

Graphical Visualization of Phase Surface of the Sprott Type A System Immersed in 4D

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Abstract. When formulating a system of differential equations, the main objective is to determine their solutions, in addition to visualizing the phase surface to observe the behavior of the physical phenomenon. In this work an algorithm is developed to graph phase surfaces and perform qualitative analysis to a four-dimensional (4D) system. The algorithm is implemented in the scientific software Octave obtaining the program called SystemSprott4D, which is applied to the Sprott type A system in 4D to be able to graph, phase surfaces, limit cycle and trajectories of initial conditions of the system. A qualitative analysis of the system is performed, such as symmetry of the vector field, sensitivity in the initial conditions, Lyapunov exponents, fractal dimension and limit cycle. It is found that it is a non-equilibrium system, this means that the 4D chaotic system can exhibit attracting limit cycles, these limit cycles are found by selecting different initial points. The program can be used to analyze non-linear 4D systems from various disciplines such as electronics, telecommunications, biology, meteorology, economics, medicine, etc.

Keywords: Sprott type A hyper attractor · Chaotic system · Geometric visualization · Octave software

1 Introduction

Chaos is a broad and universal phenomenon that exists in several nonlinear systems and has been confused with noise for a long time. In 1980, the engineering field officially announced chaos existence in electrical systems. Chaotic systems have sensitivity to initial conditions. Small changes in initial conditions can lead to significant changes in the systems' behavior. The coexisting hidden attractors of a chaotic system can be found by selecting different initial conditions. Chaotic systems are used to model various physical phenomena; for example, Lorenz systems are used to model climate behavior. The Sprott simple systems were created to simplify the Lorenz systems of equations that model climate behavior globally.

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Previous work on the Sprott system type A in four-dimensional space (4D) is presented in [5]; the authors analyzed the energy cycle of the Sprott-A system transforming it to a Kolmogorov type system, then based on the energy analysis of the Sprott-A system, a new conservative 4D chaotic system with constant Hamiltonian energy is obtained. The study presents four static images of the phase surface. Still, it doesn't show a qualitative analysis of the 4D system or deliver a complete phase surface to be analyzed.

Currently, to visualize phase surfaces and limit cycles of 4D systems, they are projected in three dimensions (3D), making zero any of the coordinate axes, resulting in four surfaces to analyze (for examples, see [1–10]). Researchers can overcome this analysis limitation by applying the Velezmoro and Ipanaque model that allows the visualization of 4D objects in a 3D system (see Sect. 2.2) by obtaining a single surface that can be rotated from different angles, allowing a better analysis of what happens in a 4D system of equations.

This study presents an Octave program called SistemaSprott4D that allows the visualization of phase surfaces and limit cycles of nonlinear systems immersed in a 4D system. The SistemaSprott4D includes the Velezmoro and Ipanaque model to visualize phase surfaces of a chaotic Sprott system type A immersed in the fourth dimension. It can be used to analyze nonlinear 4D systems from various disciplines such as electronics, telecommunications, biology, meteorology, economics, medicine, etc.

2 Preliminaries

In this Section, the necessary mathematical concepts for the work are defined, the definitions of, sensitivity in the initial conditions, chaotic invariant set and strange attractor are fundamental characteristics for a system to be chaotic. In addition, mention is also made of the Velezmoro and Robert model that allows to visualize in 3D objects that are in 4D.

2.1 Dynamical Systems

Definition 1 (Sensitive dependence on initial conditions) [13]. The flow $\phi(t, x)$ is said to possess a sensitive dependence on initial conditions in Λ (invariant compact set under $\phi(t, x)$) if there exists $\varepsilon > 0$ such that for any $x \in \Lambda$ and any neighborhood U of x, there exist $y \in U$ and $t > 0$ such that $|\phi(t, x) - \phi(t, y)| > \varepsilon$.

Definition 2 (Chaotic Invariant Set) [13]. Λ Is said to be chaotic if:

1. $\phi(t, x)$ has a sensitive dependence on initial conditions in Λ.
2. $\phi(t, x)$ is topologically transitive in Λ.
3. The periodic orbits of $\phi(t, x)$ are dense in Λ.

Definition 3 (Strange attractor) [12]. Suppose that $A \subset R^n$ is an attractor. Then A is called a strange attractor if it is chaotic.

2.2 Velezmoro and Ipanaque Immersion Model

Ricardo Velezmoro and Robert Ipanaque [11], proposed a new mathematical model that allows 4D objects to be graphed in 3D following the same method for visualizing 3D objects on a 2D screen. The mathematical model is built using a 3D hologram projector.

