Unveiling the Potential Use of Euler Equations in the Utility Additive Analysis for Mass Appraisal

Francesco Tajani[1], Francesco Sica[1], Maria Rosaria Guarini[1](✉), Pierluigi Morano[2], and Rossana Ranieri[1]

[1] Department of Architecture and Design, "Sapienza" University of Rome, 00196 Rome, Italy
mariarosaria.guarini@uniroma1.it

[2] Department of Civil, Environmental, Land, Building Engineering and Chemistry, Polytechnic University of Bari, 70125 Bari, Italy

Abstract. Using mass appraisal techniques has become strategically important in case of plans property assets, investments in technical and economic renewals, alienation of structures no longer suitable for public needs. Multiple operators implement mass appraisal procedures for residential properties. The UTility Additive (UTA) approach for mass residential property evaluation is tested in this research. The UTA considers the process of deciding a property's price as a multi-criteria typology, with the market-determining attributes of the property acting as the selection criteria to which be capable of assigning marginal utility functions. The Euler equations technique makes use of a variety of margin utility functions, including logistic, exponential, and linear ones. The purpose of this research is to provide a novel operational paradigm for figuring out a property's value in relation to UTA and optimization roles. We wished to investigate the applicability of the Euler Equation (linear, exponential, logistic) as marginal utility functions in particular. We look at the results of independently applying several marginal utility functions, illustrating the various performance stances toward the price formation process. The conclusions discuss the potential for using Euler equations as utility marginal functions throughout the UTA, with following extensions of the work in view to evaluate the performance of the operative model UTA on real case studies.

Keywords: Property Valuation · Mass Appraisal · Utility functions · Euler Equations

1 Introduction

In 2021, the global real estate market size was valued 3.69 trillion of US dollars and is expected to expand of 5.2% from 2022 to 2030 at a compound annual growth rate. The market is supposed to grow both in the residential segment and in the commercial sector, and, as happened for the 2021, commercial spaces have been the most important element driving industry expansion [1]. In Europe, real estate investments in the first half of 2022 increased by approximately 70% compared to the same period in 2021. However, high

© The Author(s), under exclusive license to Springer Nature Switzerland AG 2023
O. Gervasi et al. (Eds.): ICCSA 2023 Workshops, LNCS 14107, pp. 42–53, 2023.
https://doi.org/10.1007/978-3-031-37114-1_4

inflation levels, increasing financing rates and escalating levels of economic uncertainty - also due to rising energy costs because of the continuing Russian-Ukrainian conflict - led to a slowdown in growth. Despite this, significant economic transactions took place in the second half of 2022, causing total volumes to exceed 2021 levels, with an overall increase of around 20% [2, 3].

The World Economic Outlook Update (January 2023) estimates that global growth will fall to 2.9 percent in 2023 but increase to 3.1 percent in 2024. The 2023 forecast is 0.2 percentage point higher than predicted in the October 2022 World Economic Outlook. Furthermore, global inflation is expected to drop to 6.6 percent in 2023 and 4.3 percent in 2024, still above pre-pandemic levels [4, 5].

In addition to this complex economic and financial situation, effects of the Covid-19 pandemic are still present. In fact, the Covid-19 pandemic has globally and deeply changed the real estate market and, also, many and different aspects of everybody's daily life, causing effects on both the social dynamics and the mental well-being of the population [6]. Since the end of 2019, collective preventive measures have been taken all over the world, with numerous limitations. Social distancing, obligatory confinement, access's reduction to public places, interruption of theatre, cinema or concert programs, and reduction of national and international trade have forced almost all the world's population to carry out different functions in their homes (i.e., remote working and distance learning activities, or sports and recreational activities) by leading to considerable variation in the ways of using domestic spaces [7–9]. Furthermore, the pandemic has changed the way of living the cities, exacerbating the resilience of urban spaces and making it more difficult to plan and manage the cities in general [10].

