Wavelength Computation from RGB

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Abstract. Conversion RGB to wavelength is not a simple problem. This contribution describes a simple method for wavelength extraction for colors given by the RGB triplet. The method is simple and accurate, based on known RGB values of the rainbow. It also respects different saturation of a color.

Keywords: Color systems · RGB · wavelength · color conversion · image processing · computer vision · autonomous vehicles · physics

1 Introduction

There are many fields in which RGB images are used and processed, extracting important features in images, etc. Image processing techniques are based nearly exclusively on shadow processing and RGB image representation. It is well known that RGB does represent a fraction of natural colors contained in the rainbow spectrum [1–12].

Surprisingly, computation of a wavelength λ of the color c given in RGB is not simple if the accurate value is required. Mostly, the RGB values are converted to the HLS, resp. HSV or similar color system, and the wavelength is estimated from the HUE value, which is quite inaccurate. This contribution presents a precise method for computing the wavelength λ based on re-sampling the spectral rainbow curve, i.e. the rainbow curve with 100% color saturation. The rainbow curve samples give the precision, Table 3. The preprocessing generates a look-up table for the whole interval of wavelengths independent of images. The run-time is based on very simple computation and extraction of the wavelength using the look-up table.

2 Color and Color Representation

The RGB values cover luminosity and chromaticity. It means that chromaticity is two-dimensional as it covers a wavelength and saturation of the given color.

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The wavelengths contained in white light are given in Fig. 1 using the reference wavelengths $\lambda_R = 780$[nm] for red, $\lambda_G = 546.1$[nm] for green and $\lambda_B = 435.8$[nm] for blue colors.[1] It can be seen, that the curve for the red color is partially negative. It means, that some colors of a rainbow are not represented within the RGB color model, as a color is represented by the RGB cube $[0,1] \times [0,1] \times [0,1]$.

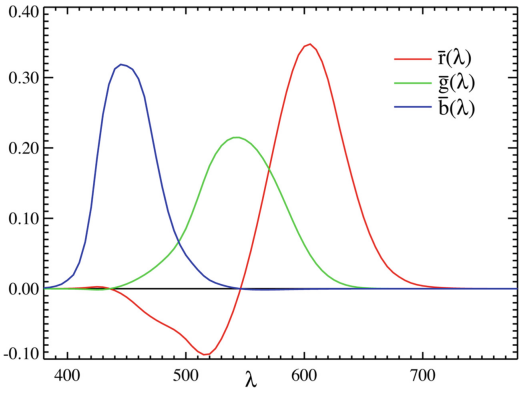


Fig. 1. RGB coefficients; courtesy WiKi [16]

To eliminate intensity, the RGB values are projected to a unitary plane $R + G + B = 1$ using Eq. 1. The chromaticity of a color is given by values r and g in the $r - g$ plane representing color saturation and wavelength together.

$$r = \frac{R}{R+G+B} \quad , \quad g = \frac{R}{R+G+B} \quad , \quad b = 1 - R - G \tag{1}$$

After projecting to the unitary plane, colors form the area in Fig. 2 (pseudo-coloring was used). Colors in the $r < 0$ are not representable within the RGB system. The rainbow curve is labeled by color wavelengths with 100% color saturation. Figure 3 represents colors available within the RGB system and E is the equal energy white light position, i.e. $(1/3, 1/3)$.

The line G-E is given as:

$$\begin{aligned} \mathbf{p}_{GE} = \mathbf{x}_G \wedge \mathbf{x}_E = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 1 & 1 \\ \frac{1}{3} & \frac{1}{3} & 1 \end{vmatrix} = [\frac{2}{3}, \frac{1}{3} : -\frac{1}{3}]^T \\ p_{GE} : \quad \frac{2}{3}x + \frac{1}{3}y - \frac{1}{3} = 0 \triangleq 2x + y - 1 = 0 \end{aligned} \tag{2}$$

where $\mathbf{x}_E = [1/3, 1/3 : 1]^T$ is the equal energy point position, $[0, 1 : 1]^T$ is the green color position and $\triangleq$ means projective equivalency.[2]

[1] The *RBG* spectral values for iso-energetic white color are specified in Table 3 [7].

[2] Details on projective geometric algebra use can be found in [12–14] and intersection computation in [15].

