

A Modified SEIR Model: Stiffness Analysis and Application to the Diffusion of Fake News

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Abstract. In this work we propose a novel and alternative interpretation of the SEIR model, typically used in epidemiology to describe the spread of a disease in a given population, to describe the diffusion of fake information on the web and the consequent truth re-affirmation. We describe the corresponding system of ordinary differential equations, giving a proper definition of the involved parameters and, through a local linearization of the system, we calculate the so-called stiffness ratio, i.e. the ratio between the real parts of the largest and smallest eigenvalues of the Jacobian matrix of the linearized problem. A large gap in the spectrum of such a Jacobian matrix (i.e., a large stiffness ratio) makes the underlying differential problem stiff. So, we study and analyze the stiffness index of the SEIR model and, through selected numerical examples on real datasets, we show that the more the model is stiff, the faster is the transit of fake information in a given population.

Keywords: SEIR model · Fake news · Stiffness index

1 Introduction

Nowadays, internet is widely used as a primary medium of information all over the world, since it is a rapidly accessible and low cost source. The availability of such a large amount of information makes digital world deeply integrated with the lifestyle habits of our society. Clearly, since veracity is a huge issue in the spread out of online information, the circulation of fake news has been extensively increasing during last years.

The concept of *fake news* concerns with information that does not coincide with the reality, and is usually transmitted through a communication environment, in general for economic and political purposes, or in order to influence public opinion on a specific aspect. Recently, this topic has become very attractive since fake news are spread out much faster than real news on internet, certainly in a more rapid way with respect to traditional media such as radio, television,

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newspapers, etc. So, internet is the ideal environment encouraging users to share information between individuals (also over long distances), included fake news (see, for instance, [5,15,22,23,27,35,38,46,48,50] and references therein).

On the one hand, one of the most natural questions about fake information is how to deal with its detection. It is an extremely important issue, since the quantity of false news circulating around us, especially online, is increasing very rapidly. Because of this, the mechanism of detection of fake news should be able to predict enough information quickly enough in order to stop the spread of fake news, almost in real time.

On the other hand, the analysis of the spreading of fake news should be addressed taking into account the different types of fake news, its role and purpose, as well as the context in which it can be found. Indeed, fake news can be presented in different ways and for different purposes: for instance, satire or parody, news based on false content for humor purposes, misleading contents, manipulated contents, etc. More details on the topic can be found, for instance, in [17,19,26,33,40,47,52,53] and references therein.

Mathematical models in epidemiology are usually considered to study the diffusion of fake news, because they are able to describe a realistic interaction between the fake information and the people who receive it. In this paper, we focus our attention on a modified version of the classical SEIR model and we apply it to describe the diffusion of fake news. During last years, the classical SEIR model applied in epidemiology has been studied and modified in order to create more realistic mathematical models capable of predicting the spread of diseases. For instance, in [49] a new SEIR model was introduced for the dynamical analysis of the COVID-19 infection in Saudi Arabia and was tested on real data. In that paper, the authors give an expression of the reproduction number and report the stability analysis. The authors of [45] have used mathematical modelling and numerical simulations to describe coronavirus dynamics with data from China, Italy, Iran, and Brazil. In order to study the evolution of COVID-19 in Italy since the beginning of the vaccination process, a new hybrid compartmental model has been introduced in [4]. Other versions of SEIR model are described, for instance, in [3,6,7,9,10,12,20,21,24,25,36,37,39,41,51] and references therein.

More recently, this kind of models have been treated also for the description of the transmission of fake news [8,16,18,30,32,34]. For example, in [16], the authors describe a SEIR rumor spreading model in online social networks, while in [18] the interaction between fake news spreading and competence of individuals has been described by means of kinetic theory arguments.

In this paper, we propose a novel and alternative interpretation of the SEIR model for the spread of fake information in a given country, first of all explaining the proposed meaning of the involved parameters. Then, through a local linearization of the system, we compute the so-called stiffness ratio, defined as the ratio between the real parts of the largest and smallest eigenvalues of the Jacobian matrix of the linearized problem. If the gap in the spectrum of such a Jacobian matrix is large and, as a consequence, the stiffness ratio is large, the

underlying differential problem is stiff. As described on a simplified prototypical model (i.e., the SIR model) in [12], we show that the stiffness ratio is related to the diffusion of fake news. More in details, in this paper it is confirmed that the higher the stiffness ratio, the faster the re-establishment of the truth after the diffusion of fake information.

