A Compact-RBF-FD Scheme for Valuing Financial Derivatives Based on Short-Rate Models

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Abstract. Modelling the term structure of interest rate is of crucial importance nowadays since during times of financial crisis, governments tend to adopt monetary policies which usually result in changes in interest rates. Therefore, it is more realistic to assume that interest rates fluctuate, even over a short time period. Numerous models have been proposed in financial engineering, to describe the stochastic dynamics of interest rates. Interest rate options are among the most liquid options that trade in derivative markets. Due to the enormous size of these markets, there is a need to develop fast and accurate numerical methods for pricing these financial derivatives. In comparison to equity derivatives where a lot of research efforts have been devoted to developing sophisticated methods, research in the area of numerical valuation of interest rate options has been quite sparse. In this paper, we propose a compact localised radial basis function method for pricing interest rate derivatives when exact solutions are not available, as in the case of European and American options on bonds. It is shown numerically that fourth-order accurate results are obtained for American bond options and the proposed method does not suffer from mispricing for long maturity problems or when the short-term rate is high.

Keywords: Stochastic interest rates · Bonds and bond options · Compact radial basis functions method

1 Introduction

A distinguishing feature of short-rate models is the need for the models to exhibit mean reversion and the volatility being dependent on the interest rate. Various stochastic models for the evolution of interest rates have been proposed, but due to the complexities of mean reversion and non-constant volatility, one-factor models are still popular among practitioners although sophisticated multi-factor models have been proposed. Following the introduction of the popular Vasicek [24] and Cox-Ingersoll-Ross (CIR) [8] models, diffusion processes have been often used in finance to model the evolution of interest rates. Recently, an interest rate model embedding the interest rate process with a non-stationary

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mean was developed by Al-Zoubi [1] and it was empirically shown that imposing a non-stationary mean in the Vasicek model is very efficient and leads to considerable amelioration in the root-mean-square error. Ascione et al. [2] introduced the Heston-CIR Lévy model for the Forex market and the jump-extended CIR model was considered in [4,16].

The models of Vasicek and Cox-Ingersoll-Ross admit analytical solutions for bond prices and European style derivatives. A comparison of different approaches for the valuation bond and options on bonds under these models can be found in [12]. However Chan, Karoyli, Longstaff and Sanders (CKLS) [5] have shown that a more general form for the stochastic differential equation may have a greater ability in capturing the dynamics of the interest rate. For such cases, analytical solutions are not available and numerical approaches are necessary for computing approximations to bond and bond-option prices.

Costabile and Massabó [7] also considered the pricing of interest rate derivatives for the square root diffusion process using their approximating binomial and trinomial tree method. Tian [22] mentioned that a problem arises when $4\kappa\theta < \sigma^2$, while implementing a discrete model by defining the transition probabilities on transformed processes which has a drift that explodes when the process reaches zero. However, Costabile and Massabó [7] showed that no such problem arises using their transformed process since the moment-matching condition is defined with respect to the drift of the original process. Choi and Wirjanto [6] proposed an analytical approximation formula for pricing bonds under the CKLS model. The authors mentioned in their paper that as time to maturity or the initial interest rate increases, the method leads to mispricing.

While there exists many numerical techniques such as lattice methods or second-order finite difference methods for pricing interest rate derivatives [7,14, 17], meshless methods have recently gained popularity for pricing equity derivatives and they are highly efficient. We proposed 5-point localised RBF methods for pricing real estate derivatives using Gaussian basis functions in Narsoo et al. [13], 3-point localised RBF method using multiquadrics basis for pricing equity derivatives under the Stochastic-Alpha-Beta-Rho model in Thakoor [18] and regime-switching stochastic volatility models with jumps was considered in Tour et al. [23]. More recently, a sixth-order non-uniform method was proposed in Thakoor [21]. In this work we propose a compact radial basis function method for pricing bonds, European and American bond options with fourth order accuracy under the generalised CKLS model. Localised radial basis function based on finite difference (RBF-FD) approximations for the discretisation of the spatial derivatives are first derived following [3,20] and these are then adapted to the high-order compact method of [15]. We demonstrate numerically that the method is fast, highly accurate and does not suffer from mispricing problems which is common for long maturity problems.

