

Mathematical Modeling of Cooling Towers-Based Refrigeration Systems for Energy Efficiency Optimization

Nadia Nedjah[1(✉)], Luiza de Macedo Mourelle[2], and Marcelo Silveira Dantas Lizarazu[1]

[1] Department of Electronics Engineering and Telecommunications, State University of Rio de Janeiro, Rio de Janeiro, Brazil
{nadia,lizarazu}@eng.uerj.br
[2] Department of Systems Engineering and Computation, State University of Rio de Janeiro, Rio de Janeiro, Brazil
ldmm@eng.uerj.br

Abstract. Cooling towers and chillers are the basis of modern refrigeration systems of large facilities, such as oil refineries, power plants and large commercial buildings. The increasing concerns about the scarcity of water and energy resources require careful optimization processes to achieve energy efficiency in industrial buildings. Energy efficiency oriented optimizations require mathematical models of real equipment that compose the refrigeration systems. In this paper, we present a complete model cooling towers and corresponding fans based on Merkel's and Braun's methods. We prove that proposed model is accurate and faithful to the real cooling cells in modern refrigeration systems. The obtained value of mean square error when applying the model and comparing the obtained results to the actual ones is minimal, hence validated the proposed model.

Keywords: Energy efficiency · Cooling tower · Chiller · Optimization

1 Introduction

In many industrial processes, heat is generated, and this heat often must be extracted and dissipated. Water is generally used as a cooling element, as can be usually seen in the petrochemical industry and in electricity generating plants. The water used in cooling systems always returns from the process at higher temperatures, which can be discarded or cooled for reuse. The latter is used whenever there is a lack of water for the application, or in cases where water consumption would be absurdly expensive in economic terms.

Cooling towers are equipment intended for application in large cooling demands, as these offer a clean and very economical way to cool the water to be reused in the cooling process. For this reason, its usage is verified in building installations, such as large shopping centers, hotels and large buildings.

© The Author(s), under exclusive license to Springer Nature Switzerland AG 2022
O. Gervasi et al. (Eds.): ICCSA 2022 Workshops, LNCS 13377, pp. 289–307, 2022.
https://doi.org/10.1007/978-3-031-10536-4_20

A refrigeration system based on cooling towers does not only involve the cooling towers themselves, but also a set of equipment that operate interdependently, such as chillers. An undue modification of a certain parameter or operational adjustment in one of these equipment can cause either a positive or negative cascading effects on the operation of the others parts of the system. This can consequently trigger a series of effects that are not necessarily satisfactory to the overall system, which includes the reduction of its energy efficiency.

As a first part, this work aims at providing a complete model of modern cooling towers to serve as the basis of optimization of many objectives, such as maximizing the thermal exchange efficiency of the cooling tower and minimizing the energy requirement of the refrigeration system, considering all its composing equipment. For the latter, a model for the chillers is required. Due to lack of space, the model of chillers and underlying processes in the refrigeration system is presented in separate paper [11].

This paper is structured into five sections. First, in Sect. 2, we give a brief description of the structure of the modeled equipment of a modern refrigeration system. Then, in Sect. 3, we review the main methods used to elaborate the proposed models. After that, in Sect. 4.1, we present the model of the cooling tower as well as that of the tower fans. Later, in Sect. 5, we present, discuss and compare the results obtained from the application of the proposed models to those collected from a real operating refrigeration system. Last but not least, in Sect. 6, the conclusions are drawn together with future work.

2 Cooling Tower in Refrigeration System

Cooling towers are a widely used equipment in oil refineries, power plants and large commercial buildings. The role of cooling tower is to recover the heat rejected by the equipment responsible for the refrigeration of the environment and/or processes. A cooling tower operates in conjunction with other equipment such as chillers and pumps to circulate and lift the water in the system. In addition, mechanical draft-type cooling towers have internal fans that operate in order to impose proper air circulation internally. Among these equipment, the chillers are responsible for the largest portion of the power consumption of the system [15]. Figure 1 illustrates the composition and operation of a typical cooling tower as modeled in this work. The function of the cooling tower is to reduce the temperature of the incoming hot water to acceptable levels for the proper functioning of the other process equipment that make up the cooling system.

As for the heat transfer method, cooling towers can be classified as dry, wet or mixed. Another way of classifying cooling towers is the type of relative flow between water and air streams. In this case, they can be classified into towers of counter-current flow or of cross-flow. Moreover, as for the type of draft, the cooling towers can be classified into: natural draft towers, also known as hyperbolic, or mechanical draft towers.

In the natural draft towers, there are no fans at the top, and the air flow occurs naturally through the tower due to the gradual increase in air temperature

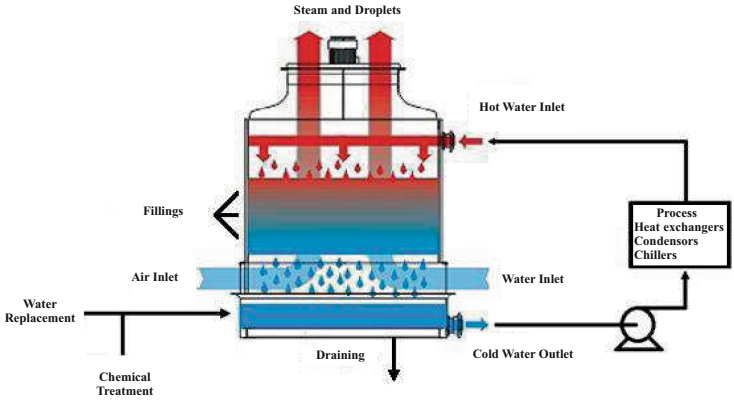


Fig. 1. Typical cooling tower operation, as modeled in this work

as it exchanges heat with the hot water (convection mechanism), and also due to their hyperbolic shape. In contrast, mechanical draft towers use fans to cause air circulation. These can be of the induced draft or forced draft type. In the proposed modeling, we consider cooling towers with mechanical air draft. More details on the cooling system's structure and operation can be found in [1].

3 Modeling Methods

The modeling of the cooling tower is necessary for any optimization simulations to be carried out, allowing the validation of the results obtained by the proposed optimization method. For this purpose, it is necessary to obtain an equation that relates the effectiveness of the tower with the speed of the fans and the temperatures of inlet and outlet of water.