The paper is structured as follows. In Sect. 2, we introduce and formulate the modified SEIR model describing the meaning of the different involved populations and parameters. Section 3 is devoted to the stiffness analysis of the underlying system of ordinary differential equations model, passing through the linearization of the problem and the computation of the stiffness ratio of the problem. Numerical experiments on real data that illustrate the behaviour of the spread of fake news are reported in Sect. 4. Finally, in Sect. 5 some conclusions and possible future developments of this research are discussed.

2 Formulation of the Modified SEIR Model

The use of compartmental models technique for modelling an infectious disease was first introduced in the early 20th century; see [28,29,42–44].

In this paper, we consider a new formulation of the classical SEIR (Susceptible-Exposed-Infective-Recovered) model, which is a mathematical model based on a system of dynamic ordinary differential equations. It was first introduced to characterize the epidemic dynamics in order to understand the behaviour of transmission of an infectious disease [49]. In this scenario (see Fig. 1), the population is considered to be divided into four sub-populations as follows

- $S(t)$ represents the Susceptible group who are healthy but can contract the disease.
- $E(t)$ acts as the Exposed population who infected but not yet infectious.
- $I(t)$ is dedicated to the people who confirmed infected.
- $R(t)$ collects the individuals who have recovered and cannot contract the disease again,

and the parameters involved are defined as

- Λ new add rate to Susceptible population.
- β contact rate from Susceptible population S to Infected population I.
- γ transmission rate of confirmed infected from the Exposed population E.
- δ transmission rate of recovery from the Exposed population E.
- α transmission rate of recovery from Infected I to Recovered R.

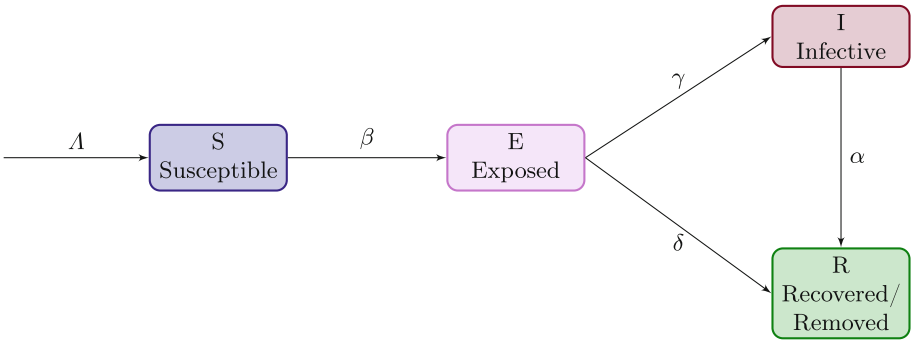


Fig. 1. Scheme of the SEIR model.

Taking into account the scheme presented in Fig. 2, the corresponding system of ODEs is then given by

$$\begin{cases} \dfrac{dS(t)}{dt} = \Lambda - \beta \dfrac{S(t)}{N} I(t) \\ \dfrac{dE(t)}{dt} = \beta \dfrac{S(t)}{N} I(t) - (\gamma + \delta) E(t) \\ \dfrac{dI(t)}{dt} = \gamma E(t) - \alpha I(t) \\ \dfrac{dR(t)}{dt} = \alpha I(t) + \delta E(t), \end{cases} \tag{1}$$

where N represents the number of total population and is defined as

$$N = S + E + I + R.$$

In this paper, we adapt the model (1) to the fake news diffusion, where $S(t)$ represents the number of internet users, who are still not aware of the news up to time t, $E(t)$ are the users who have been exposed to the item of fake news through the internet and have to decide to spread or not the news, $I(t)$ speaks for the spreader ones, who read and actively disseminate the news, and $R(t)$ are silent ones, individuals who have encountered the news item but choose not to spread it. Regarding the different parameters involved in (1), we take