The organisation of this paper is as follows. In §2 we give a review of the CKLS model and the analytical solution for bond pricing under the Vasicek and CIR models. In §3, the compact RBF scheme is derived and in §4 we present some numerical results to show the merits of the proposed method. Finally §5 contains the conclusion of this work.

2 The CKLS Model

Chan, Karoyli, Longstaff and Sanders [5] proposed a generalised framework for short rate models under which several popular models can be considered as special cases. Let r be the riskless interest rate which is considered to be stochastic. The CKLS model assumes that r satisfies the stochastic differential equation

$$dr = a(b - r)dt + \sigma r^{\gamma} dW_t, \tag{1}$$

where a is known as the reversion rate about the long run mean b, σ is the diffusion volatility and γ is the nesting parameter. Recently, Kubilius and Medžiūnas [11] investigated the fractional generalization of the CKLS model. Special cases in the CKLS term structure models are the Vasicek model which corresponds to $\gamma = 0$ and the Cox-Ingersoll-Ross (CIR) model [8] for which $\gamma = 1/2$. Only these two models admit closed-form expressions for bond and options on bond prices and for other values of the parameter γ, numerical evaluation of the prices are required.

A zero-coupon bond is the most basic interest rate derivative which promises a payment of a face value at maturity date T. If we let $B(r_0, T)$ denote the time-zero value of a zero coupon bond, then under the risk-neutral measure $\mathbb{Q}$,

$$B(r_0, T) = \mathbb{E}^{\mathbb{Q}} \left[\exp \left(- \int_0^T r(s)\, ds \right) \right].$$

Under the measure $\mathbb{Q}$, the market price of risk is zero and $B(r_0, \tau)$ is the solution of the pde

$$\frac{\partial B}{\partial \tau} = \frac{1}{2}\sigma^2 r^{2\gamma} \frac{\partial^2 B}{\partial r^2} + a(b - r)\frac{\partial B}{\partial r} - rB, \tag{2}$$

where $\tau = T - t, \quad 0 \leq \tau \leq T$ $B(r_0, 0) = FV$ as the initial condition, where FV denote the bond's face value.

Let $T_O < T$ be the date on which a European option on a zero-coupon bond can be exercised and $\tau^* \in [0, T_O]$ bethe bond option's time to maturity and K be the strike price, then the value $B(r, \tau^*)$ of a European call bond option satisfies

$$\frac{\partial B}{\partial \tau^*} = \frac{1}{2}\sigma^2 r^{2\gamma} \frac{\partial^2 B}{\partial r^2} + a(b - r)\frac{\partial B}{\partial r} - rB, \tag{3}$$

with initial condition $B(r, 0) = \max[B(r, T - T_O) - K, 0]$.

For American options on a zero-coupon bond, there is an unknown optimal exercise interest rate $r^*(\tau^*)$ at τ^*, which is the smallest interest rate value at which it becomes optimal to exercise the option.

Applying the operator splitting method [10] for solving the linear complementarity problem for American options on bonds [25], we add a penalty term, λ and the value $B_{Am}(r, \tau^*)$ of an American call option on bond satisfies

$$\frac{\partial B_{Am}}{\partial \tau^*} - \mathcal{L}_r B_{Am} - \lambda = 0, \quad \lambda \geq 0, \tag{4}$$

where the spatial operator $\mathcal{L}$ is given by

$$\mathcal{L}_r = \frac{1}{2}\sigma^2 r^{2\gamma}\frac{\partial^2}{\partial r^2} + a(b-r)\frac{\partial}{\partial r} - r,$$

subject to

$$\begin{aligned} B_{Am}(r,\,0) &= \max\left(B(r,\,0)-K,\,0\right), \quad r \geq 0, \\ B_{Am}(r,\,\tau^*) &\geq \max\left(B(r,\,\tau^*)-K,\,0\right), \quad 0 < r < r^*(\tau^*),\, 0 \leq \tau^* < T_O, \\ \left[\frac{\partial B_{Am}}{\partial \tau} - \mathcal{L}_r B_{Am} = \lambda\right] &\wedge \left[B_{Am}(r,\,\tau^*) - \max\left(B(r,\,\tau^*)-K,\,0\right)\right]. \end{aligned}$$