On the real estate market, the impact of the pandemic was visible directly in the first semester of 2020, especially from a retail point of view, due to strict lockdown measures and movement restrictions. The lockdowns imposed across the world, also, caused an interruption in new construction projects and led to slow-moving industry growth. Despite the pandemic's huge reduction in home sales, in 2022 real estate activity began to recover, returning to pre-pandemic levels. According to the National Association of Realtors [11], awaiting sales in U.S. metro areas, were down more than 30% in April 2020. After that period, potential buyers began to search again for and purchase of homes, enhancing the growth of the real estate market. Additionally, the influence of the e-commerce has increased awareness among consumers regarding online real estate services, live-streaming rooms, that are used still nowadays.

As well, the Covid-19 pandemic and the energy issue resulting from the geopolitical crisis that flare up in January 2022 have exacerbated some latent critical issues inherent in the residential building segment.

In fact, the effects of the pandemic – which, as already mentioned, have substantially altered the way of living in domestic spaces - and the effects of energy price restrictions have overlaid their effects. When analyzing the evolution of the real estate sector, it is possible to see how abnormal events of a varied nature have significantly modified market dynamics and make it complex and difficult to make appropriate market forecasts, because this type of events could increase situations of uncertainty and market instability. The real estate sector has been directly and indirectly impacted by the state of uncertainty

and evolutionary dynamics, which have at various points during the last year affected and shaped all of the real estate segments, particularly in Italy.

However, uncertainty regarding the real estate sector's direction in the near future is significant and projected to rise. Real estate markets are structurally unstable in Italy and have undergone significant changes and ongoing transformations because of social, economic, and fiscal factors. On the other hand, the real estate sector will be also affected by the development of projects using funds from National Recovery and Resilience Plan (NRRP) and National Plan of Complementary Investments (PNC) works and interventions.

In Italy the NRRP has been definitively endorsed on 13 July 2021 for a total of 222.1 billion euros and provides a general framework of reforms and measures to be implemented from 2021 to 2026 within the six pillars identified in the European Green Deal: i) green transition, ii) digital transformation, iii) social and territorial cohesion, iv) health, v) policies for the next generation and sustainable and vi) smart and inclusive growth.

It is important to highlight that the construction sector first, and the real estate sector after, are central to the effective development of the missions envisaged in the NRRP. In particular using instruments for the evaluation of property values has become crucial for the definition of effective projects that meet the needs of the community, and beyond that, for the proper allocation of public resources. This context of changing dynamics and of persisting uncertainty requires the employment of models that, in addition to having a solid theoretical and methodological foundation, can automatically capture the causal relationships between explanatory factors and prices and estimate property values to plan and develop effective projects.

2 Objectives and Sections of the Work

The strategic use of mass appraisal methodologies includes the development of management and upgrading plans for both public and private property assets [12] regarding: expenditures in technological and economic refun tionalization [13]; alienation of structures that are no longer appropriate [14] for societal demands (military barracks, hospitals, areas in disuse, etc.); risk associated with the credit institutions' supply of mortgage loans [15]. For example, Harrison et al. (2001), Lewis (2003), Hefferan and Boyd (2010), all discuss the taxation system, programming territorial transitions, and reviewing current or ex post planning choices made by the public administration [16–18].

An experimental property assessment approach has been used in the current paper. It is based on the Utility Additive (UTA) technique, which views the process of a property's price formation by means of the property's attributes serving as assessment criterion of reference market. The multi-attribute utility theory is an extension of the UTA technique, which is based on the idea that preferences may be broken down using additive utility functions. There aren't many examples of this approach being used in literature to analyze the real estate market. Gomes and Rangel (2009) explained the utility functions of variables often taken into account in the appraisal of residential properties in Volta Redonda using the UTA technique (Brazil) [19]. When there is little information on recent sales of comparable properties and their supply only partially reflects the values

of the properties, Aouni and Martel (2004) advised using the UTA approach to support property assessments [20]. The UTA technique has been employed by Greco et al. (2008) and Figueira et al. (2009) by taking into account all potential marginal value functions that are consistent with the decision-indicated maker's preferences [21, 22]. The research's novelty lies in the adaption of several additive utility function types under the UTA umbrella. The typologies were discovered using the Euler Equation technique, which is often used in consumer theory and appreciation of relative properties. The technique takes into account the most important characteristics that characterize utility functions, such as the coefficients of relative and absolute risk aversion, the intertemporal elasticity of substitution, and the coefficients of relative and absolute prudence.