- Λ new add rate to Susceptible population.
- β contact rate at which Susceptible population S are aware of the news and becomes infected population I.
- γ rate at which agents decide to spread the news.
- δ rate at which agents decide not to spread the news.
- α time life of the fake news measured in days divided by the total number of days under investigation,

where Λ is assume to be for *new internet subscriptions* and it has been chosen so that the new internet subscriptions are of order 10^{-5}, α is fixed as 0.01 since we assume 10 days for the life of the fake news and 1000 days under investigation (see Sect. 4), β and δ are calculated, respectively, as

$$\beta = \frac{\textit{percentage of individuals using internet}}{10}, \quad \delta = \frac{\textit{literacy rate}}{10^3},$$

and γ is an assumed value that we take between 0.1 and 0.9 different for each country with the aim to investigate different possible situations. These values have been provided by "ITU Committed to connecting the world" website [1] and "World Bank Open Data" website [2] for different countries in 2018, and are collected in Table 1.

Table 1. Values of the parameters for different countries in 2018.

Country	Λ	β	γ	δ	N
Morocco	2300	0.065	0.5	0.074	36029089
Guatemala	230	0.042	0.9	0.081	16346950
Argentina	2300	0.078	0.2	0.099	44494502
Thailand	2300	0.057	0.1	0.094	69428454
Portugal	230	0.075	0.7	0.096	10283822

3 Stiffness Analysis

In this section we analyze the stiffness ratio of system (1). To do this, let us consider an initial value vector $[S_0, E_0, I_0, R_0]^T = [S(0), E(0), I(0), R(0)]^T$, and let us linearize model (1) around it, obtaining

$$\begin{cases} \dfrac{dS(t)}{dt} = \Lambda + \beta\dfrac{S_0}{N}I_0 - \beta I_0\dfrac{S(t)}{N} - \beta\dfrac{S_0}{N}I(t) + \text{high order terms} \\ \dfrac{dE(t)}{dt} = -\beta\dfrac{S_0}{N}I_0 + \beta I_0\dfrac{S(t)}{N} + \beta\dfrac{S_0}{N}I(t) - (\gamma+\delta)E(t) + \text{high order terms} \\ \dfrac{dI(t)}{dt} = \gamma E(t) - \alpha I(t) + \text{high order terms} \\ \dfrac{dR(t)}{dt} = \alpha I(t) + \delta E(t) + \text{high order terms}. \end{cases} \tag{2}$$

The Jacobian matrix associated to the linear part of (2) is defined as

$$J_{\beta,\gamma,\delta,\alpha}\left(\frac{S_0}{N}, I_0\right) = \begin{bmatrix} -\beta\frac{I_0}{N} & 0 & -\beta\frac{S_0}{N} & 0 \\ \beta\frac{I_0}{N} & -(\gamma+\delta) & \beta\frac{S_0}{N} & 0 \\ 0 & \gamma & -\alpha & 0 \\ 0 & \delta & \alpha & 0 \end{bmatrix}. \tag{3}$$

By calculating the eigenvalues of the above matrix, we get one eigenvalue equal to 0 and two real eigenvalues that we use to compute the stiffness ratio (see [14,31]) of the differential equations system as

$$r_{\beta,\gamma,\delta,\alpha}\left(\frac{S_0}{N}, I_0\right) = \frac{\left|\lambda^{\max}_{\beta,\gamma,\delta,\alpha}\left(\frac{S_0}{N}, I_0\right)\right|}{\left|\lambda^{\min}_{\beta,\gamma,\delta,\alpha}\left(\frac{S_0}{N}, I_0\right)\right|}. \tag{4}$$

4 Numerical Experiments

The numerical experiments were performed on a Intel Core i7-8565U CPU @ 1.80 GHz x 8 system with 16Gb RAM, running the Debian GNU/Linux operating system and Matlab 9.9. We consider a different initial data vector for each investigated country and we report them in Table 2. The value of S_0 is obtained from [2] as the percentual of the population which has access to internet, while E_0, I_0, and R_0 have been assumed for each country.

Table 2. Values of the initial vector for each country.