2.1 Analytical Solution for Bonds

The Vasicek and CIR models admit analytical solutions for the bond price $B(r,\,t,\,T)$ in the form

$$B(r,\,t,\,T) = X(t,\,T)e^{-r(t)Y(t,\,T)}. \tag{5}$$

For the Vasicek model, $X(t,\,T)$ and $Y(t,\,T)$ are given by

$$\begin{aligned} Y(t,\,T) &= \frac{1}{a}\left(1 - e^{-a(T-t)}\right), \\ X(t,\,T) &= \exp\left(\left(b - \frac{\sigma^2}{2a^2}\right)\left(Y(t,\,T) - T + t\right) - \frac{\sigma^2}{4a}Y^2(t,\,T)\right). \end{aligned}$$

For the Cox-Ingersoll-Ross square root diffusion model, we have

$$\begin{aligned} X(t,\,T) &= \left(\frac{2\sqrt{2\sigma^2+a^2}\,e^{(\sqrt{2\sigma^2+a^2}+a)(T-t)/2}}{2\sqrt{2\sigma^2+a^2} + (\sqrt{2\sigma^2+a^2}+a)\left(e^{\sqrt{2\sigma^2+a^2}(T-t)}-1\right)}\right)^{2ab/\sigma^2}, \\ Y(t,\,T) &= \frac{2\left(e^{\sqrt{2\sigma^2+a^2}(T-t)}-1\right)}{2\sqrt{2\sigma^2+a^2} + (\sqrt{2\sigma^2+a^2}+a)\left(e^{\sqrt{2\sigma^2+a^2}(T-t)}-1\right)}. \end{aligned}$$

3 Compact-RBF Method

To develop the Compact RBF scheme for the CKLS pde (2), we first truncate the spatial domain to $[r_{\text{rmin}},\, r_{\text{rmax}}]$ and we consider a grid in the r-direction with uniform mesh spacing $h = (r_{\text{rmax}} - r_{\text{rmin}})/M$ with $r_m = r_{\min} + mh$, $0 \leq m \leq M$.

We then re-write (2) in the form

$$B_\tau = \frac{1}{2}\sigma^2 r^{2\gamma}B_{rr} + a(b-r)B_r - rB. \tag{6}$$

The derivative terms B_{rr} and B_r in (6) are approximated using a 3-node RBF-FD approximations [3,20] in the form

$$B_r(r_m, \tau) \approx \sum_{i=-1}^{1} \alpha_i^{(1)} B(r_m + ih, \tau),$$
$$B_{rr}(r_m, \tau) \approx \sum_{i=-1}^{1} \alpha_i^{(2)} B(r_m + ih, \tau),$$

for $1 \leq m \leq M-1$ and the coefficients $\alpha_i^{(1)}$ and $\alpha_i^{(2)}$ are obtained by solving

$$\phi_j'(r_m) = \sum_{i=-1}^{1} \alpha_i^{(1)} \phi_j(r_m + ih) \quad \text{for} \quad -1 \leq j \leq 1,$$

and

$$\phi_j''(r_m) = \sum_{i=-1}^{1} \alpha_i^{(2)} \phi_j(r_m + jh) \quad \text{for} \quad -1 \leq j \leq 1,$$

where ϕ is the multiquadrics basis function with shape parameter c given by

$$\phi_j(r_m) = \sqrt{c^2 + (r_m - r_{m+j})^2},$$

with

$$\phi_j'(r_m) = \frac{r_m - r_{m+j}}{\sqrt{c^2 + (r_m - r_{m+j})^2}},$$
$$\phi_j''(r_m) = \frac{c^2}{(c^2 + (r_m - r_{m+j})^2)^{3/2}}.$$