Consider a two-unit length cube whose center of gravity is the coordinate origin of the 3D Cartesian system. Four coordinate axes, x, y, z, and w are arranged in the cube shown in Fig. 1.

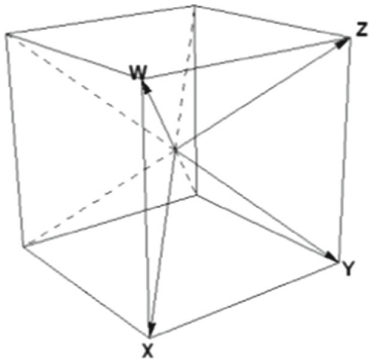


Fig. 1. Arrangement of the coordinate axes x, y, z, and w in three-dimensional space.

Consider the unit vectors:

$$e_1 = \left(\frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}}\right) \; e_2 = \left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}}\right)$$

$$e_3 = \left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right) \; e_4 = \left(\frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)$$

In the direction of the x, y, z, and w axes (see Fig. 2).

From the unit vectors, an immersion is defined as ψ, constituting the method for graphing 4D objects in a 3D environment.

Let's define immersion $\psi : R^4 \to R^3$ such that,

$$\psi(p) = p_1 e_1 + p_2 e_2 + p_3 e_3 + p_4 e_4, \text{for all} \, p \in R^4.$$

Immersion ψ can be written in the form,

$\psi(p) = \frac{1}{\sqrt{3}}\big(p_1 + p_2 + p_3 + p_4, -p_1 + p_2 + p_3 - p_4, -p_1 - p_2 + p_3 + p_4\big), \forall p \in R^4$

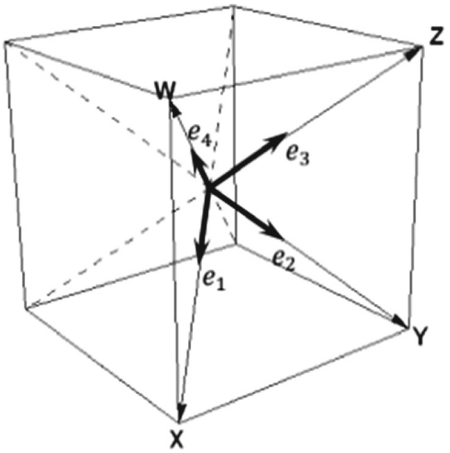


Fig. 2. Unit vectors, e_i, $i = 1, 2, 3, 4$, from which immersion ψ is defined.

3 Results and Discussion

In this section, the new Sprott type A system is shown in 4D. Also, an algorithm to plot phase surfaces and perform qualitative analysis is shown, then the algorithm is applied to the new Sprott type A system in 4D. In addition, a qualitative analysis of the system is also made and a scenario of the different hidden attractors is shown by varying the initial conditions.

3.1 Nonlinear Sprott Type A 4D System

Equation (1) is a system without equilibrium, its trajectories form attractive limit cycles. The system (1) represents the new Sprott type A system in 4D. This system is nonlinear in the second and third equations, also it has a positive control parameter and four state variables x, y, z, w as shown below:

$$\begin{cases} x' = y \\ y' = -x + ayz \\ z' = -y^2 + 1 \\ w' = -z \end{cases} \tag{1}$$

3.2 Steps to Graph Phase Surfaces and Perform Qualitative Analysis to a 4D System

Step 1. Solve the 4D system using a numerical method.
Step 2. Use the Velezmoro and Ipanaque immersion model to visualize a phase surface graph of the system that is in 4D in 3D.
Step 3. Perform a qualitative analysis of the system:

Step 3.1. Symmetry.

Step 3.2. Sensitivity in initial conditions.
Step 3.3. Lyapunov exponents.
Step 3.4. Lyapunov - Yorke dimension.
Step 3.5. Dissipation and existence of the attractor.

