Using an Economically Justified Trend for the Stationarity of Time Series in ARMA Models

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Abstract. The ARMA models are used in econometric studies to predict the behavior of a time series. In case of non-stationarity of the initial data ARIMA models get time series to stationarity by differentiation. The problem is applying differentiation provide the loss of essential information. The paper is trying to prove that ARMA model based on the differences between non-stationarity initial data and trend line can provide the same with classic ARIMA approach level of prediction force. For this purpose, the comparison of the quality indicators of the model constructed according to the ARIMA model based on the initial data and the ARMA model based on trend line was carried out. The cryptocurrency market has been chosen as the sphere of research. It was found that the two approaches give approximately the same prediction error and variations from the initial data.

Keywords: ARIMA · Bass's theory of diffusion of innovations · Stationarity · Cryptocurrencies · Differentiation

1 Introduction

Time series analysis is an important sphere in statistic and econometric. In the professional community, there are many models and methods for predicting and analyzing the behavior of time series. ARMA family models (autoregressive moving average model) are one of the most mathematically and statistically coherent models.

The ARMA method assumes that any stationary time series satisfy the criteria:

- theoretical mathematical expectation and variance of the series are independent of time,
- the covariance between its values at moments t and $t + n$ depends only on s, not on time.

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For ARMA models the stationarity of time series is critical for obtaining a reliable level of forecasting. In case of non-stationary time series, the ARIMA (autoregressive integrated moving average) model is used [7]. ARIMA uses differentiation of initial data to bring non-stationary time series. The main disadvantage of this method is that the level of differentiation carried out by simple brute force method. Besides differentiation can provide a loss of significant information.

In this paper we want to analyze an alternative approach to the problem of stationarity when using ARMA family models. We suppose the usage the difference between initial time series and economy-based trend line as a training data for ARMA model can provide the same with classic ARIMA approach level of prediction force and explanation of initial data.

To prove this assumption, we tested the efficiency of the classical ARIMA and our approach at the number of active crypto wallets of Bitcoin and Ethereum data in different time periods. The Python programming language with extended statistical packages is used for analysis.

2 Methodology

2.1 ARMA and ARIMA Models Construction Methods

To compare the effectiveness of the proposed method with the classical ARIMA, we use the following algorithm:

1. Building trend lines.
2. Calculation of the trend line deviation from initial time series.
3. Checking the obtained deviation for stationarity by the Dickey-Fuller method [12].
4. Calculation of the parameters of the classical ARIMA model based on the initial data using the pmdarima.autoarima package for the python programming language (ARIMA optimization method is the Powell Method).
5. Calculation of ARMA parameters, based on trend line deviation, using the pmdarima.auto_arima package with a given $d = 0$.
6. Calculation of ARIMA parameters using the pmdarima.auto_arima package (in this case, parameter d will not be used).
7. Calculation of the Root Mean Square Error (RMSE), the mean absolute error (MAE), as well as mean absolute percentage error (MAPE) for ARIMA and ARMA models.
8. Calculation of the predictive force indicators of models.

For all calculated quality parameters, we use the sklearn and pandas' libraries for the python programming language.

2.2 Trend Identification Approach

In case of initial data, we conduct a series of comparative tests on the number of active wallets of cryptocurrencies Bitcoin (hereinafter BTC) and Ethereum (hereinafter ETH) for two large-scale periods – from 2013 to 2019 (the period of the first speculative peak

in 2018), and from 2013 to 2021 (the period including two speculative peaks – the peak of 2018 and the peak of the beginning of 2021, caused by the speculative interventions of Elon Musk) [1]. Time series of cryptocurrency price dynamics are a popular object for research. Machine learning methods [9], correlated stochastic differential equations [2] and ARMA family models [8] are usually used for their analysis. Nevertheless, the focus of such papers is to predict the dynamics of cryptocurrencies prices. In our previous research [3–6], we focused on considering cryptocurrencies as payment system. It's not the price that interests us, but the audience. Therefore, in this paper we propose to test classical ARIMA approach and our approach on the number of cryptocurrencies' active wallets.