In the limit $c \gg h$, the coefficients $\alpha_i^{(1)}$ and $\alpha_i^{(2)}$ are given as

$$\alpha_{\pm 1}^{(1)} = \frac{1}{2h}\left(1 + \frac{h^2}{2c^2} - \frac{9h^4}{8c^4}\right)$$
$$\alpha_0^{(1)} = 0,$$
$$\alpha_{\pm 1}^{(2)} = \frac{1}{h^2}\left(1 + \frac{h^2}{c^2} - \frac{15h^4}{8c^4}\right),$$
$$\alpha_0^{(2)} = -\frac{2}{h^2}\left(1 + \frac{h^2}{c^2} - \frac{3h^4}{2c^4}\right).$$

Then letting

$$\delta_r B_m \approx \frac{1}{2h}\left(1 + \frac{h^2}{2c^2} - \frac{9h^4}{8c^4}\right)(B_{m+1} - B_{m-1}),$$

and

$$\delta_r^2 B_m \approx \frac{1}{h^2}\left(1+\frac{h^2}{c^2}-\frac{15h^4}{8c^4}\right)(B_{m+1}-B_{m-1})-\frac{2}{h^2}\left(1+\frac{h^2}{c^2}-\frac{3h^4}{2c^4}\right)B_m,$$

and replacing in (6) gives the second-order approximation scheme

$$(B_\tau)_m = \frac{1}{2}\sigma^2 r_m^{2\gamma}\delta_r^2 B_m + a(b-r_m)\delta_r B_m - r_m B_m - T_m, \tag{7}$$

where T_m is the truncation error given by

$$T_m = \frac{h^2}{12c^4}\left[2c^2 a(b-r_m)\left(3\left(B_r\right)_m + c^2\left(B_{rr}\right)_m\right) + \frac{1}{2}\sigma^2\left(r^{2\gamma}\right)_m\left(-9B_m + 12c^2\left(B_{rr}\right)_m + c^4\left(B_{rrrr}\right)_m\right)\right] + \mathcal{O}(h^4). \tag{8}$$

Differentiating the pde (6) twice gives

$$\sigma^2\left(B_{rrr}\right)_m = 2r^{-2\gamma-1}\left(r_m(B_{r\tau})_m + \left(ar_m^2 - abr_m - \gamma\sigma^2 r_m^{2\gamma}\right)\left(B_{rr}\right)_m + \left(ar_m + r_m^2\right)\left(B_r\right)_m\right),$$

$$\sigma^2\left(B_{rrrr}\right)_m = 2r^{-2\gamma-2}\left[r_m(B_{rrr})_m + \left(ar_m^3 - abr_m^2 - 2\gamma\sigma^2 r_m^{2\gamma+1}\right)\left(B_{rrr}\right)_m + \left(2ar_m^2 + r_m^3 + \gamma\sigma^2 r_m^{2\gamma} - 2\gamma^2\sigma^2 r_m^{2\gamma}\right)\left(B_{rr}\right)_m + 2r_m^2\left(B_r\right)_m\right].$$

Replacing the third and fourth order derivatives in terms of lower derivatives in (8) gives the scheme

$$\left(I + \left(\frac{h^2}{12}r_m^{2-2\gamma}D_{rr} + \frac{h^2 r_m^{-4\gamma}}{6\sigma^2}\left(-a(b-r_m)(r_m^2 - 2r_m^{2\gamma}) - 2\gamma\sigma^2 r_m^{2\gamma+1}\right)D_r\right)(B_\tau)_m = \rho_m D_{rr}B_m + \vartheta_m D_r B_m + \zeta_m B_m, \tag{9}$$

where $I \in \mathbb{R}^{(M+1)\times(M+1)}$ denotes the sparse identity matrix, $D_r, D_{rr} \in \mathbb{R}^{(M+1)\times(M+1)}$ are sparse tridiagonal matrices given by

$$D_r = \text{tridiag}\left[\alpha_{-1}^{(1)}, 0, \alpha_1^{(1)}\right],$$
$$D_{rr} = \text{tridiag}\left[\alpha_{-1}^{(2)}, \alpha_0^{(2)}, \alpha_1^{(2)}\right],$$

