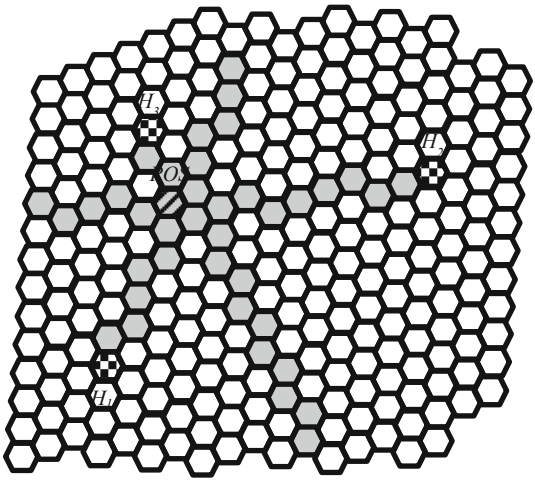


Fig. 6. Positioning by sets of hexagons based on angles.

5 Numerical Study

In the numerical study, we use application of ToA in hexagonal space for the case of civil airplane positioning. We use airplane trajectory data obtained with Automatic Dependent Surveilance-Broadcust (ADS-B) technology. ADS-B data are measured onboard of airplane and then transmitted omnidirectionally in the open data-transferring protocol.

Ground network of software-defined receivers receive transmitted ADS-B messages, decode and process them with a global network facility. Processed coordinates of airplane are saved with particular identification code separately by each flight. Nowadays, many databases provide easy access to the ADS-B database of flights, which could be used to study different transportation tasks.

We use trajectory data of DLH 1010 flight on December 16, 2023 for connection between Frankfurt am Main (EDDF) and Brussels (EBBR) airport. Flight time was 38 min. Total trajectory length is 216 NM. Trajectory data of DLH 1010 flight include only 87 points of not synchronized timestamps. Coordinates of data points are provided in angles of geodetic latitude and longitude. We use interpolation of data for 1Hz with linear regression separately for each parameter. Results of interpolation give 2280 data points.

To work with hexagonal spatial index a JavaScript library H3 was used. Transformed airplane flight path along a trajectory to the hexagonal space with 6th level of resolution is presented in Fig. 7.

The main idea of using hexagonal global spatial index is to use indexes instead of geographic coordinates. Also, measured data by on-board senor has a particular area of uncertainty that could be associated with hexagonal cell of particular resolution. This uncertainty area is represented as an ellipse or circle based on 95% confidence band.

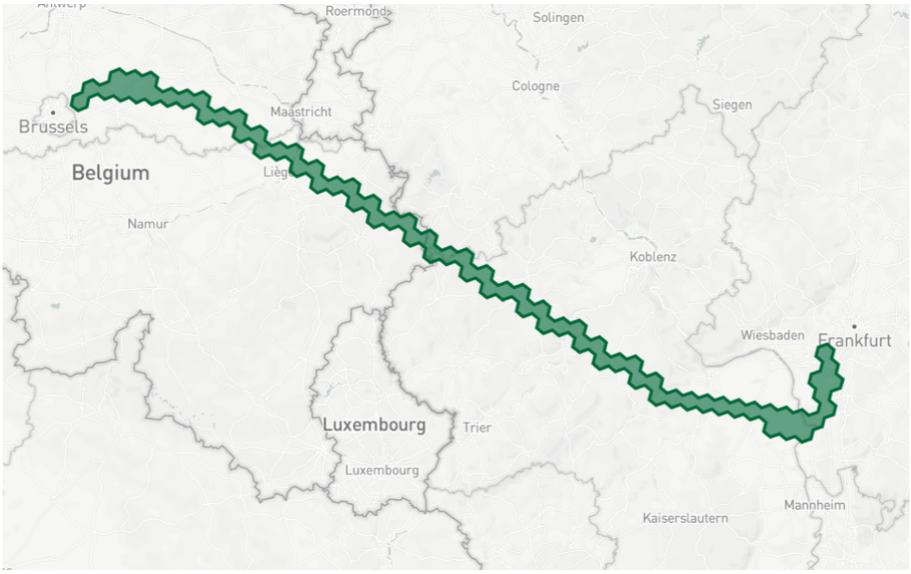


Fig. 7. Flight path of DLH 1010 in hexagons.

To simulate positioning by ToA let's consider airplane positioning by a network of navigational aids. Civil aviation uses Distance Measuring Equipment (DME) to measure ranges from airplane to ground network of DME navigational aids. Positioning by pair of two DME/DME is used as a stand-by positioning algorithm for hard back-up in case

of primary positioning sensor lock. We consider particular cases of usage DME DIK (Diekirch, 49.5168º N, 6.0778ºE) and DME FFM (Frankfurt, 50.0322ºN, 8.3823ºE).