With reference to the Italian real estate market, the application of UTA as it is impacted by the Euler method to creating the utility functions is investigated. In particular, four scenarios were developed based on the kind of utility function that was taken into account. It is of particular relevance to compare the results of UTA implementation to the case study as the utility function under examination differs. This was accomplished using AMPL software to develop an optimization model in accordance with the UTA method's principles.

The paper is set up as follows. Outlines for the UTA and Euler approach for utility function definition are presented in Sect. 3. Section 4 describes the appropriate non-linear optimization algorithms for constructing model UTA-performance. In the same section, an optimization model is presented in which the user can develop alternatives utility functions. The work is concluded in Sect. 5 with a brief summary of the conclusions.

3 Overlooking the UTA Approach

The UTA approach posits that throughout the ordering phase of the eligible alternatives, each decision-maker tries to maximize - consciously or subconsciously - all the variables that form the global utility function (U) associated with each option.

The $v_{i,j}$ shows how the i-*th* option follows the j-*th* criterion whereas a_i ($i = 1,\ldots,$ n) denotes the potential alternatives and c_j (j = 1,…, m) the criteria that characterize the alternatives. $V_{i,j}$ specifies a marginal utility ($u'_{i,j}$), which is the contribution of the alternative i-*th* in reference to the j-*th* criterion to the accomplishment of the global utility U regarding the alternative under consideration.

The UTA technique verifies the following relation by designating C and A, respectively, as the set of criteria of c_j unit (with $j = 1, \ldots, m$) and the set of alternatives of a_i unit (with $i = 1, \ldots, n$):

$$\forall a_i \in A \cup \forall c_j \in C : U(a_i^j) = f(v_{i,j}) \cong f(u'_{i,j}) \tag{1}$$

The global utility of an alternative (a_i) may be calculated using the additive technique Eq. (2), which needs the assumption of independence among the criteria studied and allows for the consideration of a constant term $\sigma(a_i)$:

$$U(a_i) = \sum_{j=1}^{m} u'_{i,j} + \sigma(a_i) \tag{2}$$

The UTA method enables one to quantify the contribution of each criteria to the price formation, otherwise global utility, of each alternative in the population of the detected sample.

In the optimization optic, the global utility functions are approximated as closely as possible to the price reported for each apartment, using a goal programming approach that aims to minimize the error $\varepsilon(a_i)$ corresponding to the difference between the value of the global utility $U(a_i)$ estimated for the apartment and its global utility detected, i.e. the respective selling price $P(a_i)$. So, the objective function takes the following form:

$$\min_{i=1,\ldots,n} \sum_{i=1}^{n} \varepsilon(a_i) = \bar{\varepsilon} = \sum_{i=1}^{n} |P(a_i) - U(a_i)| \tag{3}$$

The UTA method's restrictions are connected to the construction of the marginal utility functions for each criteria used. The marginal utility function should reflect the phenomena under investigation, followed by the connection seen between input value ($v_{i,j}$) and the appropriate marginal utility function ($u_{i,j}$). The piecewise linear function is typically thought of as the functional form that represents the marginal utility. Yet, when using an Eulerian mathematical method, it is also feasible to discover the existence of other functions that describe the marginal utility.

3.1 The Utility Functions Under an Eulerian Mathematical Approach

The most common utility functions used in the literature on the Euler problem are quadratic, exponential, and isoelastic preferences. In this part, we present an overview of these functions and discuss their key advantages and disadvantages.