Each time series was considered by us in two states. In the first case, training will be performed on the initial data (2517 days for the first period and 4517 for the second period). This allows to simulate the situation of data analysis with a high proportion of random noise. It also can show the difference of ARMA prediction force depending on the training period. In the second case, we transform the data into weekly japanese candlesticks (reducing the sample size by 7 times). Thus, we get smoothed data, which are free from random noise, but preserving the main directions of the trend development:

- BTC_1 - Dynamics of active BTC wallets for 2517 days of observations (daily data);
- BTC_1_light - Dynamics of active BTC wallets for 2517 days of observations (weekly data);
- BTC_2 - Dynamics of active BTC wallets for 4517 days of observations (daily data);
- BTC_2_light - Dynamics of active BTC wallets for 4517 days of observations (weekly data);
- ETH_1 - Dynamics of active ETH wallets for 2517 days of observations (daily data);
- ETH_1_light - Dynamics of active ETH wallets for 2517 days of observations (weekly data);
- ETH_2 - Dynamics of active ETH wallets for 4517 days of observations (daily data);
- ETH_2_light - Dynamics of active ETH wallets for 4517 days of observations (weekly data).

In our previous work [3], we demonstrated that the growth dynamics of Bitcoin and Ethereum users is well described by the Bass equation, originally developed by us to predict the dynamics of the development of payment systems.

In our previous works [4–6], we were able to establish that the development of payment systems can be described using a modified equation of Bass [10]. The advantage of this approach is that the indicators used in the model have a pronounced economic meaning. We modified classical Bass equation for the purpose of payment systems behavior forecast. The following parameters are used:

- current number of users x_0;
- the maximum number of users, for example, the entire audience of a given country, N. Therefore, the number of potential users not currently participating in the system is $N - x_0$;
- audience capture rate, which reflects the probability that a given user will start using the service: $r > 0$ (the reverse time of the decision) within a given period;

- audience fatigue coefficient, which reflects the probability that the customer will stop using the service: $b > 0$ (inverse decision-making time) during a given period.

The modified equation for predicting payment systems behavior forecast has the following form [3]:

$$\frac{dx}{dt} = a(N - x)x - bx \tag{1}$$

We also proposed a generalized equation for predicting payment systems behavior forecast [3]:

$$x = \frac{x_0 N e^{rKt}}{x_0\left(e^{rKt} - 1\right) + N} \tag{2}$$

3 Construction and Quality Analysis of ARMA and ARIMA Models

3.1 Construction and Analysis of Trend Lines for Time Series

First, we built trend lines for the initial time series. The trend lines are shown in Table 2. On the next step we calculated deviation of the trend line from initial data. The obtained time series are presented in Table 3. Received time series was tested by The Dickey-Fuller criterion [12]. This is a popular method in machine learning for checking the stationarity of a time series. The main idea of the test is searching for single roots. If a single root exists, the series is non-stationary, if there is no such root, it is stationary.

The coefficients for the Bass equation and the presence or absence of stationarity according to the Dickey-Fuller test are presented in Table 1.

Table 1. Bass equation coefficients

	K	r	x_0	Is stationarity achieved by the Dickey-Fuller criterion
BTC_1	705 027	0,00240	29 540	Yes
BTC_1_light	551 209	0,00260	29 540	Yes
BTC_2	729 526	0,00236	29 540	Yes
BTC_2_light	651 667	0,00238	29 540	Yes
ETH_1	396 582	0,00823	959	Yes
ETH_1_light	381 595	0,00812	959	Yes
ETH_2	445 028	0,00804	959	No
ETH_2_light	478 891	0,00773	959	No