In [10], a theory for evaluating the performance of cooling towers is presented, and this method has been used since then by manufacturers of cooling towers to evaluate the achieved performance. However, Merkel's method relies on several approximations in order to simplify the necessary computations. For instance, water loss by evaporation is not considered, and the *Lewis factor*, which is related to heat and mass transfer in the tower, is set to 1 [4].

In [12], Poppe presented a more precise method without resorting to the approximations made in [10]. The performance comparison between the two methods is studied in [6] and in [7]. The analysis indicated that Merkel's approximations do not compromise the results, proving their effectiveness.

In [14], a model based on a more rigorous analysis of Merkel's model is also proposed. In [2], the effectiveness method is developed, which is also known as e-NTU (Number of Transfer Units). It uses some simplifications based on in Merkel's model. The model proposed therein considers the linearization of the enthalpy of saturated air and a modification in the definition of NTU.

In the literature on cooling towers, two terminologies are commonly found for the parameter of evaluation of cooling towers' performance: the effectiveness

and the efficiency of the tower. According to [5], the effectiveness of a cooling tower is defined as the ratio between the energy that is effectively exchanged and the maximum possible amount of energy to be transferred. Merkel and Braun's methods are at the heart of the proposed model for the refrigeration system. Therefore, they are briefly described in Sects. 3.1 and 3.2, respectively.

3.1 Merkel's Method

Inside the cooling towers, heat and mass transfer processes occur simultaneously. Heat is transferred from water to the atmosphere by two mechanisms: convection (heat transfer) and evaporation (mass transfer). This can be expressed by Eq. 1

$$dQ_{overall} = dQ_{conv} + dQ_{evap}, \tag{1}$$

wherein $dQ_{overall}$ is the total heat exchanged, dQ_{conv} is the heat exchanged by convection, and dQ_{evap} is the amount of heat exchanged by evaporation. In convective heat transfer, heat is transferred from water to air through the temperature gradient established with the flow of air in contact with water. Therefore, we have Eq. 2:

$$dQ_{conv} = h_{conv}(T_{water} - T_{air}dA), \tag{2}$$

where h_{conv} is the convection heat transfer coefficient in $W/m^2.°C$, T_{water} is the temperature of the water, T_{air} is the air temperature and dA is the heat exchange area.

Figure 2 illustrates the heat and mass transfer process considered by in [10]. The indicated variables will be defined in the modeling presented in the sequel. Here we consider that the temperature of the drop or water film is constant from its center to its surface.

The process of heat exchange by evaporation occurs due to mass transfer between water and air. In this case, the energy necessary for the state change is taken from the water itself, thus causing its temperature to decrease. Note that thermal exchange by evaporation is responsible for most of the heat exchanged in the cooling tower.

The mass of water that evaporates is transferred to the air through two basic mechanisms: the first is the diffusion of water molecules into the air, along the interface formed between water and air, during contact; the second is the dragging of water molecules by the air current, which is mechanically induced through the use of a fan in the cooling tower. Therefore, we can establish Eq. 3:

$$dQ_{evap} = h_m L_c(UE_2 - UE_1)dA, \tag{3}$$

wherein h_m is the mass transfer coefficient, in kg/m^2.s, L_c is the latent heat of vaporization of water, in kcal/kg, UE_2 and UE_1 represents the specific humidity of saturated air and the specific humidity of the air before contact with water, respectively in $kg\ steam/kg\ ar$, and dA is the heat exchange area, in m^2. The heat received by the air can be expressed through Eq. 4:

$$dQ = \dot{m}_{air}dh_{air}, \tag{4}$$

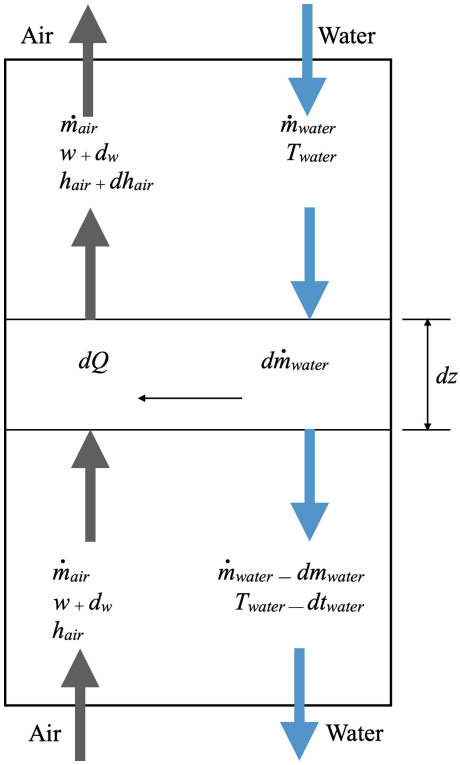


Fig. 2. Energy balance in Merkel's model.

wherein $\dot{m}_{air}$ is the mass flow rate of air, in kg/s, and dh_{air} is the enthalpy change of air, in kcal/kg. The enthalpy variation of the air occurs due to the portions of sensible heat and latent heat received through the heat exchange with water. Therefore, the value of h_{ar} can be approximated as in Eq. 5:

$$h_{air} = c_p T_{air} + L_c U E_{air}, \tag{5}$$

where c_p is the specific heat of air at constant pressure, T_{air} is the temperature of the air, and UE_{air} is the specific humidity of the air. It is also known that the sensible heat transferred from water to air is given by Eq. 6:

$$dQ = \dot{m}_{water} c_{water} dT_{water}, \tag{6}$$

where c_{water} is the specific heat of water, $\dot{m}_{water}$ is the mass flow rate of water, and dT_{water} is the temperature variation of the water. Considering that there is no loss of water mass by evaporation, that is, that $\dot{m}_{water}$ is constant in the cooling tower, and using the equations defined above, we have Eq. 7:

$$dQ = \dot{m}_{water} c_{water} dT_{water} = h_{conv}(T_{water} - T_{air}) dA + h_m L_c (UE_2 - UE_1) dA. \tag{7}$$