The marginal utility, for each functional form, is shown in the Table 1. After the suggested summary table, specifics of each type of utility function and its marginal formulation are described separately.

Table 1. Overview of utility functions typologies.

Utility functions types U (A)	Marginal utility function $u'_{i,j}$
a) *Quadratic*	
$U_{(A)} = qc \pm \frac{m}{2}c^2$	$q \pm mc$
b) *Exponential*	
$U_{(A)} = -\frac{e^{-ac}}{a}$	e^{-ac}
c) *Isoelastic*	
$U_{(A)} = \frac{c^{1-\gamma}}{1-\gamma}$	$\frac{\gamma}{c}$

a) **Quadratic function**

In earlier studies, the particular situation of quadratic preferences was used to evaluate the conventional model of consumption. The Permanent Income model with confidence equivalence is the name given to this situation in the literature [23].

In light of this supposition, the Euler equation is as follow:

$$E_{j-1}[u'(C_j)] = u'(C_{j-1}) \tag{4}$$

updates to:

$$\mathrm{C_j} = \mathrm{C_{j-1}} + \varepsilon_\mathrm{j} \tag{5}$$

where the ε_j term is representative that every new knowledge concerning the consumer's sources of uncertainty has an influence on consumption. As a result, in the exceptional situation of quadratic preferences, consumption acts like a martingale: ex ante, present consumption is the greatest predictor of next period's consumption; ex post, consumption changes only if expectations are not met.

When expectations are replaced by realizations, the consumption function is the same as under certainty, which is what quadratic utility implies. The person spends as much as they would if their future wages were guaranteed to match their means. Because there is no precautionary saving in the confidence equivalence situation, the model can be quite deceptive when there is uncertainty.

The quantity of consumption that people are prepared to forgo in order to eliminate a specific degree of consumption uncertainty climbs as people get wealthier since quadratic utility implies growing absolute risk aversion. As riskier portfolios are often owned by richer households, this trait is undesirable theoretically and strongly counterfactually.

b) **Exponential function**

In the literature on consumption, exponential utility has been frequently employed, notably in the well-known model put out by Caballero (1990), where a closed form solution for consumption is found [24].

This functional type stands out because it Consistently exhibits Absolute Risk Aversion (CARA). On the other hand, consumption of relative risk aversion is rising, which is a completely erroneous premise. Intertemporal elasticity of substitution is also declining, just as it was in the case of quadratic preferences.

Because of these characteristics, exponential utility is an unrealistic description of individual preferences.

c) **Isoelastic function**

Since Hansen and Singleton's works from 1982 and 1983, a highly common preference definition known as isoelastic utility has been utilized in the consumption literature. It has also been used extensively in numerous theoretical asset pricing studies [25, 26]. In reality, the isoelastic utility assumption and the lognormality of the joint distribution of consumption and stock returns lead to a closed-form, empirically tractable specification of the model's constraints.

Such a specification, however, also places severe constraints on preferences: in this case, the elasticity of consumption's ability to be substituted over time is constant and equal to the reciprocal of the degree of risk aversion. As a result, there are two functions for the curvature parameter. It summarizes consumers' attitudes toward risk since it is equivalent to the coefficient of relative risk aversion on the one hand. Conversely, its reciprocal measures changes in consumption growth when the relative prices of current and future consumption vary since it is equal to the elasticity of intertemporal substitution.

4 Non-linear Optimization Algorithms for Operative Models UTA-Performance

Using approaches from operational research, it is possible to address the many aspects of the smart city using mathematical formalizations that are responsive to changes in the analysis system. When it comes to manage dynamic flow in optimization optic, it is helpful to apply programming approaches. Due to the true functional relationships between the variables that govern the system, it is feasible to handle the multi-dimensionality of the analysis relatively simply.