Country	S_0	E_0	I_0	R_0
Morocco	23059000	12941000	29089	0
Guatemala	6784000	9516000	46950	0
Argentina	34572000	9900000	22502	0
Thailand	39449000	29900000	79454	0
Portugal	7677900	2000000	605922	0

By following the numerical procedure described in Sect. 3 and taking into account the initial vectors in Table 2, we compute the different values of the stiffness ratio from (4) corresponding to the different countries; see Table 3.

Figure 2, 3, 4, 5, 6 contain, on the right, a graph of the behavior of each population involved in our SEIR model obtained by the standard Matlab built-in

Table 3. Values of the stiffness ratio for each country and their corresponding parameters.

Country	$r_{\beta,\gamma,\delta,\alpha}\left(S_0, \frac{I_0}{N}\right)$
Morocco	34.5155
Guatemala	65.5248
Argentina	27.2741
Thailand	14.1569
Portugal	71.9384

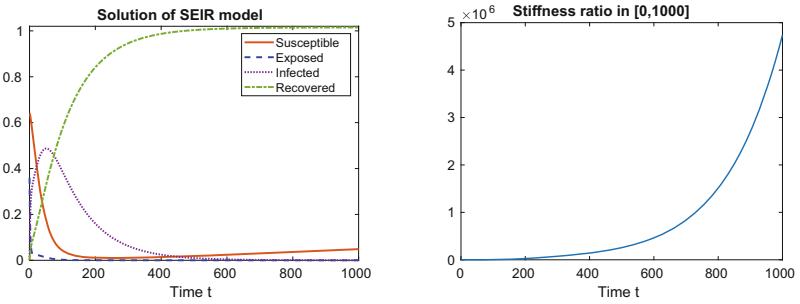


Fig. 2. Solution for Morocco to the SEIR model (left) together with the corresponding stiffness ratio (right).

function ode23s. On the left, we have reported the pattern of the ratio $r_{\beta,\gamma,\delta,\alpha}$ at time t. For each numerical simulation, we investigate problem (1) on the interval time $[0, 1000]$, where the time unit is measured in days. Each plot confirm that when the stiffness index is higher (see Table 3), the transition of fake news is faster.

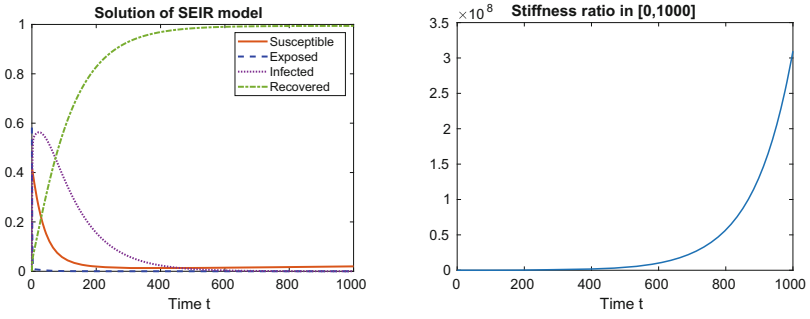


Fig. 3. Solution for Guatemala to the SEIR model (left) together with the corresponding stiffness ratio (right).

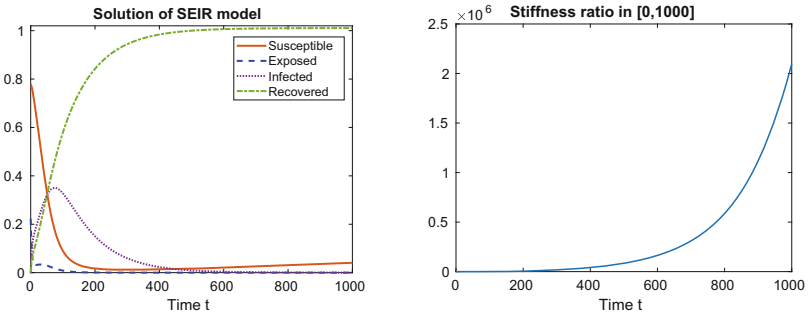


Fig. 4. Solution for Argentina to the SEIR model (left) together with the corresponding stiffness ratio (right).