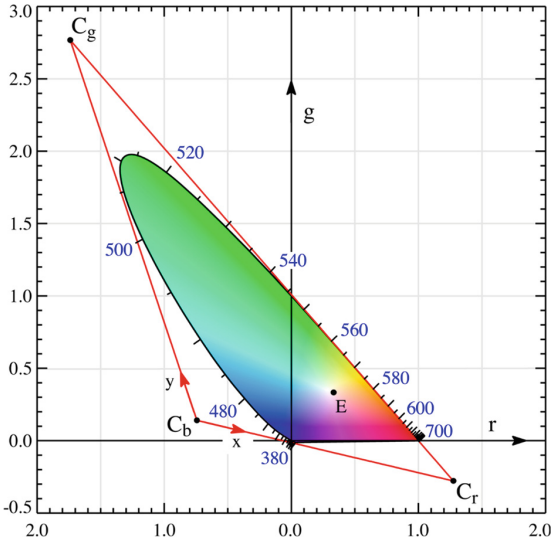


Fig. 2. Colors in RGB and XYZ coordinate systems; courtesy WiKi [16]

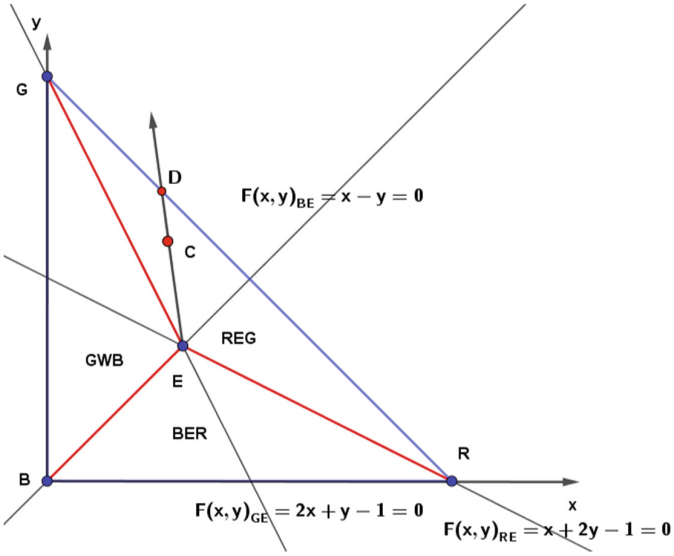


Fig. 3. RGB - color sectors

It should be noted that positions are given in homogeneous coordinates[3] of the white light, $\mathbf{x}_G = [0, 1 : 1]^T$ is a position of the pure green color, see Fig. 2.

[3] A value (X, Y) in Euclidean space can be expressed as $[wx, wy : w]^T$ in the projective space & $w \neq 0$; $w = 1$ is used in this case [13].

The function $F(x,y)_{GE}$ forms a separation function, which is positive for colors in the Red(R)-White(E)-Green(G) sector of colors.

$$F(x,y)_{GE} = 2x + y - 1 \tag{3}$$

The lines R-E and B-E are given similarly as:

$$\begin{aligned} \mathbf{p}_{RE} = \mathbf{x}_R \wedge \mathbf{x}_E = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & 0 & 1 \\ \frac{1}{3} & \frac{1}{3} & 1 \end{vmatrix} = [-\frac{1}{3}, -\frac{2}{3} : \frac{1}{3}]^T \\ p_{RE}: \quad -\frac{1}{3}x - \frac{2}{3}y + \frac{1}{3} = 0 \triangleq x + 2y - 1 = 0 \end{aligned} \tag{4}$$

$$\begin{aligned} \mathbf{p}_{BE} = \mathbf{x}_B \wedge \mathbf{x}_E = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 1 \\ \frac{1}{3} & \frac{1}{3} & 1 \end{vmatrix} = [-\frac{1}{3}, \frac{1}{3} : 0]^T \\ p_{BE}: \quad -\frac{1}{3}x + \frac{1}{3}y = 0 \triangleq x - y = 0 \end{aligned} \tag{5}$$

where $\mathbf{x}_R = [1,0:1]^T$, $\mathbf{x}_B = [0,0:1]^T$ and $\mathbf{x}_E = [1/3,1/3:1]^T$ are positions in homogeneous coordinates of the red, blue and white colors in the diagram Fig. 2. If the given color C is in the BER color sector, the color is not a spectral color, e.g. purple or brown colors are not present in a rainbow, and complementary wavelength is to be used. In the case of light, only the sectors REG and GEB are valid.

Therefore, if $F(x,y)_{GE} \geq 0$, the sector REG is to be used, otherwise the sector GEB.

$$F(x,y)_{GE} \begin{cases} \geq 0 & \text{the sector REG is to be used} \\ < 0 & \text{the sector GEB is to be used} \end{cases}$$

Now, edges of the sectors have to be labeled by the relevant wavelength λ.