On the other hand, the Lewis factor is defined in Eq. 8:

$$L_{ef} = \frac{h_{conv}}{h_m c_p}. \tag{8}$$

As in the Merkel model, we must assume that $L_{ef} = 1$, then we have Eq. 9:

$$h_m c_p = h_{conv}. \tag{9}$$

In general terms, assuming $L_{ef} = 1$, it is considered that the degree of difficulty (or ease) for heat transfer by convection is the same as for mass transfer by evaporation. That is, the variation in temperature obtained is equal to the variation in the specific humidity of the air. Using Eqs. 7 and 9 and after that applying Eq. 5, we have Eq. 10:

$$\begin{aligned} dQ &= \dot{m}_{water} c_{water} dT_{water} \\ &= h_m c_p (T_{water} - T_{air}) dA + h_m L_c (UE_2 - UE_1) dA \\ &= h_m \left[(c_p T_{water} + L_c UE_2) - (c_p T_{air} + L_c UE_1) \right] dA \\ &= h_m (h_{air_{sat}} - h_{air_{dry}}) dA, \end{aligned} \tag{10}$$

where $h_{air_{sat}}$ and $h_{air_{dry}}$ represent the enthalpy of saturated air (after heat exchange) and the enthalpy of dry air (before heat exchange), from Eq. 10 we get Eq. 11:

$$\frac{h_m dA}{\dot{m}_{water}} = \frac{c_{water} dT_{water}}{h_{air_{sat}} - h_{air_{dry}}}. \tag{11}$$

This equation is known as the Merkel equation. With its integration, we obtain Eq. 12:

$$\frac{KaV}{L} = \frac{h_m a_p A_p z}{\dot{m}_{water}} = \int_{T_1}^{T_2} \frac{c_{water}\, dT_{water}}{h_{air_{sat}} - h_{air_{dry}}}, \tag{12}$$

wherein a_p represents the specific area of the tower infill, in $\mathrm{m}^2/\mathrm{m}^3$, A_p represents the frontal cross-sectional area of the infill, in m^2, and z represents the height of the cooling tower, in meters.

It is noteworthy that in Eq. 12, the term $\frac{KaV}{L}$ is dimensionless and represents the overall coefficient of performance of the tower. The term Ka, in $\mathrm{kgs}^{-1}\ \mathrm{m}^{-3}$, is the result of the product of the total heat coefficient K, in $\mathrm{kgs}^{-1}\ \mathrm{m}^{-2}$, and the heat transfer area per unit volume a, in $\mathrm{m}^2\ \mathrm{m}^{-3}$. The term L represents the mass flow rate of water in the tower, in $\mathrm{kg\ s}^{-1}$. The term $\frac{KaV}{L}$ is also found in the literature as Me_M, *Merkel's number* or NTU (Number of Transfer Units) [3,7]. The term NTU is the most used, and represents the number of times the enthalpy difference between saturated and dry air indicated in the equation is transferred to water.

3.2 Effectiveness Method

The effectiveness method has been introduced in [2], where heat and mass transfers are considered to occur inside the cooling tower as indicated in Fig. 3.

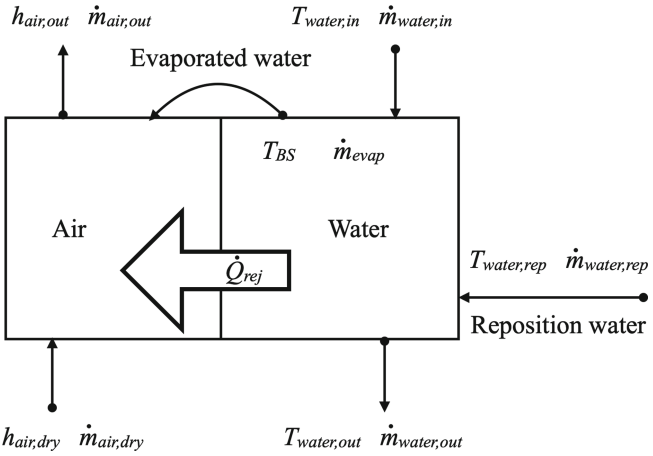


Fig. 3. Energy balance with the effectiveness method.

The energy balance of the control volume considered in Fig. 3 can be analyzed through the Eqs. 13–16, which represent the conservation of air mass, heat of air, mass of water and heat of water, respectively.

$$\dot{m}_{air_{dry}} + \dot{m}_{evap} = \dot{m}_{air_{sat}}, \tag{13}$$

$$\dot{m}_{air_{dry}} h_{air_{dry}} + Q_{rej} - Q_{evap} = \dot{m}_{air_{sat}} h_{air_{sat}}, \tag{14}$$

$$\dot{m}_{water_{in}} - \dot{m}_{evap} = \dot{m}_{water_{out}}, \tag{15}$$

$$\dot{m}_{water_{in}} T_{water_{in}} c_{water} - Q_{rej} + \dot{m}_{water_{rep}} T_{water_{rep}} c_{water} = \dot{m}_{water_{out}} T_{water_{out}} c_{water}, \tag{16}$$

where $\dot{m}_{air_{dry}}$ represents the mass flow rate of dry air entering the tower, $\dot{m}_{air_{sat}}$ is the mass flow rate of saturated air leaving the tower, $\dot{m}_{evap}$ is the mass flow rate of the air leaving the tower due to evaporation, $\dot{m}_{water_{rep}}$ is the flow reposition water mass required due to the volume of water leaving the tower due to evaporation, $T_{water_{rep}}$ is the reposition water temperature, $\dot{m}_{water_{in}}$ is the mass flow rate of hot water entering the tower, $T_{water_{in}}$ is the temperature of hot water entering the tower, $\dot{m}_{water_{out}}$ is the mass flow rate of the cold water leaving the tower, $T_{water_{out}}$ is the temperature of the cold water leaving the tower, T_{BS} is the dry bulb temperature of the air, and Q_{rej} represents the amount of heat that is transferred from water to dry air, making it saturated, and Q_{evap} represents the amount of heat that is removed from the water by the process of evaporation. Note that Fig. 3 illustrates the processes described by Eqs. 13–16.