and

$$\rho_m = \frac{1}{12}\left(\frac{2a^2h^2(b-r_m)^2\left(r_m^2 - 2r_m^{2\gamma}\right)r_m^{-4\gamma}}{\sigma^2} + \frac{2ah^2\left(r_m^{2-2\gamma}(3b\gamma - 3\gamma r_m + r_m) + 2\gamma(r_m - b)\right)}{r_m} + h^2\left(\frac{6\sigma^2 r_m^2}{c^2} + \gamma(2\gamma+1)\sigma^2 + r_m^{3-2\gamma}\right) - 6\sigma^2 r_m^2\right),$$

$$\vartheta_m = \frac{r_m^{-4\gamma}}{6c^2\sigma^2}\left[-a^2c^2\,h^2(b-r_m)\left(r_m^2-2r_m^{2\gamma}\right)-c^2(2\gamma-1)h^2\sigma^2r_m^{2\gamma+2}\right.$$
$$+a\left(-bc^2\left(h^2\left(r_m^3-2r_m^{2\gamma+1}\right)+6\sigma^2r_m^{4\gamma}\right)+3bh^2\sigma^2r_m^{4\gamma}\right.$$
$$\left.\left.+c^2r_m\left(h^2\left(-2\gamma\sigma^2r_m^{2\gamma}-2r_m^{2\gamma+1}+r_m^3\right)+6\sigma^2r_m^{4\gamma}\right)-3\,h^2\sigma^2r_m^{4\gamma+1}\right)\right],$$

and

$$\zeta_m = \frac{1}{24}\left(-\frac{4ah^2(b-r_m)\left(r_m^2-2r_m^{2\gamma}\right)r_m^{-4\gamma}}{\sigma^2}-\frac{9\,h^2\sigma^2r_m^2}{c^4}+8r_m\left(3-\gamma h^2r_m^{-2\gamma}\right)\right).$$

At the left and right boundaries, we use RBF-FD one-sided approximations and the first and last of the matrices D_r and D_{rr} are replaced by the one sided coefficients as follows

$$D_r(1,1:3) = \left[-\frac{3}{2h}\left(1-\frac{h^2}{c^2}\right),\ \frac{4}{2h}\left(1-\frac{h^2}{c^2}\right),\ -\frac{1}{2h}\left(1-\frac{h^2}{c^2}\right)\right],$$
$$D_r(M+1,M-1:M+1) = -\text{flip}[D_r(1,1:3)],$$
$$D_{rr}(1,1:4) = \left[\frac{2}{h^2}\left(1-\frac{41h^2}{4c^2}\right),\ -\frac{5}{h^2}\left(1-\frac{101h^2}{10c^2}\right)\right.$$
$$\left.\frac{4}{h^2}\left(1-\frac{79h^2}{8c^2}\right),\ -\frac{1}{h^2}\left(1-\frac{19h^2}{2c^2}\right)\right],$$
$$D_{rr}(M+1,M-2:M+1) = \text{flip}[D_{rr}(1,1:4)].$$

The semi-discrete scheme (9) can then be written in the form

$$PB'(\tau) = QB(\tau) \quad \text{for} \quad 0 \le \tau \le T, \tag{10}$$

where $P, Q \in \mathbb{R}^{(M+1)\times(M+1)}$ are the resulting matrix from the left hand side and right hand side of Eq. (9) respectively.

For the time discretisation, let $\Delta\tau = T/N$ and $\tau_n = n\Delta\tau$ for $0 \le n \le N$ and let the vector of bond prices be denoted by $B^n = [B_0^n, B_1^n, B_2^n, \ldots, B_M^n]$. Using an implicit Euler scheme we require the solution of

$$(P - \Delta\tau Q)\,B^{n+1} = PB^n, \quad 0 \le n \le N-1, \tag{11}$$

with initial condition $B^0 = [B_0^0, B_1^0, \ldots, B_M^0]^T$. The accuracy is further enhanced with an $s = 6$ stage extrapolation as in [9,19] with $N = 20$ initial time steps.