3 Wavelength Computation

In the case of the sector REG, the wavelength of the color RGB is determined as the intersection D of a ray $\mathbf{p}_{EC}$ from the point E passing the point representing the color C with the edge of the relevant color sector, i.e. $\mathbf{p}_{RG}$. The intersection point $\mathbf{x}_D$ is given as:

$$\begin{aligned} \mathbf{p}_{EC} = \mathbf{x}_E \wedge \mathbf{x}_C \quad , \quad \mathbf{p}_{RG} = [1,1:-1]^T \\ \mathbf{x}_D = \mathbf{p}_{RG} \wedge \mathbf{p}_{EC} = [x_D, y_D : w_D]^T \triangleq (\frac{x_D}{w_D}, \frac{y_D}{w_D}) \end{aligned} \tag{6}$$

It should be noted, that lines representing the triangle edges are constant, i.e. $\mathbf{p}_{RG} = [1,1:-1]^T$, $\mathbf{p}_{GB} = [1,0:0]^T$ and $\mathbf{p}_{BR} = [0,1:0]^T$.

Similarly, in the case of the sector GEB. The intersection point $\mathbf{x}_D$ is given as:

$$\begin{aligned} \mathbf{p}_{EC} = \mathbf{x}_E \wedge \mathbf{x}_C \quad , \quad \mathbf{p}_{GB} = [1, 0 : 0]^T \\ \mathbf{x}_D = \mathbf{p}_{GB} \wedge \mathbf{p}_{EC} = [x_D, y_D : w_D]^T \triangleq (\frac{x_D}{w_D}, \frac{y_D}{w_D}) \end{aligned} \tag{7}$$

In the actual implementation, the code should be optimized as general formulations are used.

The RGB values of the spectrum given in Table 3 have to be recomputed and relevant wavelengths on $\mathbf{p}_{RG}$ and $\mathbf{p}_{GB}$ are obtained. In the case of REG sector projection to the x-axis is made, while in the case of GEB sector projection to the y-axis is made.

Now, the relevant points on the x-axis and y-axis have wavelengths projected from the spectral curve. As the distance between consecutive wavelengths in Table 3 is $5[nm]$ the axes x and y can be finely uniformly re-sampled and relevant wavelengths can be linearly interpolated. It means, that if a color $C = (r, g)$ is given, relevant position of the point D is to be computed and the r, resp. g value is index to a table with the wavelengths using a scaling factor ξ

$$index = \lfloor \xi * r \rfloor \text{ , resp. } index = \lfloor \xi * g + k \rfloor \tag{8}$$

where ξ is a scaling (re-sampling) factor[4], e.g. $\xi = 100$, $k = \xi + 1$ is the index of the first row in the table with the interpolated wavelengths for the y-axis. It should be noted that $r \in [0, 1]$ $g \in [0, 1]$.

Table 1. Example of wavelength reconstructed



Original image



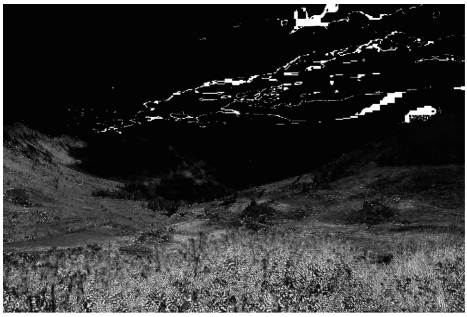
Pseudo-colored wavelengths

[4] $\xi = 100$ means 100 sub-intervals on the x, resp. y axis.

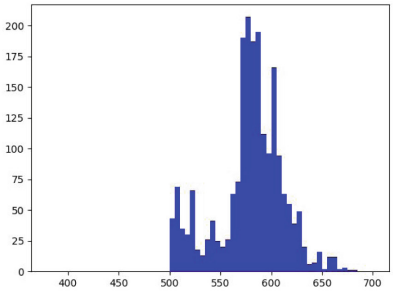
4 Experimental Results

The presented approach of the wavelength reconstruction from acquired RGB images are proving, that wavelength can be determined as described above. Table 1 presents wavelengths using a gray, i.e. it is pseudo-colored, and Table 2 presents saturation and histogram of wavelengths.

Table 2. Example of saturation reconstructed, and wavelength histogram.