According to this method, the effectiveness of the tower is given by Eq. 17:

$$\epsilon_a = \frac{1 - e^{-NTU(1-m^*)}}{1 - m^* e^{-NTU(1-m^*)}}, \quad \text{with} \tag{17}$$

$$NTU = c\left(\frac{\dot{m}_{air}}{\dot{m}_{water}}\right)^{-(1+n)} \quad \text{and} \tag{18}$$

$$m^* = \frac{\dot{m}_{air}}{\dot{m}_{water}} \cdot \frac{C_S}{c_{water}}, \tag{19}$$

where C_S represents the rate of change in enthalpy of air with respect to change in water temperature. In [2], it is considered that this variation is linear for small temperature intervals, as defined in Eq. 20:

$$C_S = \frac{dh_{air_{sat}}}{dT_{water}} \approx \frac{h_{air_{sat},T_{water_{in}}} - h_{air_{sat},T_{water_{out}}}}{T_{water_{in}} - T_{water_{out}}}. \tag{20}$$

In Eqs. 17 and 18, the term NTU is dimensionless and represents the global performance of the cooling tower. This term is similar to the $\frac{KaV}{L}$ indicated in Eq. (12).

In Eq. 18, c and n are empirical constants associated with each particular cooling tower. These constants are obtained by the cooling tower manufacturers after performing the performance tests, before starting the tower operation, for a given wet bulb temperature. In Eq. 20, the terms $h_{air_{sat},T_{water_{in}}}$ and $h_{ar_{sat},T_{water_{out}}}$ represent the enthalpy of saturated air, respectively, at the temperatures of the inlet water and the water leaving the tower.

Moreover, from the effectiveness, one can obtain the heat rejected by the cooling tower, Q_{rej}, which is given by Eq. 21:

$$Q_{rej} = \epsilon_a \dot{m}_{air}(h_{air_{sat},T_{water_{in}}} - h_{air_{dry},T_{BS}}). \tag{21}$$

Note that in the effectiveness method, obtaining C_S depends on the conditions at the exit of the tower (cold water temperature and saturated air enthalpy), which requires the use of recursive techniques to solve Eqs. 17–20, from the definition of approximate initial values for the output variables.

4 Cooling Tower's Proposed Model

In this section, we present the modeling of the process performed by the cooling tower and the ventilators. Based on Merkel's and Braun's methods, in Sect. 4.1, we provide a precise and faithful model of the cooling tower and in Sect. 4.2, we do so for the used fans.

4.1 Tower

The modeling adopted for the cooling tower is available in [9]. This defines that the effectiveness of a cooling tower can be approximated by a second order function, from the Merkel's and Braun's models, based on the fundamental laws of heat and mass transfer.

In Braun's model, according to Eq. 20, a linear approximation is performed for the variation of the enthalpy of saturated air with respect to temperature.

In this case, the variation of the enthalpy of saturation is obtained based on the difference in temperature between the hot water that arrives at the tower and the cold water that leaves it [2]. This is the premise adopted by Braun for the calculation of C_S, as indicated in Eq. 20.

From a control and optimization point of view, the leaving water temperature $T_{water_{out}}$ and the enthalpy of saturated air leaving the tower $h_{air_{sat},T_{water_{out}}}$, both used in Eq. 20, are output variables. Therefore, they are variables that need to be controlled and should not be used as input variables, as indicated in Eqs. 17–20. Let us consider the illustration presented in Fig. 4.

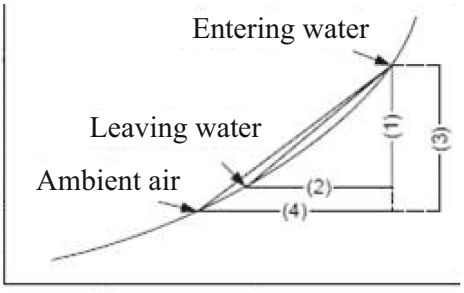


Fig. 4. Enthalpy of saturated air versus temperature.

Based on Fig. 4, Eq. 20 can be approximated as in Eq. 22:

$$C_S = \frac{dh_{air_{sat}}}{dT_{water}} \approx \frac{(h_{air_{sat},T_{water_{in}}} - h_{air_{sat},T_{BU}}) + f_1(\Delta h)}{(T_{water_{in}} - T_{BU}) + f_2(\Delta T)}, \tag{22}$$

where ΔT is defined as the *approach* of the cooling tower. Graphically, this is represented in Fig. 4 as the difference between the length of lines (4) and (2), that is, between the temperature of the water leaving the tower and the local wet bulb temperature. Variable Δh is the enthalpy change of saturated air related to the approach ΔT. Note, as indicated in Fig. 4, that the enthalpy of air saturation varies with approach, ΔT, so that the higher ΔT is, the higher the variation of the enthalpy of saturation.

The term C_S, as indicated in Eq. (20), is defined in Braun's modeling as a linear approximation of the curve that relates the inlet and outlet temperatures of water with the enthalpy of saturation. It is represented in Fig. 4 as the ratio between the lengths of lines (1) and (2), which represent, respectively, the variation of the enthalpy of saturation of the air, $h_{air_{sat},T_{water_{in}}} - h_{air_{sat},T_{water_{out}}}$, and the change in water temperature, $T_{water_{in}} - T_{water_{out}}$. Therefore, Fig. 4 illustrates the behavior of the tower with regard to the variation in enthalpy of air saturation as a function of variations in water and air temperatures. Observe that the approximation of the value of C_S based on the wet bulb temperature, T_{BU},

instead of the leaving water temperature of the tower, $T_{water_{out}}$, unlike the approach adopted in Braun's model (as indicated in Eq. 20), requires a correction, which is obtained by using $f_1(\Delta h\ )$ and $f_2(\Delta T)$ in Eq. 22.