Table 2. Investigated time series and trend lines based on the Bass equation

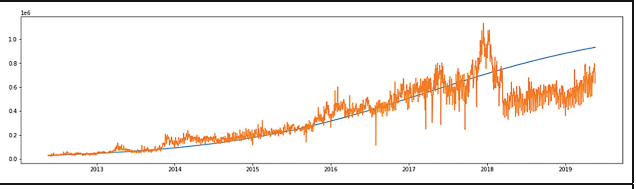
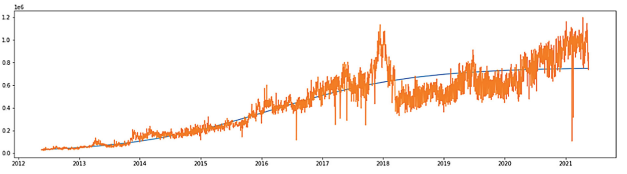
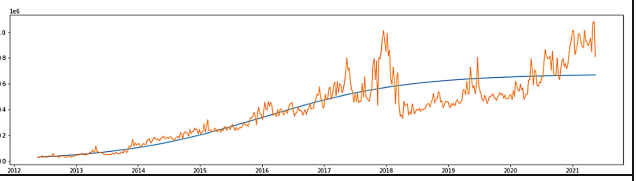
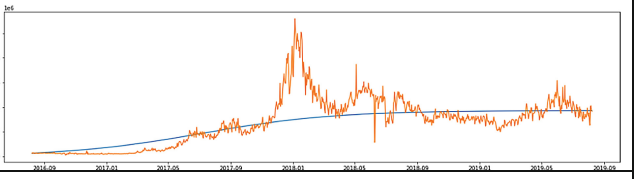
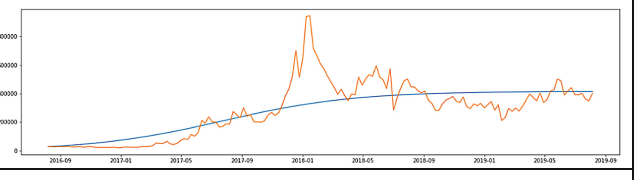
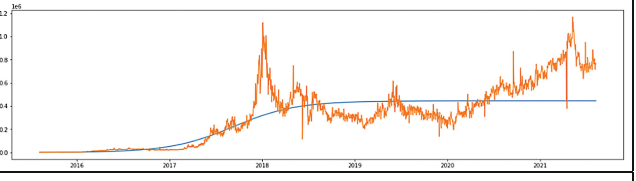
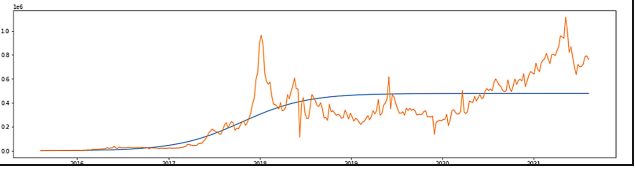
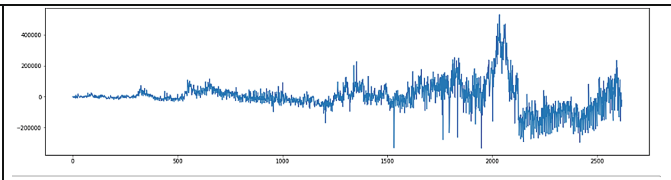
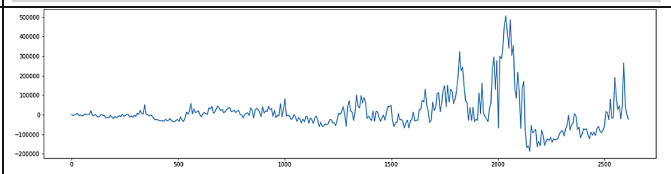
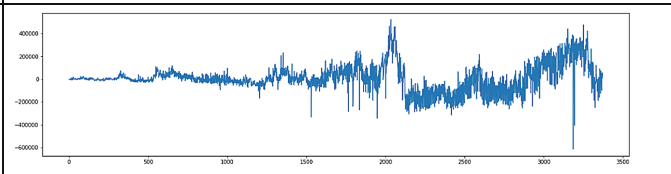
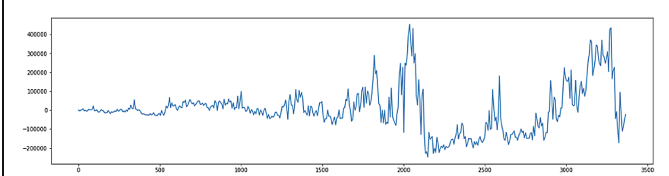
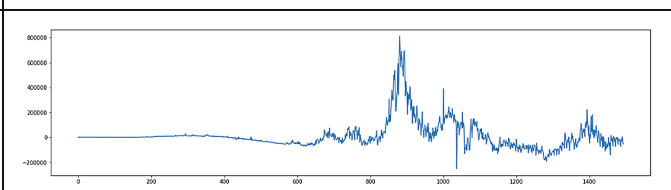
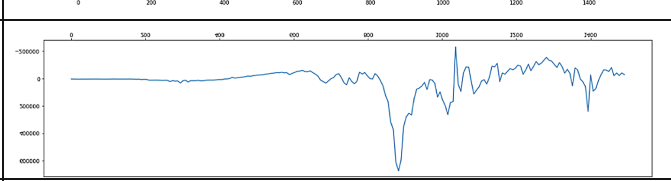
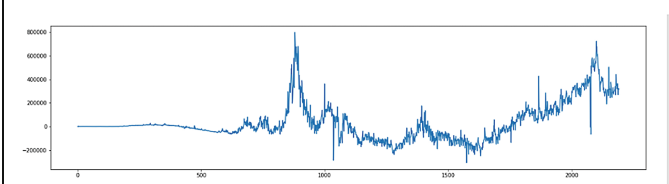
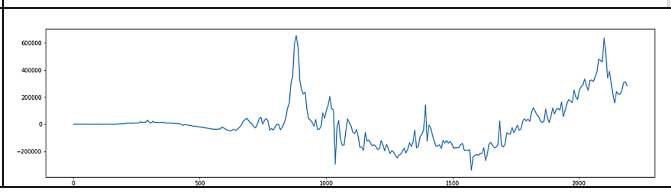
BTC_1	
BTC_1_light	
BTC_2	
BTC_2_light	
ETH_1	
ETH_1_light	
ETH_2	
ETH_2_light	