3.3 SystemSprott4D

The SystemSprott4D program is an implementation of the algorithm of Sect. 3.2 in the Octave scientific software, this program allows graphing, phase surfaces and trajectories of initial conditions, to perform the qualitative analysis of the Sprott type A system in 4D.
Case Study to the New 4D Sprott Type 1.

```
a=1;
func=@(t,x)[x(2);-x(1)+a*x(2)*x(3);1-x(2)*x(2);-x(3)];
x0=[-0.0561 1.5652 1.0484 1.0543]; % condiciones iniciales.
t=linspace(0,300,10000);
[t, x]=ode45(func,t,x0);
x(1:20,:) % vector que muestra las 20 primeras iteraciones.
```

Implementation of Step 2.

```
a=1;
func=@(t,x)[x(2);-x(1)+a*x(2)*x(3);1-x(2)*x(2);-x(3)];
x0=[-0.0561 1.5652 1.0484 1.0543];
t=linspace(0,300,10000);
[t,x]=ode45(func,t,x0);
z=[x(:,1) x(:,2) x(:,3) x(:,4)];
p1=(1/sqrt(3)).*[z(:,1)+z(:,2)+z(:,3)+z(:,4)];
p2=(1/sqrt(3)).*[-z(:,1)+z(:,2)+z(:,3)-z(:,4)];
p3=(1/sqrt(3)).*[-z(:,1)-z(:,2)+z(:,3)+z(:,4)];
disp([p1(1:20,:),p2(1:20,:),p3(1:20,:)]);
plot3(p1,p2,p3,'b','linewidth',2).
xlabel('X'); ylabel('Y'); zlabel('Z').
```

Implementation of Step 3.
Subprogram that allows visualizing the trajectories by varying the first component.

```
sistem4D=@(t,x)[x(2);-x(1)+x(2)*x(3);-x(2)^2+1;-x(3)];
x0=[-0.0561 1.5652 1.0484 0];
X0=[-0.2561 1.5652 1.0484 0];
t=linspace(0,80,800);
[tout,xout]=ode45(sistem4D,t,x0);
plot(tout,xout(:,1),'b').
hold on.
[tout,Xout]=ode45(sistem4D,t,X0);
plot(tout,Xout(:,1),'k').
xlabel('time').
```

```
ylabel('X vs x').
grid.
```

Subprogram that allows visualizing the trajectories by varying the second component.

```
sistem4D=@(t,x)[x(2);-x(1)+x(2)*x(3);-x(2)^2+1;-x(3)];
x0=[-0.0561 1.5652 1.0484 0];
X0=[-0.0561 1.9652 1.0484 0];
t=linspace(0,100,1000);
[tout,xout]=ode45(sistem4D,t,x0);
plot(tout,xout(:,2),'b').
hold on.
[tout,Xout]=ode45(sistem4D,t,X0);
plot(tout,Xout(:,2),'k').
xlabel('time').
ylabel('Y vs y').
grid.
```

Subprogram that allows visualizing the trajectories by varying the third component.

```
sistem4D=@(t,x)[x(2);-x(1)+x(2)*x(3);-x(2)^2+1;-x(3)];
x0=[-0.0561 1.5652 1.0484 0];
X0=[-0.0561 1.5652 1.1884 0];
t=linspace(0,100,1000);
[tout,xout]=ode45(sistem4D,t,x0);
plot(tout,xout(:,3),'b').
hold on.
[tout,Xout]=ode45(sistem4D,t,X0);
plot(tout,Xout(:,3),'k').
xlabel('time').
ylabel('Z vs z').
grid.
```

Subprogram that allows visualizing the trajectories by varying the fourth component.

```
sistem4D=@(t,x)[x(2);-x(1)+x(2)*x(3);-x(2)^2+1;-x(3)];
x0=[-0.0561 1.5652 1.0484 0];
X0=[-0.0561 1.5652 1.1884 0.123];
t=linspace(0,100,1000);
[tout,xout]=ode45(sistem4D,t,x0);
plot(tout,xout(:,4),'b').
hold on.
[tout,Xout]=ode45(sistem4D,t,X0);
plot(tout,Xout(:,4),'k').
xlabel('time').
ylabel('W vs w').
grid.
```