The approach that should be used is expressed as an optimization process in the construction of an analytical mathematical model, which including:

$$\begin{cases} \max(\text{or } \min) C(x_1, \ldots, x_n) \\ \varphi_m(x_1, \ldots, x_n) \leq b_m \\ x \in X \end{cases} \tag{6}$$

with the set of variables x, the objective function C(x), the system of constraints φ_m.

There are two types of programming problems that can be distinguished:

1. linear programming problems, where the objective function and the functions that define the constraints are both linear, and
2. non-linear programming problems, where at least one of the functions that define the problem is not linear.

The algorithm for addressing the problem must be chosen in every case. The more popular resolution strategies in the context of all linear programming include dynamic programming, implicit enumeration techniques like Branch&Bound (B&B), cutting plane algorithms, and the Brunch & Cut approach [27–31].

The purpose of the section that follows is to give an operational tool UTA-performance based that can be customized according to the kind of utility function the user wants to provide in the analysis. This is accomplished by developing a simple rational model using the syntax of AMPL software, which is capable of resolving various optimization issues while taking into account and respecting the relationships between inputs and outputs. About this, AMPL contains a number of solvers (KNITRO, CPLEX, MINOS, etc.) that are intended to resolve linear, as opposed to optimization problems, while taking into account a variety of different types of predetermined constraints. The real estate market's features, which might affect how prices are built, are taken into consideration while calibrating the suggested analytical model.

4.1 Describing UTA-Performance Model for Mass Appraisal

The AMPL syntactic rules were employed for developing the proposed optimization model [32]. The system has the same structure as in (6), and the AMPL tool was used to implement the Simplex optimization approach, which allows for the resolution of both linear and non-linear programming problems.

The use of AMPL enables the following:

- construction of a parametric model through the file.mod;
- writing of the data of the problem with a file.dat separated from the corresponding file.mod;
- characterization of the system's components such as a series of objects (set);
- establishment of the unknown values,
- to articulate the objective function using an equation based on linear algebra that maximizes/minimize the utility function.

The m criteria are used to evaluate the n alternatives (set CASES). The division PARAMETERS contains the numerical values for each factor used to characterize the mass appraisal instance. These are the price of each option (param PRZ) and the j-*th* Criterion (C) for evaluating each option (param CRITERION).

The problem's unknown values are determined by the type of margin utility function used (linear, exponential, isoelastic) in the respect to the Euler Equation approach. In accordance with Eq. (3), the objective function is expressed as follows:

$$\min imize\ \overline{\varepsilon}:\ \text{sum\{i in CASES\}(PRZ[i]}--(\text{U[i]} = \text{f}(\text{u}_{\text{i}}^{\text{j}})) \tag{7}$$

It makes sense to associate the i-th marginal utility function with numerical intervals of defined extremes of the j-*th* characteristics because each marginal utility function is denoted by a different mathematical formula and therefore has a different numerical significance depending on the type of analysis variable under consideration. To put it another way, it makes logical to discretize the j-*th* variable's numerical series into discrete k-*th* intervals and connect the marginal utility function of study with each. This implies that the mass appraisal optimization problem should include algebraic formulations of continuity of characteristics at the end ($^-$) and beginning ($^+$) extremes of each interval, as follow:

$$u_{ijk}^{-} = u_{ijk+1}^{+} \tag{8}$$

Depending on the marginal utility function to be used, the (8) might be described by foreseeing various mathematical structures. The three different types of Euler Equations (Linear, Exponential, and Isoelastic) are then used to arrange the potential algebraic configurations of the number (8), as in Table 2.

Table 2. Overview of constraints per margin utility functions typologies.