Thus, based on the laws of conservation of mass and energy, the approach, ΔT, can be approximated as a function of $\frac{\dot{m}_{air}}{\dot{m}_{water}}$ and $(T_{water_{in}} - T_{BU})$. Therefore, we have Eq. 23:

$$f_1(\Delta T) = f_1\left(\frac{\dot{m}_{air}}{\dot{m}_{water}}, (T_{water_{in}} - T_{BU})\right). \tag{23}$$

Still based on the laws of conservation of mass and energy, Δh can be defined as a function of ΔT, which in turn is also a function of $\frac{\dot{m}_{air}}{\dot{m}_{water}}$ and $(T_{water_{in}} - T_{BU})$. So we have Eq. 24:

$$f_2(\Delta h) = f_2\left(\frac{\dot{m}_{air}}{\dot{m}_{water}}, (T_{water_{in}} - T_{BU})\right). \tag{24}$$

Thus, as C_S in Eq. 22 is given as a function of the enthalpy change of the air with respect to the change of water temperature, based on the Eqs. 23 and 24, we conclude that C_S can be described in Eq. 25:

$$C_S = f_3\left(\frac{\dot{m}_{air}}{\dot{m}_{water}}, (T_{water_{in}} - T_{BU})\right). \tag{25}$$

Through Eqs. 17–19, we can deduce that the effectiveness ϵ_a is a function of NTU and m^*, where NTU is a function of $\frac{\dot{m}_{air}}{\dot{m}_{water}}$ and C_S. Therefore, applying Eq. 25 to Eqs. 17–19, we obtain a general expression for the effectiveness ϵ_a as presented in Eq. 26:

$$\epsilon_a = f\left(\frac{\dot{m}_{air}}{\dot{m}_{water}}, (T_{water_{in}} - T_{BU})\right) = f(x, y), \tag{26}$$

where variables x and y are be used, respectively, to represent $\frac{\dot{m}_{air}}{\dot{m}_{water}}$ and $(T_{water_{in}} - T_{BU})$, in order to simplify the following equations. So we have $x = \frac{\dot{m}_{air}}{\dot{m}_{water}}$ and $y = T_{water_{in}} - T_{BU}$.

Note that Eq. 26 can be viewed as a function in which the effectiveness of the heat exchange of the cooling tower ϵ_a can be modeled as a function depending only of input variables, differently from what occurs in Merkel's and Braun's methods. So, considering that the effectiveness ϵ_a is a continuous function under normal operating conditions of the cooling tower, it can be considered derivable. Therefore, one can use the Taylor series as an approximation to solve Eq. 26. Due to the non-linearity characteristic of the cooling tower response, the Taylor series will be used up to the second order, to represent it correctly and precisely.

$$f(x,y) = f(x_0,y_0) + \left[\frac{\partial f(x_0,y_0)}{\partial x}(x-x_0) + \frac{\partial f(x_0,y_0)}{\partial y}(y-y_0)\right] + \frac{1}{2!}\left[\left(\frac{\partial^2 f(x_0,y_0)}{\partial x^2}\right)\cdot(x-x_0)^2 + \left(\frac{\partial^2 f(x_0,y_0)}{\partial y^2}\right)\cdot(y-y_0)^2 + \left(\frac{\partial^2 f(x_0,y_0)}{\partial y^2}\right)\cdot(y-y_0)^2 + 2\left(\frac{\partial^2 f(x_0,y_0)}{\partial x \partial y}\right)\cdot(x-x_0)(y-y_0)\right], \tag{27}$$

wherein the point (x_0,y_0) represents any cooling tower operating point close to a hypothetical condition (x,y). The point (x_0,y_0) can be determined experimentally, allowing to consider the terms $f(x_0,y_0)$, $\frac{\partial f(x_0,y_0)}{\partial x}$, $\frac{\partial f(x_0,y_0)}{\partial y}$, $\frac{\partial^2 f(x_0,y_0)}{\partial x^2}$, $\frac{\partial^2 f(x_0,y_0)}{\partial y^2}$ and $\frac{\partial^2 f(x_0,y_0)}{\partial x \partial y}$ as constants. These will be represented by coefficients c_0, c_1, c_2, c_3, c_4 and c_5, respectively. So, replacing $\frac{\dot{m}_{air}}{\dot{m}_{water}}$ and $(T_{water_{in}} - T_{BU})$, respectively, by the variables x and y in Eq. 27, we obtain Eq. 28:

$$\epsilon_a = c_0 + c_1 x + c_2 y + c_3 x^2 + c_4 y^2 + c_5 xy, \tag{28}$$

where coefficients c_0 to c_5 are constants to be determined according to the behavior of the refrigeration process via collected data.

The actual performance curve of a cooling tower is usually provided by manufacturers or surveyed in the field during performance tests. It is then possible, from the data collected in the field, and with the use of the Eq. 29:

$$min \sum_{i=1}^{N} \frac{1}{2} \left(\epsilon_a - F_{data_i}\right)^2, \tag{29}$$

to determine the coefficients c_0 to c_5 through the minimization of the difference between the surveyed curve and the proposed model, using any optimization algorithm. So, for each point of a total of surveyed points, the coefficients of Eq. 28 must be obtained in order to minimize the error between the surveyed curve and the proposed model, as indicated in Eq. 29. Note that F_{data_i} represents a point on the real curve obtained through field surveys, and N is the number of points used to obtain F_{data}.

In order to obtain good results, the number of points acquired must be greater than the number of coefficients, that is, $N \geq 6$. In addition, the points surveyed must have the widest possible distribution within the operating range of the cooling tower. Gauss-Newton and Levemberg-Marquardt methods are used to obtain the coefficients c_0 to c_5 [9]. In practice, the measurement of air mass flows at the inlet and outlet of the tower is difficult to perform, as this type of instrumentation is generally not available in these locations. However, this problem can be solved considering the principle of conservation of energy, replacing $\dot{m}_{air}(h_{air_{sat},T_{water_{in}}} - h_{air_{in}})$ by $c_p(T_{water_{in}} - T_{BU})$ and applying to Eqs. 21 and 16. So to obtain ϵ_a, we have Eq. 30:

$$\epsilon_a = \frac{\dot{m}_{water_{in}} T_{water_{in}} + \dot{m}_{water_{rep}} T_{water_{rep}} - \dot{m}_{water_{out}} T_{water_{out}}}{\dot{m}_{water_{out}}(T_{water_{in}} - T_{BU})}. \tag{30}$$

Using Eqs. 28 and 30, we can obtain the temperature of the water leaving the cooling tower, as defined by Eq. 31:

$$T_{water_{out}} = \left(\frac{\dot{m}_{water_{in}}}{\dot{m}_{water_{out}}}\right) T_{water_{in}} + \left(\frac{\dot{m}_{water_{rep}}}{\dot{m}_{water_{out}}}\right) T_{water_{rep}} - y(c_0 + c_1 x + c_2 y + c_3 x^2 + c_4 y^2 + c_5 xy). \tag{31}$$

From Eq. 31, it is possible to estimate how the temperature of the water leaving the cooling tower varies as a function of the mass flow of air induced in the tower, *i.e.*, as a function of the fan speed. Equation 31 takes into account the outlet temperature of the chiller condenser, $T_{water_{in}}$, which is a function of the thermal load of the system.