Subprogram to calculate the Lyapunov exponents.

```
Function[Texp,Lexp]=lyapunov(n,rhs_ext_fcn,
fcn_integrator,tstart,stept,tend,ystart);
n1=n; n2=n1*(n1+1);
%  Number of steps
nit = round((tend-tstart)/stept);
% Memory allocation
y=zeros(n2,1); cum=zeros(n1,1); y0=y;
gsc=cum; znorm=cum;
% Initial values
y(1:n)=ystart(:);
for i=1:n1 y((n1+1)*i)=1.0; end;
t=tstart;
% Main loop
for ITERLYAP=1:nit
% Solution of extended ODE system
[T,Y] = feval(fcn_integrator,rhs_ext_fcn,[t t+stept],y);
 t=t+stept;
  y=Y(size(Y,1),:);
for i=1:n1
     for j=1:n1 y0(n1*i+j)=y(n1*j+i); end;
  end;
znorm(1)=0.0;
  for j=1:n1 znorm(1)=znorm(1)+y0(n1*j+1)^2; end;
znorm(1)=sqrt(znorm(1));
for j=1:n1 y0(n1*j+1)=y0(n1*j+1)/znorm(1); end;
 for j=2:n1
     for k=1:(j-1)
        gsc(k)=0.0;
        for l=1:n1 gsc(k)=gsc(k)+y0(n1*l+j)*y0(n1*l+k); end;
     end;
     for k=1:n1
        for l=1:(j-1)
           y0(n1*k+j)=y0(n1*k+j)-gsc(l)*y0(n1*k+l);
        end;
     end;
     znorm(j)=0.0;
     for k=1:n1 znorm(j)=znorm(j)+y0(n1*k+j)^2; end;
     znorm(j)=sqrt(znorm(j));
     for k=1:n1 y0(n1*k+j)=y0(n1*k+j)/znorm(j); end;
  end;
  for k=1:n1 cum(k)=cum(k)+log(znorm(k)); end;
  for k=1:n1
     lp(k)=cum(k)/(t-tstart);
  end;
```

```
% Output modification
  if ITERLYAP==1
     Lexp=lp;
     Texp=t;
  else
     Lexp=[Lexp; lp];
     Texp=[Texp; t];
  end;
  for i=1:n1
      for j=1:n1
          y(n1*j+i)=y0(n1*i+j);
      end;
  end;
end;

function f=exp_lyapunov4(t,X)
       x=X(1); y=X(2); z=X(3);w=X(4);
       Y= [X(5), X(9), X(13),X(17); X(6), X(10), X(14), X(18);...
          X(7), X(11), X(15), X(19); X(8), X(12), X(16), X(20)];
       f=zeros(16,1);
       f(1) = y;
       f(2) = -x+y*z;
       f(3) = -y^2+1;
       f(4) = -z;
       Jac=[0 1 0 0;-1 z y 0;0 -2*y 0 0;0 0 -1 0];
       f(5:20)=Jac*Y;
end

[T,Res]=lyapunov(4,@exp_lyapunov4D,@ode45,0,0.5,200,[0.0561,1.5652,1.0
484,0);
plot(T,Res(:,1),'r','linewidth',1);
hold on
plot(T,Res(:,2),'b','linewidth',1);
plot(T,Res(:,3),'k','linewidth',1);
plot(T,Res(:,4),'g','linewidth',1);
xlabel('Tiempo'); ylabel('Exponentes de Lyapunov');
Res(length(Res),:)
```

Subprogram that shows different scenarios for different values of a.

```
% a=[0.01, 0.1, 0.5, 1, 10] different values of the parameter a to obtain the.
% phase surfaces.
a=1;
func=@(t,x)[x(2);-x(1)+a*x(2)*x(3);1-x(2)*x(2);-x(3)];
x0=[-0.0561 1.5652 1.0484 1.0543];
t=linspace(0,300,10000);
[t,x]=ode45(func,t,x0);
```

```
z=[x(:,1) x(:,2) x(:,3) x(:,4)];
p1=(1/sqrt(3)).*[z(:,1)+z(:,2)=z(:,3)+z(:,4)];
p2=(1/sqrt(3)).*[-z(:,1)+z(:,2)+z(:,3)-z(:,4)];
p3=(1/sqrt(3)).*[-z(:,1)-z(:,2)+z(:,3)+z(:,4)];
disp([p1(1:20,:),p2(1:20,:),p3(1:20,:)]);
plot3(p1,p2,p3,'b','linewidth',2).
xlabel('X'); ylabel('Y'); zlabel('Z').
```

3.4 Applying the Algorithm as a Case Study to the New 4D Sprott Type A System

In [5], a new 4D Sprott type A system was proposed (see Eq. 2)

$$\begin{cases} x' = y \\ y' = -x + yz \\ z' = -y^2 + 1 \\ w' = -z \end{cases} \tag{2}$$

We solve the new system of Sprott type A in 4D using the numerical method of Runge Kutta (library ode45 in Octave). Table 1 shows the approximate solution for the first 20 iterations.