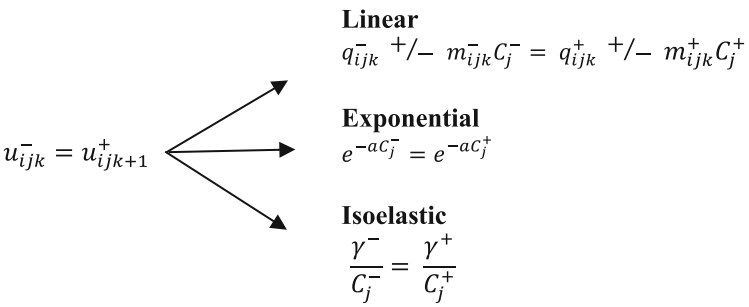

Table 3 then shows the entire layout of the AMPL programming standards-compliant UTA-performance based optimization system. The characteristics of the model in Table 2 are then provided while taking into account the linear, exponential and isoelastic marginal utility functions. A system example with four instances and one criterion (SURF) is suggested in the Fig. 1 below.

Table 3. Structuring UTA-performance model according to AMPL editing-rules.

#Sets-system#
set CASES: = 1..n;
#Parameters#
param PRZ {*i* in CASES};
param CRITERION {*i* in CASES};
#Objective function clarification#
minimize $\overline{\varepsilon\varepsilon}$: : sum{i in CASES}(PRZ[i] – (U[i] = f(u_i^j));
#Constraints#
subject to (s.t.) constraints: $u_{\overline{ijk}} = u_{ijk+}^{+}$

Linear

```
set CASI := 1..4;

param PRZ{CASI};
param SURF{CASI};

var q1;
var m1;
var q2;
var m2;
var q3;
var m3;
var q4;
var m4;

minimize somma: sum{i in CASI}(PRZ[i] - (q1 + m1*SURF[1]) - (q2 + m2*SURF[2]) - (q3 + m3*SURF[2])- (q4 + m4*SURF[4]))^2;

s.t. vincolo_0 {i in CASI}: q1 + m1*SURF[1] - q2 - m2*SURF[1] = 0;
s.t. vincolo_1 {i in CASI}: q2 + m2*SURF[2] - q3 - m3*SURF[2] = 0;
s.t. vincolo_2 {i in CASI}: q3 + m3*SURF[3] - q4 - m4*SURF[3] = 0;
```

Exponential

```
set CASI := 1..4;

param PRZ{CASI};
param SURF{CASI};

var a1;
var a2;
var a3;
var a4;

minimize somma: sum{i in CASI}(PRZ[i] - (exp(-a1*SURF[1])) - (exp(-a2*SURF[2])) - (exp(-a3*SURF[3]))- (exp(-a4*SURF[4])))^2;

s.t. vincolo_0 {i in CASI}: exp(-a1*SURF[1]) - exp(-a2*SURF[2]) = 0;
s.t. vincolo_1 {i in CASI}: exp(-a2*SURF[2]) - exp(-a3*SURF[2]) = 0;
s.t. vincolo_2 {i in CASI}: exp(-a3*SURF[3]) - exp(-a4*SURF[3]) = 0;
```

Isoelastic

```
set CASI := 1..4;

param PRZ{CASI};
param SURF{CASI};

var γ1;
var γ2;
var γ3;
var γ4;

minimize somma: sum{i in CASI}(PRZ[i] - (γ1/SURF[1]) - (γ2/SURF[2]) - (γ3/SURF[3]) - (γ4/SURF[4])^2;

s.t. vincolo_0 {i in CASI}: (γ1/SURF[1]) - (γ2/SURF[2]) = 0;
s.t. vincolo_1 {i in CASI}: (γ2/SURF[2]) - (γ3*SURF[2]) = 0;
s.t. vincolo_2 {i in CASI}: (γ3*SURF[3]) - (γ4*SURF[3]) = 0;
```

Fig. 1. AMPL files. Mod for linear, exponential and isoelastic functions

5 Conclusions

The application potential of the proposed UTA approach for mass appraisal, which is capable of producing an efficient management of both public and private property, is connected to the consequences of this research for valuation practice. To improve the properties with the most convenience and to reduce management costs in order to streamline the management of public assets, public administrations may use the suggested operative models. They may also determine the effects of property tax policies and investments in urban redevelopment, as expressed in variations in property prices, and update the property values of their assets over time.