At each cell of flight path, grid distances are calculated to DMEs. These distances have been used to model measurements in the space of cells from vehicle side. Simulated grid distances to both DMEs have been used for positioning by proposed ToA method in hexagons. Configuration of hexagons for vehicle position in 100th data point of trajectory at 8th level of resolution is presented in Fig. 8. A hexagonal distance from vehicle position to DME FFM is 51 hexagons. Hexagonal ring for FFM includes a set of 306 hexagons. Hexagonal ring of DIK is much bigger (171 hexagons range and includes 1026 hexagons) than FFM for 100th data point. Also, a configuration of both hexagons for this point is not optimal due to one leg of FFM hexagonal ring coincides with a leg of DIK. Thus solution of (13) for this case gives 51 hexagons of possible vehicle position. The exact position could be identified as the closest hexagon to the previously detected position from the set of possible solutions obtained by (13). A case of only two solutions is shown in Fig. 9 for the 300th trajectory point of 8th level of resolution. A case for 12th level of precision for 400th data point is shown in Fig. 10.

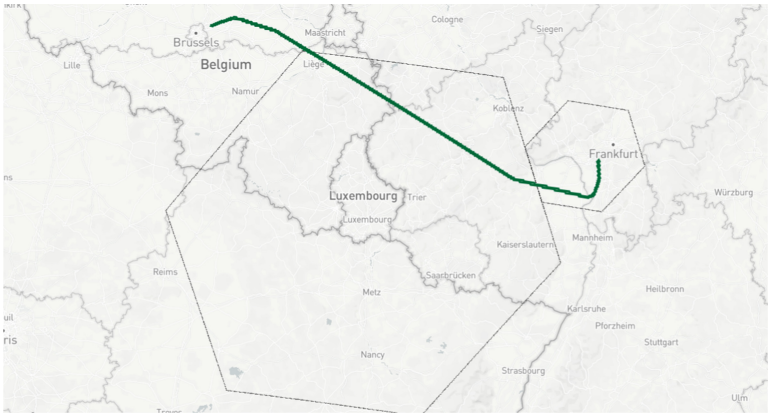


Fig. 8. Positioning by ToA for 100th point of airplane trajectory.

Solution of navigation set (13) at 12th level of resolution works with hexagons of 10.8 m side length. Navigation ring from FFM includes 10104 (with k = 1684) and from FFM includes 74748 (with k = 10774). Technically positioning in hexagons for a case of m = 2 grounds on a simple comparison of two sets of hexagons on rings to search identical indexes. In most cases, we get two or a set of identical indexes. Exact index is chosen based on the closest distance from previous positioning step.

Results of positioning at particular data points for different levels of resolutions are presented in Table 1.

Results in Table 1 show that by increasing the grid distance the number of hexagons at the hexagonal ring is increased. Results of number of hexagons calculation for different grid distances are shown in Fig. 11. An identification of hexagon side could be applied to minimize length of set of possible positions based on vehicle velocity vector. Only

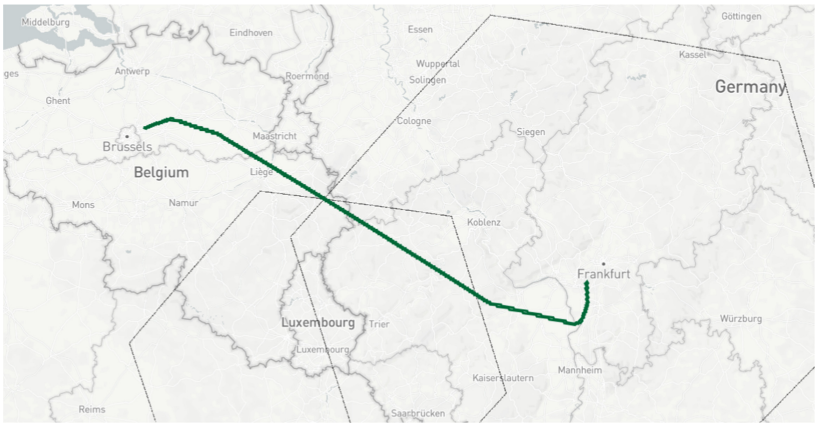


Fig. 9. Positioning by ToA for 300th point of airplane trajectory.