Table 1. Numerical solution of the system (3.2)

N°	x	y	z	w
1	-5.6100e-02	1.5652e+00	1.0484e+00	1.0543e+00
2	-8.3853e-03	1.6151e+00	1.0025e+00	1.0235e+00
3	4.0793e-02	1.6627e+00	9.5190e-01	9.9419e-01
4	9.1359e-02	1.7075e+00	8.9668e-01	9.6645e-01
5	1.4322e-01	1.7489e+00	8.3704e-01	9.4043e-01
6	1.9627e-01	1.7866e+00	7.7325e-01	9.1626e-01
7	2.5038e-01	1.8199e+00	7.0565e-01	8.9407e-01
8	3.0542e-01	1.8485e+00	6.3466e-01	8.7395e-01
9	3.6125e-01	1.8718e+00	5.6080e-01	8.5601e-01
10	4.1769e-01	1.8896e+00	4.8463e-01	8.4032e-01
11	4.7458e-01	1.9016e+00	4.0676e-01	8.2695e-01
12	5.3174e-01	1.9075e+00	3.2787e-01	8.1593e-01
13	5.8898e-01	1.9072e+00	2.4864e-01	8.0727e-01
14	6.4613e-01	1.9006e+00	1.6981e-01	8.0099e-01
15	7.0297e-01	1.8879e+00	9.2110e-02	7.9706e-01
16	7.5934e-01	1.8690e+00	1.6216e-02	7.9544e-01
17	8.1506e-01	1.8442e+00	-5.7237e-02	7.9607e-01
18	8.6995e-01	1.8139e+00	-1.2766e-01	7.9886e-01
19	9.2385e-01	1.7783e+00	-1.9450e-01	8.0371e-01
20	9.7661e-01	1.7378e+00	-2.5727e-01	8.1050e-01

With the application of the Velezmoro and Ipanaque model, the 4D phase surface is displayed in 3D (see Fig. 3).

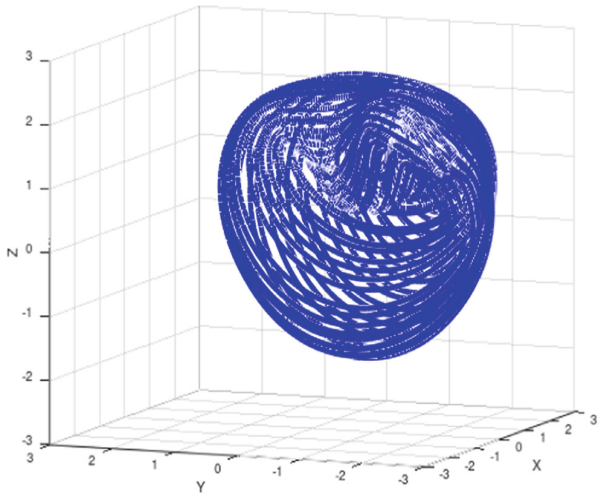


Fig. 3. Phase surface of the chaotic Sprott type A system immersed in 4D.

3.5 Qualitative Analysis of the System

3.5.1 Symmetry

The coordinate transformation is applied to the system of Eqs. (2), if the equations of state remain unchanged, it means that the system is symmetric with respect to the analyzed axis.

- Symmetry with the x-axis: It exists because when replacing –y by y, –z by z, and –w by w in the system (2), equality is obtained.

$$x^{'} = y = -y = -x'$$

$$y^{'} = -x + (-y)(-z) = y'$$

$$z^{'} = -(-y)^2 + 1 = z'$$

$$w^{'} = -(-z) = -w'$$

- Symmetry with the y-axis: It exists because when replacing –x by x, –z by z, and –w by w in the system (2), equality is obtained.

$$x^{'} = y = x'$$

$$y^{'} = -(-x) + y(-z) = -y'$$

$$z^{'} = -y^2 + 1 = z'$$

$$w^{'} = -(-z) = -w'$$

- Symmetry with the z-axis: It exists because when replacing –x by x, -y by y, and –w by w in the system (2), equality is obtained.

$$x^{'} = -y = -x^{'}$$

$$y^{'} = -(-x) + (-y)z = -y^{'}$$

$$z^{'} = -(-y)^2 + 1 = z^{'}$$

$$w^{'} = -z = w^{'}$$

- Symmetry with the w axis: It doesn't exist, because when replacing –x by x, -y by y, and –z by z in the system (2), equality is not obtained.

$$x^{'} = -y = -x^{'}$$

$$y^{'} = -(-x) + (-y)(-z) \neq -y^{'}$$

$$z^{'} = -(-y)^2 + 1 = z^{'}$$

$$w^{'} = -(-z) = -w^{'}$$

The analysis shows that the vector field of the system (2) has symmetry with respect to the axes x, y, z and it has no symmetry in the w-axis. When the flow of the system of equations crosses the axes x, y, z, the state variable w is canceled, i.e., $w' = 0$. This result indicates that every orbit that passes through the axes x, y, z cannot leave it; therefore, the system is invariant.