Table 3. Graphs of the difference between real time series data and trend lines based on the Bass equation

BTC_1	
BTC_1_light	
BTC_2	
BTC_2_light	
ETH_1	
ETH_1_light	
ETH_2	
ETH_2_light	

The stationarity of the time series based on deviation of the trend line from initial data is achieved in almost all cases. The exception is a series based on the dynamics of ETH fluctuations in the period from 2012 to 2021. This is because ETH is close to investment instrument, unlike BTC, which keep characteristics of the payment system. In addition, the price movement of many cryptocurrencies, as well as the level of investor confidence in the market, depend on fluctuations in BTC prices. Thus, in 2018, following the fall in the price of BTC, faith in ETH and other cryptocurrencies has been seriously decreased. This behavior is atypical for payment systems.

This example shows the meaning of trend line equation choice. In our case, increasing the number of ETH active wallets since 2018 has a high market effect and can't be explained by modified Bass equation. Nevertheless, the dynamics of another time series is generally described by the Bass equation, which allows us to continue our research.

3.2 Building Models

Based on the initial data and the trend lines constructed using the modified Bass equation, we calculated the ARIMA and ARMA models. The calculation is based on pmarima.autoarima Python library with a fixed value of $d = 0$ for ARMA models, and computed d for ARIMA.

Table 3 shows the best combinations of parameters obtained by pmarina.autoarima for ARMA and ARIMA models.

Table 4. ARMA and ARMA models parameters

	ARIMA models parameters	ARMA models parameters
BTC_1	*p: 0 d: 1 q: 2*	*p: 5 d: 0 q: 4*
BTC_1_light	*p: 0 d: 1 q: 1*	*p: 1 d: 0 q: 1*
BTC_2	*p: 5 d: 1 q: 3*	*p: 5 d: 0 q: 4*
BTC_2_light	*p: 0 d: 1 q: 1*	*p: 1 d: 0 q: 1*
ETH_1	*p: 2 d: 1 q: 4*	*p: 5 d: 0 q: 5*
ETH_1_light	*p: 2 d: 1 q: 3*	*p: 2 d: 0 q: 1*

ARMA and ARIMA models' parameters have a significant spread. Basically, large spread is fixed for data with a high random noise. In case of the weekly candle data, the q indicator is quite close. The largest deviations are observed in the parameter p – the behavior of the autoregressive model.