Still referring to Eq. 31, the wet bulb temperature, T_{BU}, is extremely important to obtain the optimal fan speed, as it represents the minimum theoretical temperature value that can be reached by the water leaving the tower, $T_{water_{out}}$. The temperature of this water can never be lower than the wet bulb temperature, due to the saturation of the air molecules during the mass transfer process that takes place between the hot water and the dry air induced in the tower. The wet bulb temperature can be calculated as a function of ambient temperature and relative humidity, both of which are measured at the cooling tower installation site.

Moreover, the water that leaves the cooling tower corresponds to the water that enters the chiller condenser. Disregarding the thermal losses in the section between the outlet of the cooling tower and the inlet of the condenser, it can be concluded that the temperature of the water leaving the tower is equal to the temperature of the water entering the chiller condenser. The latter directly influences the performance of the chiller itself, and consequently, its energy consumption. Thus, the tower outlet temperature defined by Eq. 31 must be evaluated in relation to its influence on the energy consumption of the chiller, in order to obtain the best operational condition of the chiller and of the cooling tower, aiming at energy efficiency with the least detriment to the tower's effectiveness.

This modeling, in comparison with the Merkel's and Braun's models, offers the advantage that all variables are measurable and correspond to the input variables of the cooling tower. Therefore, there is no need to use recursive techniques. In terms of computational implementation, this modeling is more convenient, as ϵ_a is described as a polynomial function, unlike in Braun's model, where NTU and ϵ_a are described as exponential functions, which substantially increases the computational effort.

4.2 Tower's Fans

Among the equipment that compose the refrigeration system considered in this work, only the tower fans allow the speed variation, through the use of frequency converters. Therefore, condensed water and chilled water circulation pumps, as well as chillers, operate at their fixed speed.

The electrical power demanded by each of the cooling tower fans depends on their speed setting. It is known that the air moved by the fan represents a

load with resistant torque, quadratic C_{res} *i.e.* $C_{res} = kn^2$ wherein n represents the fan speed and k a proportionality constant. The electrical power required to drive the fan, at a given speed, is defined by $P(n) = C_{res} \cdot n = kn^3$. Thus, when reducing the engine speed from speed n_1 to n_2, we get Eq. 32:

$$P_2 = P_1 \cdot \left(\frac{n_2}{n_1}\right)^3, \tag{32}$$

where it is possible to verify that the demanded power drops with the cubic power of the reduction of the engine speed. Therefore, reducing the fan speed of the cooling tower cells should significantly reduce their energy consumption.

Thus, the electrical power demanded by each cooling tower fan, P_v, in kW, for a given speed, can be defined as in Eq. 33:

$$P_v = \sqrt{3} V_n I_n \cdot FP(n) \cdot \left(\frac{\dot{m}_{air}}{\dot{m}_{air_n}}\right)^3, \tag{33}$$

where V_n and I_n represent, respectively, the rated voltage and current of the fan motors, $FP(n)$ represents the motor power factor, which is a function of its rotation speed n, $\dot{m}_{air}$ represents the mass flow rate of air induced by the fan, and $\dot{m}_{air_n}$, the mass flow rate of air induced by the fan.

The speed set for the fan is proportional to the air flow obtained. Therefore, the Eq. 33 can also be defined as a function of the fan speed. The optimal speed value to be adopted for the cooling tower fans will be defined by the optimization system, aiming to obtain the best compromise between minimizing the global energy consumption of the cooling system and maximizing the effectiveness of the cooling tower.

The power factor of an electric motor, as well as its efficiency, are non-linear regarding the motor's rotation n. This is mainly due to the increase in internal heating for speeds below the rated speed. Therefore, in order to estimate the energy consumption of the fan motors for a given speed, taking into account these non-linearities, we adopt the model of Eq. 34:

$$P_v = \sqrt{3} V_n I_n (d_0 n^3 + d_1 n^2 + d_2 n + d_3), \tag{34}$$

where the coefficients $d_i, 0 \geq i \leq 5$ can be obtained using non-linear regression techniques.

The fans' consumption is small when compared to the value of consumption of chillers. However, the operation of the fans at low speed provides considerable energy savings, since it is an equipment that must operate continuously. For instance, running at 50% of the rated speed provides energy savings of around 87.5%, since the motor consumes around 1/8 of the nominal power.

5 Model Validation Results

In order to complete and validate the cooling tower proposed model, it is necessary to collect data from the cooling system and used them to obtain the proper values of the required coefficients c_i and d_i as required in Eq. 31 regarding the cooling tower and Eq. 34 regarding the tower's fans, respectively. The dataset considered for carrying out this operation is composed of points that represent different situations of thermal load. These were collected on different days of operation. The choice of days for the collection of operational data was carried out in order to obtain data that represent the various thermal load conditions present throughout the day and for different climatic conditions. We contemplated both sunny and rainy days, as these conditions influence the performance of the cooling system considered in this work.

Overall, 21,385 operating points of the cooling system were collected, corresponding to 29 h and 42 min of operation. The points were collected on four different days, in order to represent the operation in daytime and nighttime hours. These represent, respectively, conditions of high and low thermal load. The dataset represented by the collected data is divided into two subsets, both equally distributed *i.e.* balanced and corresponding to 50% of the total points collected, The first subset is called the estimation dataset and the second the validation dataset.

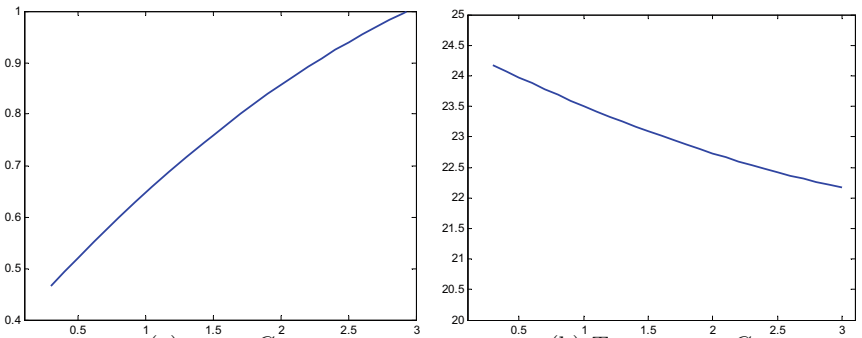
The coefficients of the adopted models are obtained using the estimation dataset. Then, the validation dataset is applied to the obtained models. All obtained models for the refrigeration system equipment are evaluated using the same estimation and validation datasets. The evaluation of the final result is performed based on the values of Mean Square Error (MSE) and the Coefficient of Determination, called R^2, computed. The coefficient of determination varies between 0 and 1, and allows assessing how much the adopted model is capable of estimating the actual collected data. Note that the closer the value of R^2 to 1 is, the better the evaluation of the obtained modeling [8].