3.5.2 Sensibility in Initial Conditions

A characteristic of a chaotic system is sensitivity to initial conditions (see Definition 1). This work graphically shows that small variations in systems' initial conditions produce significant variations in their trajectories.

In Fig. 4, a small variation from -0.0561 to -0.2561 was made in the first component of the initial condition x_0. The varying trajectory is shown in black color and the non-varying trajectory is shown in blue color. The results show that black and blue color trajectories remain close to the value of $x_0 = 40$. For values of x_0 greater than 40, the black color trajectory diverges, thus showing sensitivity in this component.

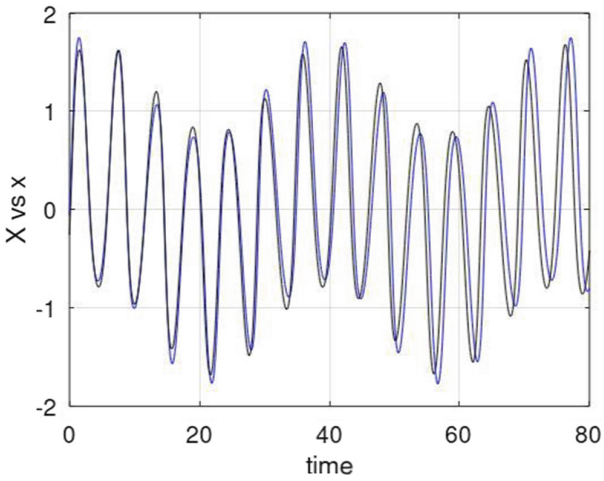


Fig. 4. The blue color trajectory corresponds to the first component without variation and the back color trajectory corresponds to the component's variation. (Color figure online)

In Fig. 5, a small variation from 1.5652 to 1.9652 was made in the second component of the initial condition y_0. The varying trajectory is shown in black color and the non-varying trajectory is shown in blue color. The results show that black and blue color trajectories remain close to the value of $y_0 = 40$. For values of y_0 greater than 40, the black color trajectory diverges, thus showing sensitivity in this component.

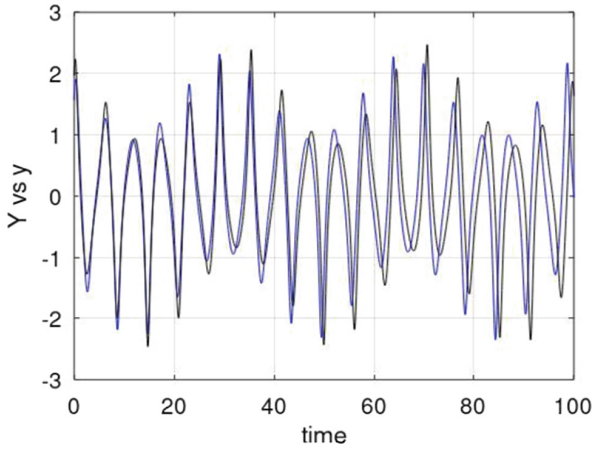


Fig. 5. The blue color trajectory corresponds to the second component without variation and the back color trajectory corresponds to the component's variation. (Color figure online)

In Fig. 6, a small variation from 1.0484 to 1.1884 was made in the third component of the initial condition z_0. The varying trajectory is shown in black color and the non-varying trajectory is shown in blue color. The results show that black and blue color

trajectories remain close to the value of $z = 30$. For values of z_0 greater than 30, the black color trajectory diverges, thus showing sensitivity in this component.