4 Numerical Results

In this section we present some numerical results to validate the accuracy of the compact RBF method. In all our numerical examples, we use $r_{\min} = 0$ and $r_{\max} = 1$ except for the Vasicek model we use $r_{\min} = -0.5$ and $r_{\max} = 1$ since the interest can go negative in the latter case. The face value (FV) of the bond is taken as 100. The accuracy of the Compact RBF scheme is first established for the case of $\gamma = 0$ corresponding to the Vasicek interest rate model which

Table 1. Bonds prices under the Vasicek model.

$T = 0.5$, $a = 0.5$, $b = 0.08$, $\sigma = 0.1$, $r_0 = 0.05$								
	RBF-3pt				C-RBF			
M	Price	Error	Rate	Cpu(s)	Price	Error	Rate	Cpu(s)
20	97.379268	2.0e-4	–	0.009	97.379413	7.4e-5	–	0.006
40	97.379433	5.0e-5	2.000	0.011	97.379494	6.1e-6	3.616	0.075
80	97.379474	1.0e-5	2.000	0.032	97.379487	3.5e-7	4.105	0.105
160	97.379484	3.0e-6	2.000	0.084	97.379488	2.3e-8	3.907	0.224
320	97.379487	9.0e-7	2.000	0.181	97.379488	1.7e-9	3.778	0.468
Exact	97.3794875586436							

admits analytical bond prices. Consider the pricing of a zero-coupon bond with a principal of 100 and a short time to maturity of 0.5 year. Numerical results given by the solution of the pde (11) when $a = 0.5$, $b = 0.08$ and $\sigma = 0.1$ are given in Table 1. The initial value of the short rate is taken as $r_0 = 0.05$.

Comparison with the exact solution shows that a three-node localised radial basis function method (RBF-3point) yields the expected second order convergence prices while the Compact-RBF method (C-RBF) is able to yield fourth-order accurate results.

Consider a short-maturity bond under the CIR model using the Compact-RBF method. Comparison is drawn against the exact analytical solution and the analytical approximation of Choi and Wirjanto and the results are reported in Table 2. For this short maturity problem, we observe close agreement between all the different methods.

Table 2. Bonds prices for $\gamma = \frac{1}{2}$ for a short maturity problem.

$T = 0.5$, $a = 0.5$, $b = 0.08$, $\sigma = 0.1$, $r_0 = 0.05$						
	C-RBF				Choi-Wirjanto Approx.	
M	Price	Error	Rate	Cpu(s)	Price	Error
20	97.363491948	1.8e-5	–	0.002	97.363475825303	2.0e-6
40	97.363474897	1.1e-6	4.085	0.056		
80	97.363473893	6.4e-8	4.060	0.097		
160	97.363473832	3.0e-9	4.421	0.162		
320	97.363473829	7.3e-11	5.359	0.326		
Exact	97.363473829394					

We next consider the case of a longer maturity bond under the CIR model and comparison is done against the Choi-Wirjanto Approximation [6] and the Box methods of Sorwar et al. [14]. In the previous example it was observed that

for the shorter maturity problem, the Choi-Wirjanto approximation yielded a solution with an error of 10^{-6} while for this longer maturity problem of $T = 5$ years, the solution is less accurate with an error of 10^{-2} as shown in Table 3. A similar accuracy is observed for the Box methods by Sorwar et al. [14]. In contrast, the pde methods, that is the RBF-3point and the C-RBF methods are still very accurate with second and fourth order convergence.