Saturation



Wavelength histogram

5 Conclusions

A precise methods for RGB conversion to the wavelength is presented. It consist of a look-up table generation, which is constant for all images and can be generated once forever. The run-time is simple as it needs only simple linear interpolation and actual wavelength extraction using the look-up table. The run-time is fast and convenient for processing of large images. The expected applications can be seen within image processing, computer vision. Standard techniques used in image processing, e.g. edge detection, feature detection etc., using the wavelength instead of grey, resp. RGB will be explored in future work. In future, XYZ color system will be explored as it eliminates negative r, g, b and values, but not appearing in captured images using RGB representation.[5]

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[5] (Responsibilities: Skala, V.: theory, algorithm design, algorithm implementation design, manuscript preparation; Bellot, T.C.L., Berault, X.: implementation and experimental verification).

RGB Trichromatic Coefficients

To avoid numerical instability in Eq. 1 $r = 0.00001$ was set for $\lambda = 780$[nm].

Table 3. RGB spectral trichromatic values.

λ	r	g	b
380	0,00003	−0,00001	0,00117
385	0,00005	−0,00002	0,00189
390	0,00010	−0,00004	0,00359
395	0,00017	−0,00007	0,00647
400	0,00030	−0,00014	0,01214
405	0,00047	−0,00022	0,01969
410	0,00084	−0,00014	0,03707
415	0,00139	−0,00070	0,06637
420	0,00211	−0,00110	0,11541
425	0,00266	−0,00143	0,18575
430	0,00218	−0,00119	0,24769
435	0,00036	−0,00021	0,29012
440	−0,00261	0,00149	0,31228
445	−0,00673	0,00379	0,31860
450	−0,01213	0,00678	0,31670
455	−0,01874	0,01046	0,31166
460	−0,02608	0,01485	0,29821
465	−0,03324	0,01977	0,27295
470	−0,03933	0,02538	0,22991
475	−0,04471	0,03183	0,18592
480	−0,04939	0,03914	0,14494
485	−0,05364	0,04713	0,10968
490	−0,05814	0,05689	0,08257
495	−0,06414	0,06948	0,06246
500	−0,07173	0,08536	0,04776
505	−0,08120	0,10593	0,03688
510	−0,08901	0,12860	0,02698
515	−0,09356	0,15262	0,01842
520	−0,09264	0,17468	0,01221
525	−0,08473	0,19113	0,00830
530	−0,07101	0,20317	0,00549
535	−0,05316	0,21083	0,00320
540	−0,03152	0,21466	0,00146
545	−0,00613	0,21487	0,00023
550	0,02279	0,21178	−0,00058
555	0,05514	0,20588	−0,00105
560	0,09060	0,19702	−0,00130
565	0,12840	0,18522	−0,00138
570	0,16768	0,17087	−0,00135
575	0,20715	0,15429	−0,00123
580	0,24526	0,13610	−0,00108

λ	r	g	b
580	0,24526	0,13610	−0,00108
585	0,27989	0,11686	−0,00093
590	0,30928	0,09754	−0,00079
595	0,33184	0,07909	−0,00063
600	0,34429	0,06246	−0,00049
605	0,34756	0,04776	−0,00038
610	0,33971	0,03557	−0,00030
615	0,32265	0,02583	−0,00022
620	0,29708	0,01828	−0,00015
625	0,26348	0,01253	−0,00011
630	0,22677	0,00833	−0,00008
635	0,19233	0,00537	−0,00005
640	0,15968	0,00334	−0,00003
645	0,12905	0,00199	−0,00002
650	0,10167	0,00116	−0,00001
655	0,07857	0,00066	−0,00001
660	0,05932	0,00037	0,00000
665	0,04366	0,00021	0,00000
670	0,03149	0,00011	0,00000
675	0,02294	0,00006	0,00000
680	0,01687	0,00003	0,00000
685	0,01187	0,00001	0,00000
690	0,00819	0,00000	0,00000
695	0,00572	0,00000	0,00000
700	0,00410	0,00000	0,00000
705	0,00291	0,00000	0,00000
710	0,00210	0,00000	0,00000
715	0,00148	0,00000	0,00000
720	0,00105	0,00000	0,00000
725	0,00074	0,00000	0,00000
730	0,00052	0,00000	0,00000
735	0,00036	0,00000	0,00000
740	0,00025	0,00000	0,00000
745	0,00017	0,00000	0,00000
750	0,00012	0,00000	0,00000
755	0,00008	0,00000	0,00000
760	0,00006	0,00000	0,00000
765	0,00004	0,00000	0,00000
770	0,00003	0,00000	0,00000
775	0,00001	0,00000	0,00000
780	0,00000	0,00000	0,00000

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