The smaller stiffness ratio corresponds to Thailand (see Table 3), for which we can see in Fig. 5, that maximum value of the Infected population is smaller in comparison with the other cases. Moreover, the width on the curve shape represents the slow transmission of fake news in this country. On the contrary, the countries which have the highest stiffness value are Guatemala and Portugal. For the case of Guatemala we can see in Fig. 3 that the maximum of the Infected population is immediately acceded, while in the case of Portugal, see Fig. 6, even if is the country with the biggest stiffness index, the Infected population reaches the maximum later in time with respect to Guatemala. This could be justified by the literacy rate parameter which is higher for Portugal, which means that the portion of Exposed population that arrives to the Recovered compartment is bigger, and this fact could influence on the behavior of the Infected population.

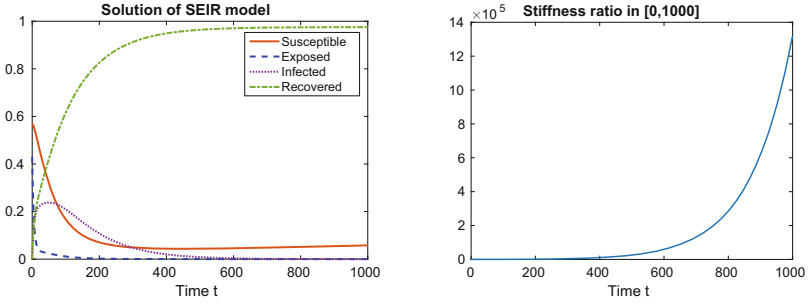


Fig. 5. Solution for Thailand to the SEIR model (left) together with the corresponding stiffness ratio (right).

After the obtained numerical results that show the relation between the stiffness index of model (1) and the behavior of the transmission of fake news, we have compared the solution of our SEIR model and the SIR model investigated in [12].

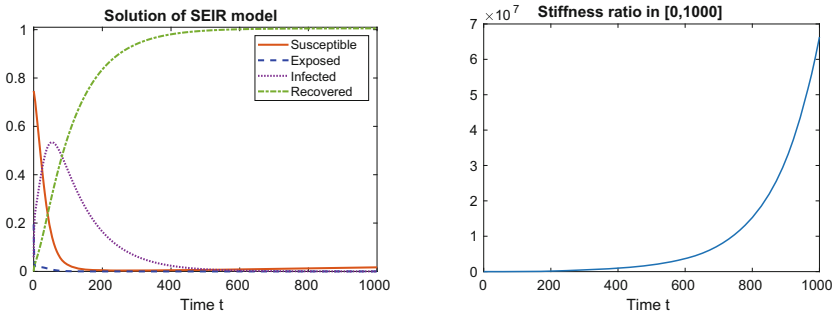


Fig. 6. Solution for Portugal to the SEIR model (left) together with the corresponding stiffness ratio (right).

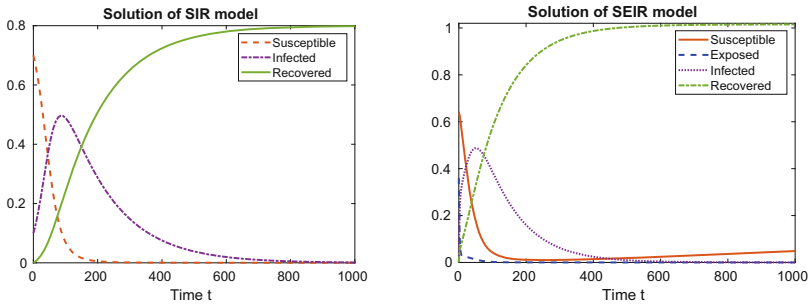


Fig. 7. Comparison between the solution of the SIR (left) and the SEIR (right) model for Morocco.

We can see in Fig. 7, 8, 9, 10, 11 that the Recovered/Removed population corresponding to the SEIR model takes values until 1, while the one corresponding to the SIR model studied in [12] takes values until 0.8. This happens because in our model, the Recovered/Removed compartment receives people from Exposed and Infected population.

Regarding the Infected population, the maximum of the curve for the SIR model is always for each country bigger than the maximum of the Infected people in the SEIR model. This is because of the structure of our compartmental model. In our scheme presented in Fig. 2, the Exposed population is divided in two groups, the Infected and the Recovered ones. This means that the proportion of population that arrives to the Infected one is smaller for the SEIR model than for the SIR model.