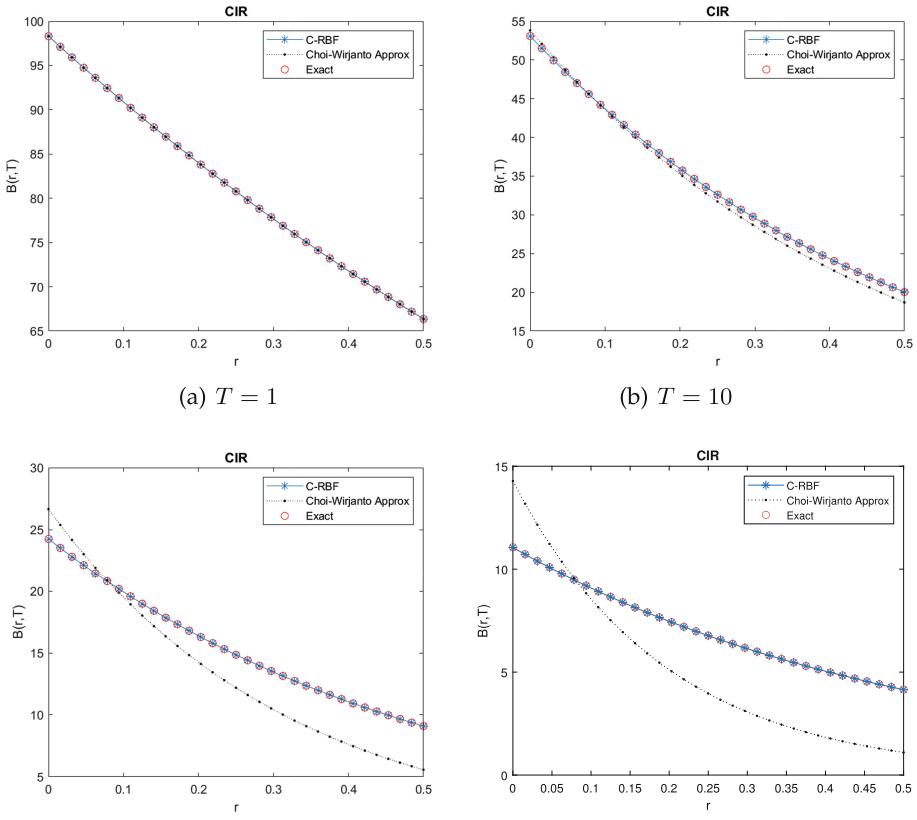
Table 3. Bonds prices under the CIR model for a longer maturity problem.

$T = 5$, $a = 0.5$, $b = 0.08$, $\sigma = 0.1$, $r_0 = 0.05$								
	RBF-3pt				C-RBF			
M	Price	Error	Rate	Cpu(s)	Price	Error	Rate	Cpu(s)
20	71.036123	1.8e-4	–	0.002	71.038108	1.7e-4	–	0.006
40	71.037552	3.9e-4	2.233	0.004	71.037947	8.9e-6	4.261	0.056
80	71.037840	9.8e-5	1.981	0.009	71.037938	6.1e-7	3.867	0.096
160	71.037913	2.4e-5	2.000	0.010	71.037938	4.3e-8	3.810	0.168
320	71.037932	6.1e-6	2.000	0.029	71.037938	3.1e-9	3.803	0.322
	Choi-Wirjanto Approx.				Box Methods			
	Price	Error			Time Steps	Price	Error	
	71.074008366	3.6e-2			300	71.0614	2.3e-2	
					500	71.0603	2.2e-2	
					1000	71.0595	2.2e-2	
Exact	71.0379377726464							

The observations made from the results in Table 3 are further illustrated in Fig. 1 where it is shown that for small maturity problems, all the methods agree closely with the exact solution. As maturity increases, the Choi-Wirjanto approximation leads to mispricing while the C-RBF method is still in good agreement with the exact solution even for the very long maturity case of $T = 30$ years. Having established the accuracy of the C-RBF method, we also report on computed bond prices for different values of the parameter $\gamma = \{0.2, 0.4, 0.6, 0.8, 1\}$ for which analytical expressions are not available in Table 4.

Table 4. Bonds prices under the general CKLS model.

$T = 5$, $a = 0.5$, $b = 0.08$, $\sigma = 0.1$, $r_0 = 0.05$					
	γ				
Method	0.2	0.4	0.6	0.8	1.0
RBF-3	71.80179	71.18419	70.95074	70.86919	70.84145
C-RBF	71.80162	71.18419	70.95074	70.86919	70.84145



(a) $T = 1$ (b) $T = 10$

(c) $T = 20$ (d) $T = 30$

Fig. 1. Mispricing With Choi-Wirjanto approximations for parameters $a = 0.5$, $b = 0.08$, $\sigma = 0.1$.