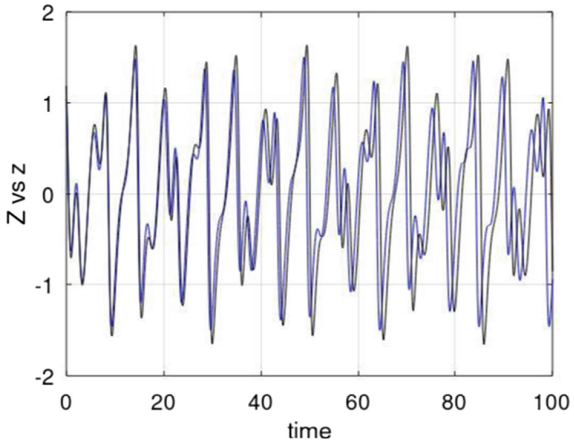


Fig. 6. The blue color trajectory corresponds to the third component without variation and the back color trajectory corresponds to the component's variation. (Color figure online)

In Fig. 7, a small variation from 0 to 0.123 was made in the fourth component of the initial condition w_0. The varying trajectory is shown in black color and the non-varying trajectory is shown in blue color. The results show that black and blue color trajectories remain close to the value of $w = 20$. For values of w_0 greater than 20, the black color trajectory diverges, thus showing sensitivity in this component.

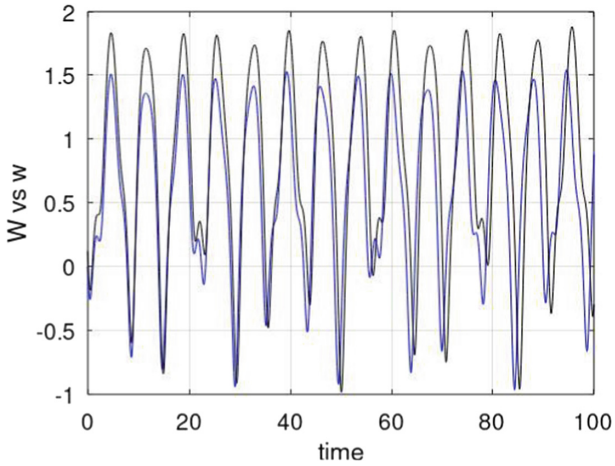


Fig. 7. The blue color trajectory corresponds to the fourth component without variation and the back color trajectory corresponds to the component's variation. (Color figure online)

3.5.3 Lyapunov Exponents

In system (2) the initial condition is $(x, y, z, w) = (-0.0561, 1.5652, 1.0484, 0)$; the Sprott System type A in the fourth dimension, presents three negative and one positive Lyapunov exponents.

$$\lambda_1 = 0.016635, \lambda_2 = -0.0004872, \lambda_3 = -0.0032606y, \lambda_4 = -0.016125$$

3.5.4 Lyapunov-Yorke Dimension

The Lyapunov-Yorke dimension of the hyper-chaotic attractor is fractional with initial conditions $(x, y, z, w) = (0.0561, 1.5652, 1.0484, 0)$.

$$D_{LY} = j + \frac{1}{|\lambda_{j+1}|} \sum_{i=1}^{j} \lambda_i$$

$$D_{LY} = 3 + \frac{1}{|-0.016125|}(0.016635 - 0.0004872 - 0.0032606)$$

$$D_{LY} = 3.7992$$

3.5.5 Dissipation and Existence of the Attractor

System (2) divergence is defined by:

$$\nabla V = \frac{\partial x^{'}}{\partial x} + \frac{\partial y^{'}}{\partial y} + \frac{\partial z^{'}}{\partial z} + \frac{\partial w^{'}}{\partial w}$$

and it measures how quickly the volume changes under the flow of the system.

$$\nabla V = z$$

$$V(t) = V(0)e^{zt}$$

When $t \to \infty$ and $z < 0$, the volume is exponentially reduced to zero. Any flow that enters the attractor will remain in time.

3.5.6 4D Type A Sprott System Scenarios

When the parameter the $a = 0.01$, the trajectories rotate forming a closed surface, the flow of the vector field forms an attractor limit cycle. By varying a, different phase surfaces are obtained (see Figs. 8, 9, 10, 11 and 12).

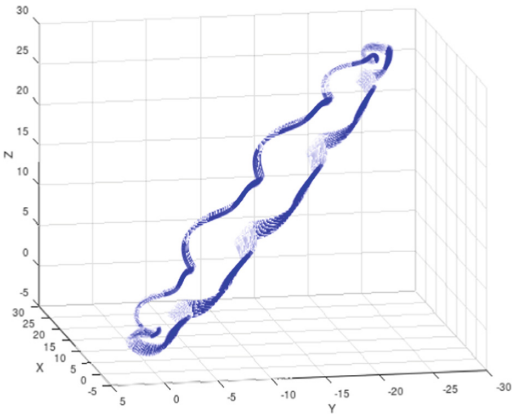


Fig. 8. When $a = 0.01$, the flow of the vector field rotates around an attractor limit cycle.