Private operators, on the other hand, could use the proposed methodologies with relative assessment tools for a variety of purposes, including: updating and periodically verifying the market values of properties as securities for credit exposures; allowing the adoption of possible balancing measures aimed at reducing credit risks; real estate funds, periodically assessing the market values of their assets; and private companies, monitoring the congruency of their assets.

The models, whose structures resemble those in Fig. 1, will be adopted to various marginal utility functions obtained using the Euler Equation method. The Euler approach's benefit of referencing mass appraisal experiences is tractable on evaluating how well the three models—linear, exponential, and isoelastic—perform in terms of the utility goal following the application of the specific marginal function. To this, the model will be applied to several case studies in order to evaluate its efficacy, highlight its benefits, and draw attention to its drawbacks.

Additional results from this research could be related to the potential for combining together the marginal utility function of each section of the UTA model taking in practice more than one assessment criterion, in order to obtain functional forms rather than necessarily linear functions that are more closely related to the empirical phenomenon under study. A further area that needs to be investigated is whether it is possible to define a universal valuation function for all markets. This research was unable to do so because the examined markets were opaque, the influencing factors for selling prices were too specific, and the sample size was too small.

References

1. CBRE: https://www.cbre.it/insights/reports/italy-real-estate-market-outlook-2023, last 01 March 2023
2. Gupta, A., Mittal, V., Peeters, J., Van Nieuwerburgh, S.: Flattening the curve: pandemic-induced revaluation of urban real estate. J. Financ. Econ. **146**(2), 594–636 (2022)
3. PWC: https://www.pwc.com/gx/en/industries/financial-services/real-estate/emerging-trends-real-estate/europe-2023.html#report, last 01 March 2023
4. IMF: https://www.imf.org/en/Publications/WEO, 21 February 2023
5. MSCI: https://www.msci.com/documents/10199/8f62c2a3-8374-cbf9-a7d2-a8c2c5e63e62, last accessed 28 February 2023
6. Inoue, H., Todo, Y.: The propagation of economic impacts through supply chains: the case of a mega-city lockdown to prevent the spread of Covid-19. In: PLoS ONE **15**(9 September), 1–10 (2020)
7. Li, H.Y., Cao, H., Leung, D.Y.P., Mak, Y.W.: The psychological impacts of a Covid-19 outbreak on college students in China: a longitudinal study. In: International Journal of Environmental Research and Public Health **17**(11) (2020)
8. Tajani, F., Liddo, F.D., Guarini, M.R., Ranieri, R., Anelli, D.: An assessment methodology for the evaluation of the impacts of the COVID-19 pandemic on the Italian housing market demand. Buildings **11**(12), 592 (2021)
9. Tajani, F., Morano, P., Di Liddo, F., Guarini, M.R., Ranieri, R.: The Effects of Covid-19 Pandemic on the Housing Market: A Case Study in Rome (Italy). In: Gervasi, O., et al. (eds.) ICCSA 2021. LNCS, vol. 12954, pp. 50–62. Springer, Cham (2021). https://doi.org/10.1007/978-3-030-86979-3_4
10. Anelli, D., Ranieri, R.: Resilience of complex urban systems: a multicriteria methodology for the construction of an assessment index. In: New Metropolitan Perspectives: Post COVID Dynamics: Green and Digital Transition, between Metropolitan and Return to Villages Perspectives, pp. 690–701. Springer International Publishing, Cham (2022)
11. NAR REALTOR: https://www.nar.realtor/blogs/economists-outlook/where-people-moved-in-2022, last accessed 28 February 2023
12. Bourassa, S.C., Cantoni, E., Hoesli, M.: Spatial dependence, housing submarkets, and house price prediction. The Journal of Real Estate Finance and Economics **35**(2), 143–160 (2007)