The modeling of the cooling tower consists of obtaining the coefficients c_0–c_5 indicated in Eq. 31, which estimate the effectiveness of the cooling tower as a function of the terms x and y, which represent, respectively, $\frac{\dot{m}_{air}}{\dot{m}_{water}}$ and $(T_{water_{ent}} - T_{BU})$, thus allowing to obtain the effectiveness of the tower as a function of the speed of its fans. To obtain the coefficients, the Levemberg-Marquardt optimization method is used, based on the use of data collected in the field, as defined in Eq. 29. The use of the Levemberg-Marquardt method is due to the wide application of this regression technique to applications involving nonlinear systems [13]. Table 1 shows the values obtained.

Table 1. Obtained values for the coefficient required by the cooling tower's model.

Coefficient	Value
c_0	+0.0262
c_1	+0.4935
c_2	+0.14350
c_3	−0.0289
c_4	−0.0129
c_5	−0.0533

The tower effectiveness modeling occasioned an MSE of 3.5×10^{-3} and a R^2 of 0.9924. The values presented are quite satisfactory and validate the model adopted. Figure 5(a) indicates the effectiveness curve in terms of G, obtained from Eq. 28, where G is the ratio between the air and water flows inside of the cooling tower. In this case, the air flow varies with the speed of the tower fans. This curve depends on the temperature values of the water entering the tower and the wet bulb temperature. Here, a tower inlet temperature of 25.9 °C and a wet bulb temperature of 22.2 °C is considered. Moreover, Fig. 5(b) indicates the variation in the temperature of the water leaving the cooling tower as a function of the variation in the ratio between the air and water flows, G, obtained from the Eq. 31, considering the application of the coefficients c_0–c_5 from Table 1. In modeling the temperature of the water leaving the tower, an MSE of 1.95×10^{-2} and R^2 of 0.9546 was obtained, a result that validates the adopted model.



(a) ϵ_a *vs.* G (b) $T_{water_{out}}$ *vs.* G

Fig. 5. Performance results of the cooling tower's model for $T_{water_{in}} = 25.9$ °C and $T_{BU} = 22.2$ °C.

Analyzing the curves in Fig. 5, it can be seen that as the ratio between the mass flow rates of air and water increases, there is a reduction in the water temperature of condensation that leaves the cooling tower. This causes an increase in its effectiveness, as indicated in Eq. 30.

Figure 6 illustrates the comparison between the real values and the values obtained from the proposed model to estimate the temperature of the water leaving the cooling tower, $T_{water_{out}}$, using the Eq. 31. In this case, the model is applied to a dataset obtained during 29 h and 42 min of operation of the cooling system, with readings every 5 s. Hence, it includes a total of 21,385 points. The oscillations shown in Fig. 6 indicate the temperature variation of the water leaving the cooling tower during field data collection. Note that during the considered time period, values are recorded for $T_{water_{out}}$ between 21.5 °C and 27 °C. Moreover, in Fig. 6, points can be observed where the modeling curve presents slight peaks compared to the curve of the real values. However, the recorded peaks represent a maximum deviation of 1.75 °C for the modeling with respect to the actual values, which represents a maximum relative deviation, in modulus, of 7.20%. This deviation is acceptable, since it occurs for a small range of the curve. Furthermore, the average relative deviation obtained for the 21,385 points is, in modulus, 0.005%.

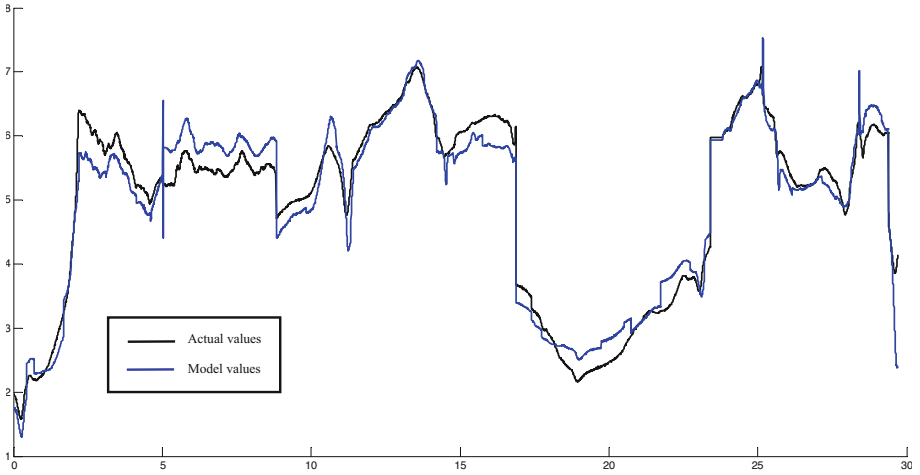


Fig. 6. Real *vs.* estimated value of the temperature leaving the cooling tower, $T_{water_{out}}$ during 30 h interval.

Regarding the energy consumption of the cooling tower fans, the coefficients d_0–d_3 indicated in Eq. 34 are also obtained using the Levemberg-Marquardt method. The values obtained are shown in Table 2. In this case, the points of the used dataset for the modeling were obtained by varying the fan speed, followed by reading the respective demanded electrical power. Overall, 12 operating points are obtained, including the nominal speed, with all other speeds being lower. Figure 7 presents the result of the modeling obtained for the demanded power by one of the cooling tower fans, using the Eq. 34 and the coefficients given in Table 2. An MSE of 2.6505×10^{-4} is occasioned by the model, which is an excellent result.

Table 2. Obtained values for the coefficient required by the fans' model.