The difference in parameters may indicate a different quality of the model's description of real data. To check this, we trained ARMA and ARIMA modes used model parameters from Table 4 by ARIMA.predict_in_sample. Obtained data was compared with initial data by RMSE, MAE, and MAPE indicators. These indicators are often used in studies of the cryptocurrency market predictions methods [2, 11].

The results of comparing ARMA and ARMA models are presented in Table 5.

Table 5. Quality analysis of ARMA and ARIMA models

	Criteria	ARIMA models parameters	ARMA models parameters
BTC_1	RMSE	29 924,54	31 794,92
	MAE	19 047,39	20 224,71
	MAPE	0,06	0,06
BTC_1_light	RMSE	23 947,90	25 406,85
	MAE	14 591,15	15 552,61
	MAPE	0,05	0,05
BTC_2	RMSE	44 985,68	44 752,18
	MAE	28 461,57	28 192,13
	MAPE	0,07	0,07
BTC_2_light	RMSE	24 715,78	26 104,84
	MAE	15 645,47	17 224,74
	MAPE	0,04	0,04
ETH_1	RMSE	11 938,82	12 889,05
	MAE	6 192,17	6 616,09
	MAPE	0,05	0,04
ETH_1_light	RMSE	8 353,00	17 101,50
	MAE	4 851,60	9 444,31
	MAPE	0,13	0,10

The analysis shows that the ARIMA and ARMA models have approximately the same percentage of error in comparison with initial data. For weekly candles, ARIMA models describe initial data generally better than ARMA (in the corridor from 0.1 to 1% accuracy). In case of unprepared initial data with high random noise, ARMA models give a slightly more accurate distribution (in a similar corridor). It means that, in general, the quality of ARMA models based on trend deviation from the initial time series is comparable to ARIMA models based on initial data.

3.3 Prediction Force

In scientific research, ARIMA-type models are mainly used to obtain short-term forecasts [8, 11]. The reason is ARMA and ARMA models in the long-term predictions is trying to balance itself into some stationary.

To test the predictive force of the models, we train them on a part of the initial time series using the pindorama.arima package for Python and the ARIMA.predict_in_sample method. According to another scientific research, the ARIMA family models lose their predictive power at the 4th prediction period [11]. Therefore, to obtain the dynamics of the loss of predictive force by models, we consider 3 cases relative to the last observation period (T):

- Prediction for the next period (for first day for models trained on the full variation series and 1 week for "light" models).
- Prediction for the period $T+2$ (for second day for models trained on the full variation series and 2 weeks for "light" models).
- Prediction for the period $T+3$ (for third day for models trained on the full variation series and 3 weeks for "light" models).

For each case, let's make a prediction for 5 periods. For each period, we train the model on initial data (or on the difference between real data and the Bass equation) using pmdarima.auto_arima and build a forecast for $T+1(T+2, T+3)$ using ARIMA.predict_in_sample. The obtained value is compared with the real number of wallets in this period (this period did not participate in the training of the model). If the number of BTC wallets increased in the period $T+1$, and the predicted value also shows an increase, we think that there is a correct prediction. If the value predicted by the model shows a decrease, when real number of wallets growth a conclusion is made about an incorrect prediction. Then the real value of the time series is added to the trained data and the process repeats.

The results of model predictions are presented in Tables 6 and 7.

It is easy to see that increase in the prediction distance provides a decrease in the number of correct predictions (for example, for ARMA models in the period $T+1$ there are 23 times of correct prediction, but in the period $T+3$ there are only 14 correct predictions). This data coincides with the ARIMA models behaviour which was identified by other researchers on analyzing cryptocurrencies [8, 11].