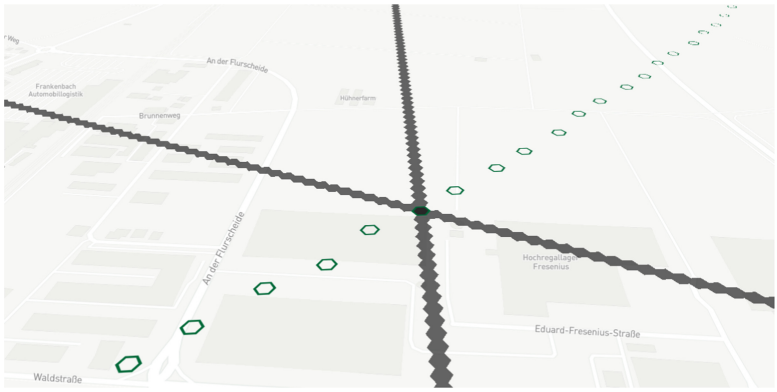


Fig. 10. Positioning by ToA for 400th point of airplane trajectory at 12th level of resolution.

hexagons on one side of hexagonal ring could participate in navigation process. It could reduce the set size by six times. Also, a geometrical configuration of both navigational aids is important due to possible cases of hexagonal rings touching one side of each other.

Positioning process in hexagonal space does not consider accuracy, because a set of global hexagons with defined size are used. It means that we operate with cells and the solution of navigation task is a cell (or group of cells) too. If we set up a level of resolution that corresponds with a particular parameter precision level then obtained cell could be associated with the uncertainty level of positioning process. Thus, level of resolution for solving navigation task should be selected based on standard deviation errors of data measuring device.

Table 1. Results of positioning by ToA in hexagons for selected data points

Point of trajectory	level of resolution	Grid distance to DME DIK	Grid distance to DME FFM	Index of airplane position
100	8	171 (1026)	51 (306)	881faecc83fffff
170	8	108(648)	114 (684)	881fa3b0ddfffff
300	8	143(858)	210(1260)	881fa03113fffff
400	8	191(1146)	306 (1836)	881fa09ae5fffff
400	9	1399(8394)	263(1578)	8a1fae08d91ffff
400	10	3610(21660)	715(4290)	8b1fae0c83a5fff
400	12	10774(74748)	1684(10104)	8c1fae0c8304bff

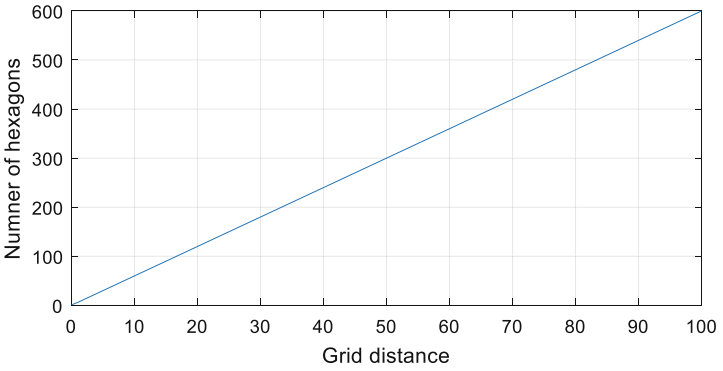


Fig. 11. Number of hexagons for different grid distances.

6 Conclusions and Future Works

Global spatial indexing system is a useful tool for geodata processing and visualization. Results of the study show the perspective of positioning methods usage in the theory of global spatial indexing. Paper presents a theoretical background of vehicle positioning based on indexes (addresses) instead of usage latitude and longitude during solution of navigation problems. The main advantage of proposed positioning methods (AoA and ToA) for spatial indexing system is a replacement of classical navigation equations (Eq. (7) for ToA and Eq. (15) for AoA) with equations based on logical operations only (Eq. (13) for ToA and Eq. (18) for AoA). In practical realization, these methods request a comparison of vehicle location sets to search identical addresses. Search in addresses is computationally more efficient than usage of numerical methods of navigation equations solution for both positioning methods. Therefore proposed methods could be useful for application in a variety of vehicle types with strong requirements for size of computation equipment used to support navigation and automatic vehicle control.

Performance of positioning methods in spatial indexing system is associated with a cell size of a particular level of resolution. Also, regular hexagonal shape of cell in spatial

indexing system gives many advantages for practical implementation in navigation. Regular hexagon gives elementary equations for cell geometry calculation. Grid distance in a hexagonal spatial indexing system is useful to identify a hexagonal line of vehicle position based on particular level of resolution. Also, grid distance parameter makes it easy to get a set of cell indexes in which vehicle could be located.

A numerical study with real airplane trajectory for flight "DLH 1010" on Dec 16, 2023, shows the potential of H3 software library for airplane positioning. Airplane trajectory data could be coded by hexagon addresses of particular resolution levels. Position of airplane could be identified by ToA method based on measured grid distance to the reference points (a pair of DMEs DIK and FFM have been used as reference points).

Performance of positioning methods in hexagons could be a perspective for future study as well as application of hyperbolic positioning method in the spatial indexing system. Also, practical implementation of proposed positioning method based on global spatial indexing is at the development stage for light unmanned aerial vehicles.

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