4.1 Bond Options

We now consider two European call options with maturities $T_O = 2$ and $T_O = 5$ years and strike price $E = 35$ on a $T = 10$-year zero-coupon bond. The other parameters are chosen as $a = 0.5$, $b = 0.08$, $\sigma = 0.1$ and $r_0 = 0.08$. The bond prices and errors computed by the RBF-3point and Compact RBF methods for the case of the CIR model are given in Table 5. The European bond option prices are shown to be converging with smooth fourth-order convergence rate with the C-RBF method.

Given the high accuracy of the C-RBF method, in Table 6 we report the prices of European bond option for different values of the parameter γ for which no analytical solutions are known.

In this final numerical example, an American put option with a maturity of $T_O = 5$ years with strike price $E = 55$ on a bond with maturity $T = 10$ is priced. Table 7 gives the option prices and the corresponding errors. Since it is

Table 5. European call bond options under CIR.

$K = 35$, $T = 10$, $a = 0.5$, $b = 0.08$, $r_0 = 0.08$, $\sigma = 0.1$							
		RBF-3pt			C-RBF		
	M	Price	Error	Rate	Price	Error	Rate
$T_O = 5$	20	21.8789823	1.2e-3	–	21.8780248	2.2e-3	–
	40	21.8799699	2.2e-4	2.438	21.8800599	1.3e-4	4.021
	80	21.8801380	5.6e-5	2.010	21.8801855	7.9e-6	4.073
	160	21.8801795	1.4e-5	1.985	21.8801940	4.8e-7	4.049
	320	21.8801900	3.5e-6	1.990	21.8801935	3.0e-8	4.017
Exact		21.880193482973					
$T_O = 2$	20	15.5834337	2.9e-3	–	15.5837021	2.6e-3	–
	40	15.5856776	6.3e-4	2.194	15.5861313	1.7e-4	3.906
	80	15.5861491	1.6e-4	2.009	15.5862946	1.0e-5	4.069
	160	15.5862658	3.9e-5	1.993	15.5863056	6.5e-7	3.994
	320	15.5862951	9.8e-6	1.997	15.5863049	4.0e-8	4.021
Exact		15.5863049533542					

Table 6. European call bond options under general CKLS

T_O	$\gamma = 0.2$	$\gamma = 0.4$	$\gamma = 0.6$	$\gamma = 0.8$
5	23.150081	22.135136	21.721509	21.563779
2	17.077626	15.885238	15.400344	15.215627

Table 7. American put bond options for $T = 10$ and $T_O = 5$.

$E = 55$, $a = 0.5$, $b = 0.08$, $\sigma = 0.1$, $r_0 = 0.08$				
American Put Options ($\gamma = 0.5$)				
M	Price	Error	Rate	Cpu(s)
20	9.5754748	2.8e-3	–	0.011
40	9.5728934	2.0e-4	3.805	0.019
80	9.5727064	1.2e-5	4.065	0.053
160	9.5726936	9.3e-7	3.684	0.128
320	9.5726944	5.9e-8	3.983	0.245
Ref Values	9.57269450			
γ	0.2	0.4	0.6	0.8
160	8.027698	9.263475	9.764938	9.955800

well known that for American bond options there are no analytical formulas, the reference values are computed on a refined mesh of $M = 1280$ spatial steps. Similar conclusions as in the previous examples can be reached where the C-RBF

is observed to yield fourth-order accurate results. We also report on the prices of some American put bond options for $\gamma = \{0.2, 0.4, 0.6, 0.8\}$.

5 Conclusion

We propose a high-order compact localised radial basis function method for pricing interest rate derivatives under the CKLS model. Fourth-order accurate results are obtained for bonds, European and American bond options and it is also shown the method does not suffer from mispricing problems when maturity increases. The methodology can be extended to the pricing of contingent claims under two-factor interest rate models.

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