We also can see that the prediction force for models based on short time series (BTC_1, BTC_1_light, ETH_1, ETH_1_light) is stronger that for models based on full data set (BTC_2, BTC_2_light, ETH_2, ETH_2_light). This behaviour can be explained by the fact that cryptocurrencies have a high speculative effect. The dynamics of interest in cryptocurrencies strongly depends on the price of coins and can vary greatly in different time periods. ARMA predictions based on a complete time series try to describe the behavior of the number of wallets during a period of strong volatility in the markets. At the same time, short time series predict the dynamics in a calm market. This is an important conclusion that should be considered when determining the amount of data to train the ARMA model.

Table 6. The number of model predictions coinciding with the actual data of the time series

		$T+1$ prediction	$T+2$ prediction	$T+3$ prediction
		Prediction force:	Prediction force:	Prediction force:
BTC_1	ARIMA	4	3	3
	ARMA	3	3	3
BTC_1_light	ARIMA	3	2	2
	ARMA	3	2	1
BTC_2	ARIMA	3	2	0
	ARMA	2	1	1
BTC_2_light	ARIMA	3	3	2
	ARMA	3	3	2
ETH_1	ARIMA	2	2	2
	ARMA	2	2	2
ETH_1_light	ARIMA	3	2	1
	ARMA	3	2	2
ETH_2	ARIMA	4	3	3
	ARMA	4	3	3
ETH_2_light	ARIMA	1	1	1
	ARMA	0	0	0
Total correct predictions	ARIMA	**23**	**18**	**14**
	ARMA	**20**	**16**	**14**

In general, the classical ARIMA model and the ARMA model based on the Bass equation both demonstrate similar predictive force. For the $T+1$ period models guess the correct direction of the time series in 3 or 4 cases out of 5. The weakest prediction is on the ETH_2_light time series.

It is also necessary to mention the dynamics of the decline in forecasting efficiency. If in the $T+1$ period ARIMA models were better than ARMA models based on the Bass equation (23 correct forecasts versus 20). In the $T+3$ period the predictive power of all models has become comparable. At the same time, ARMA (except for ETH_2_light) was generally more accurate.

Table 7. Absolute error of model predictions

		$T+1$ prediction	$T+2$ prediction	$T+3$ prediction
		prediction_mistake:	prediction_mistake:	prediction_mistake:
BTC_1	ARIMA	5.31%	5.92%	4.93%
	ARMA	5.60%	4.71%	4.42%
BTC_1_light	ARIMA	17.35%	17.80%	15.46%
	ARMA	16.68%	16.46%	13.98%
BTC_2	ARIMA	3.43%	4.53%	4.99%
	ARMA	4.48%	4.81%	4.60%
BTC_2_light	ARIMA	7.51%	12.63%	7.45%
	ARMA	7.43%	11.99%	6.63%
ETH_1	ARIMA	4.56%	7.49%	6.69%
	ARMA	4.49%	6.70%	6.86%
ETH_1_light	ARIMA	5.88%	4.26%	6.45%
	ARMA	4.85%	3.96%	4.61%
ETH_2	ARIMA	3.09%	4.06%	4.52%
	ARMA	2.65%	3.10%	3.20%
ETH_2_light	ARIMA	3.22%	4.77%	6.42%
	ARMA	4.37%	7.15%	11.53%

4 Conclusions

The paper showed that the usage of the difference between the initial data and the economically justified trend as a stationary time series for making of ARMA model allows to obtain a level of accuracy like classical ARIMA. It allows us to describe the behavior of the time series without differentiation. Moreover, the study shows in case of high data noise, the usage of a trend live is preferable, because the differentiation of the initial time series leads to more significant errors.

Nevertheless, the study showed that the quality of proposed method depends on the quality of the explanation of real data by an economically justified trend. The example with ETH demonstrates how weak explanation force of the trend can provide the loss of ARMA prediction potential. Nevertheless, we believe that this problem can be overcome by adjusting the model. Ultimately, this method opens additional opportunities for empirical prediction and analysis of time series.

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