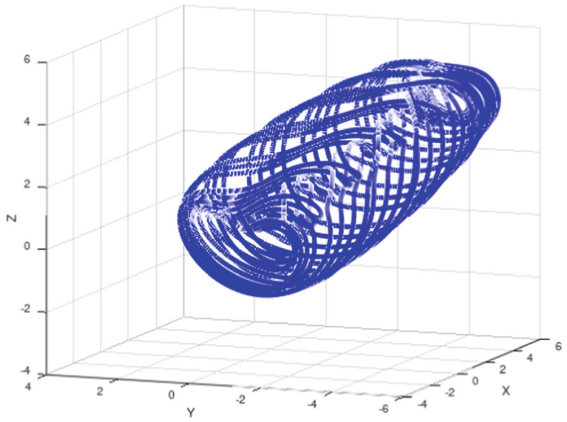


Fig. 9. When the $a = 0.1$, the flow of the vector field is compact and invariant.

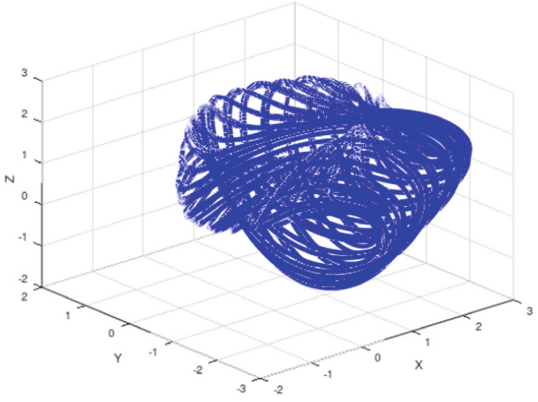


Fig. 10. When $a = 0.5$, the flow is compact and invariant

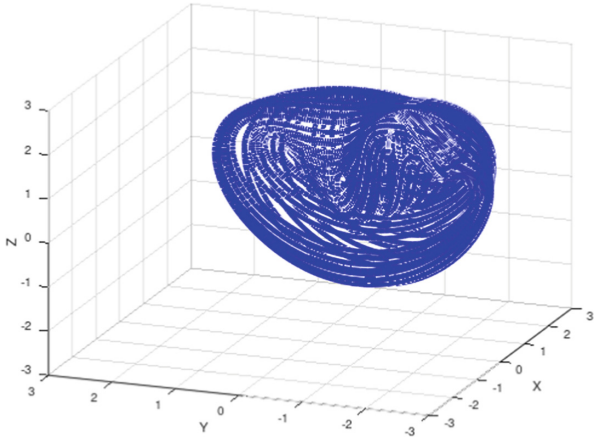


Fig. 11. When $a = 1$, a compact and invariant, and chaotic flow is observed.

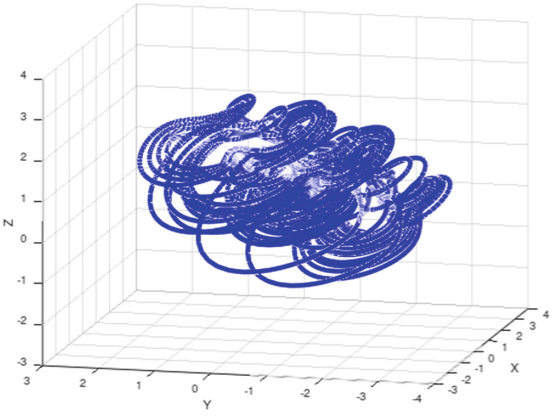


Fig. 12. When the $a = 10$, the flow of the vector field is invariant and chaotic.

4 Conclusions and Recommendations

The SystemSprott4D program implemented in Octave allowed graphical visualization of the phase surfaces of the Sprott type A system in four dimensions. In addition, the program allows graphing trajectories of initial conditions, attractor limit cycles to later observe the qualitative behavior of the system flow.

The Sprott system type A in 4D shows sensitivity in the initial conditions, presents three negative and one positive Lyapunov exponents, and has a fractional dimension. When a $= 0.0$, the flow forms a limit cycle, and when the parameter a values approach one, the flow is compact and invariant, and its volume is maximum. The flow is invariant for values a > 1, and its volume tends to zero for values of $z < 0$.

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