13. McCluskey, W., Deddis, W., McBurney, R.D., Mannis, A., Borst, R.: Interactive application of computer assisted mass appraisal and geographic information systems. J. Prop. Valuat. Invest. **15**(5), 448–465 (1997)
14. Lee, S., Park, I., Choi, J.K.: Spatial prediction of ground subsidence susceptibility using an artificial neural network. Environ. Manage. **49**(2), 347–358 (2012)
15. Lentz, G., Wang, K.: Residential appraisal and the lending process: a survey of issues. Journal of Real Estate Research **15**(1), 11–39 (1998)
16. Harrison, D.T., Smersh, G., Schwartz, A.: Environmental determinants of housing prices: the impact of flood zone status. Journal of Real Estate Research **21**(1/2), 3–20 (2001)
17. Lewis, B.D.: Property tax in Indonesia: measuring and explaining administrative (under-) performance. Public Administration and Development **23**(3), 227 (2003)
18. Hefferan, M.J., Boyd, T.: Property taxation and mass appraisal valuations in Australia adapting to a new environment. Prop. Manag. **28**(3), 149 (2010)
19. Gomes, L.F.A.M., Rangel, L.A.D.: Determining the utility functions of criteria used in the evaluation of real estate. Int. J. Prod. Econ. **117**(2), 420–426 (2009)
20. Aouni, B., Martel, J.M.: Property assessment through an imprecise goal programming model. Info. Sys. Operat. Res. **42**(3), 189–200 (2004)
21. Greco, S., Mousseau, V., Słowinski, R.: Ordinal regression revisited: multiple criteria ranking using a set of additive value functions. Eur. J. Oper. Res. **191**(2), 416–436 (2008)
22. Figueira, J.R., Greco, S., Słowinski, R.: Building a set of additive value functions representing a reference preorder and intensities of preference: GRIP method. Eur. J. Oper. Res. **195**(2), 460–486 (2009)
23. Jappelli, T., Pistaferri, L.: The consumption response to income changes. Annual Review of Economics **2**, 479–506 (2010)
24. Caballero, R.J.: Consumption puzzles and precautionary savings. J. Monet. Econ. **25**(1), 113–136 (1990)
25. Merton, R.B.: Rational theory of option pricing. Bell J. Econ. Manag. Sci. **4**, 141–183 (1973)
26. Rubinstein, M.: The strong case for the generalized logarithmic utility model as the premier model of financial markets. Journal of Finance (1976)
27. Vercellis, C.: Ottimizzazione: teoria, metodi, applicazioni. McGraw-Hill, Milano (2008)
28. Guarini, M.R., Morano, P., Sica, F.: Eco-system Services and Integrated Urban Plan-ning. A Multi-Criteria Assessment Framework for Ecosystem Urban Forestry Projects. In: Values and Functions for Future Cities, pp. 201–216. Springer, Cham (2020)
29. Guarini, M.R., Morano, P., Sica, F.: Integrated ecosystem design: an evaluation model to support the choice of eco-compatible technological solutions for residential building. Energies **12**(14), 2659 (2019). https://doi.org/10.3390/en12142659
30. Morano, P., Tajani, F., Di Liddo, F., Amoruso, P.: The public role for the effectiveness of the territorial enhancement initiatives: a case study on the redevelopment of a building in disuse in an Italian small town. Buildings **11**(3), 87 (2021)
31. Calabrò, F.: Integrated programming for the enhancement of minor historical centres. The SOSTEC model for the verification of the economic feasibility for the enhancement of unused public buildings. [La programmazione integrata per la valorizzazione dei centri storici minori. Il Modello SOSTEC per la verifica della fattibilità economica per la valorizzazione degli immobili pubblici inutilizzati]. ArcHistoR **13**(7), 1509–1523 (2020). https://doi.org/10.14633/AHR280
32. Bruglieri, M., Cordone, R., Liberti, L., Iuliano, C.: Manuale essenziale di AMPL. Dipartimento di Elettronica e Informazione. Politecnico di Milano (2010)