Coefficient	Value
d_0	+0.7931
d_1	+0.0330
d_2	+0.0557
d_3	+0.0039

The obtained coefficients are applied to the dataset composed of 21,835 points, obtaining the result shown in Fig. 8. Differently from that indicated in Fig. 7, it illustrates the application of the modeling to the set of fans that are in operation during field data collection. In Fig. 8 it is possible to compare the real values with those obtained via the proposed model. In this case, an MSE of 1.1609 and a R^2 of 0.9947 are obtained. This result validates the model. Note in Fig. 8 that the values obtained using the proposed model follow the real curve of required power by the fans in all the points. Also, it can be observed in the points where there is a sudden reduction or increase in the demanded power. These points indicate the turning off and restarting of the fans, as well as their operation with speeds below the nominal.

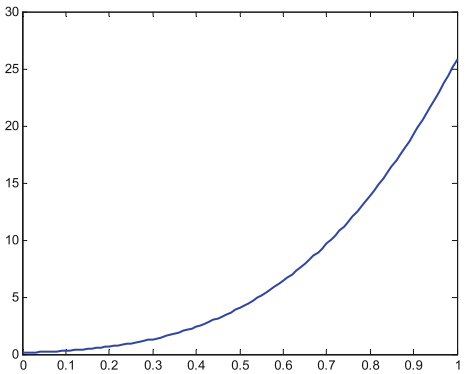


Fig. 7. Result of modeling the power consumption of cooling tower fans in terms of fractions of the nominal rotation speed.

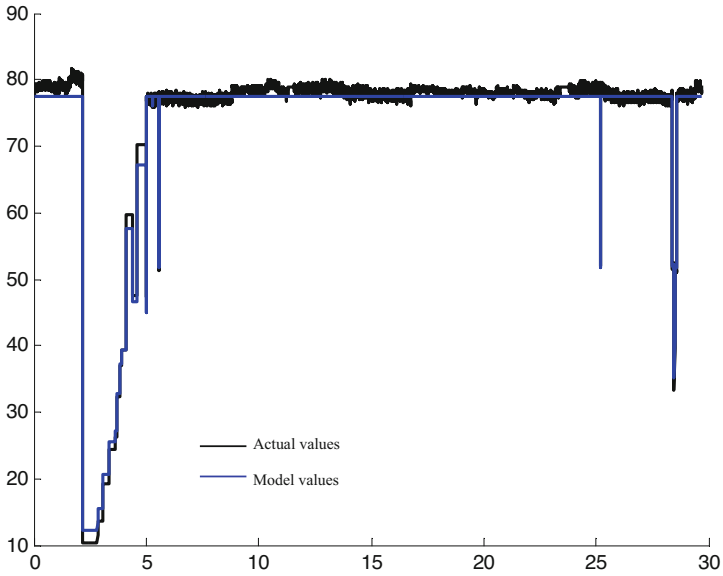


Fig. 8. Actual consumption of cooling tower fans *vs.* values obtained using the proposed model.

6 Conclusions

In this paper, the models of the main equipment that compose an industrial refrigeration system is presented: the cooling tower together with its fans. Due to lack of space, the proposed model for the chiller will be presented in [11]. The models obtained are applied to real data, collected from a refrigeration system. The models of the cooling tower and fans present satisfactory results. This allows their use in any multi-objective optimization, aiming at energy efficiency.

Acknowledgments. This work is supported by Conselho Nacional de Desenvolvimento Científico e Tecnológico (CNPq - Brazil) and by Fundação Carlos Chagas Filho de Amparo à Pesquisa do Estado do Rio de Janeiro (FAPERJ - Brazil 203.111/2018). We are most grateful for their continuous financial support.

References

1. Alpina: Torres de resfriamento de água. Tech. rep., Alpina S/A Indústria e Comércio, São Bernardo do Campo, SP (1978)
2. Braun, J.E., Klein, S.A., Mitchel, J.W.: Effectiveness models cooling towers cooling coils. ASHRAE Trans. **95**(2), 164–174 (1989)
3. Fahmy, A.A., Shouman, L.A.: Deterministic study on the optimum performance of counter flow cooling tower. Int. J. Sci. Eng. Res. **3**(6), 1–5 (2012)
4. Kern, D.Q.: Process Heat Transfer. McGraw-Hill Book Company Inc., Tokyo (1983)

5. Khan, J.R., Zubair, S.M.: An improved design and rating analyses of counter flow wet cooling towers. J. Heat Trans. **123**, 770–778 (2001)
6. Kloppers, J.C., Kroger, D.G.: A critical cooling tower performance evaluation. In: 12th IAHR Symposium in Cooling Towers and Heat Exchangers, UTS, pp. 108–115 (2001)
7. Kloppers, J.C., Kroger, D.G.: Cooling tower performance: a critical evaluation of the Merkel assumptions. Res. Dev. J. Incorporated SA Mech. Eng. **20**(1), 24–29 (2004)
8. Leontaritis, I.J., Billings, S.A.: Model selection and validation methods for non-linear systems. Int. J. Control **45**(1), 311–341 (1987)
9. Lu, L., Cai, W.: A universal engineering model for cooling towers. In: International Refrigeration and Air Conditioning Conference, Indiana, USA (2002)
10. Merkel, F.: Verdunstungskuhlung, pp. 576–583. VDI Forschungsarbeiten, Berlin (1925)
11. Nedjah, N., Mourelle, L.M., Lizarazu, M.S.D.: Mathematical modeling of chiller-based refrigeration systems for energy efficiency optimization. In: Gervasi, O., et al., B.M. (eds.) International Conference on Computational Science and Its Applications, pp. 1–18 (2022, accepted)
12. Poppe, M., Rogener, H.: Berechnung von ruckkuhlwerken, VDI-Warmeatlas (1991). pp. 1001–1015
13. Seber, G.A.F., Wild, C.J.: Nonlinear Regression. Willey-Interscience, Hoboken (2003)
14. Sutherland, J.W.: Analysis of mechanical draught counter flow air water cooling tower. J. Heat Trans. **105**, 576–583 (1983)
15. Yu, K., Hung, Y., Hsieh, S., Hung, S.: Chiller energy saving optimization using artificial neural networks. J. Appl. Sci. **16**(11), 3008–3014 (2011). https://doi.org/10.3923/jas.2011.3008.3014