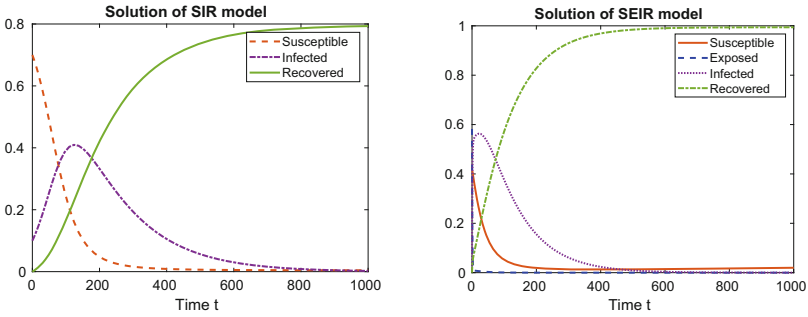


Fig. 8. Comparison between the solution of the SIR (left) and the SEIR (right) model for Guatemala.

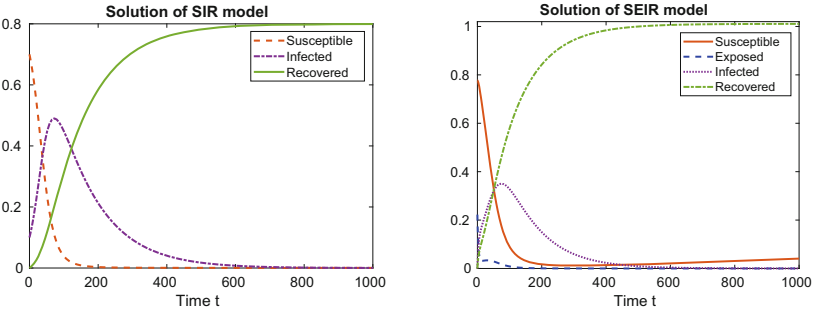


Fig. 9. Comparison between the solution of the SIR (left) and the SEIR (right) model for Argentina.

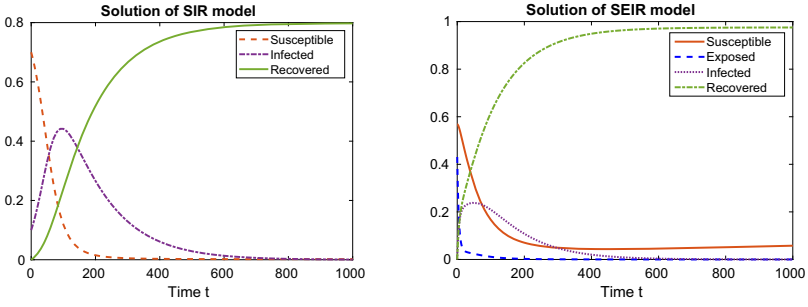


Fig. 10. Comparison between the solution of the SIR (left) and the SEIR (right) model for Thailand.

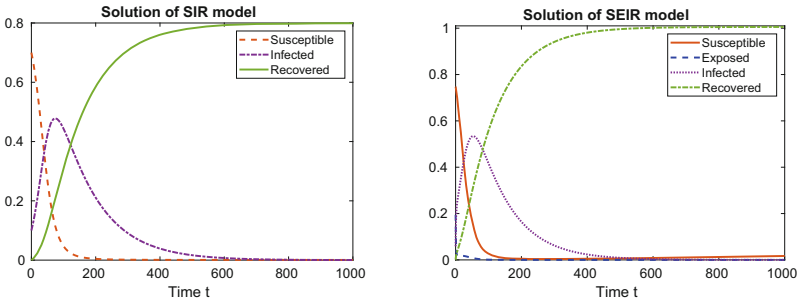


Fig. 11. Comparison between the solution of the SIR (left) and the SEIR (right) model for Portugal.

5 Conclusions and Future Developments

In this paper, we have proposed a study of the transmission of fake news in internet through a modified SEIR model. Stiffness analysis of the underlying differential problem helps predicting how fast the fake news diffusion is, as well as the corresponding truth re-affirmation. The numerical evidence, given on selected countries and based on real data, shows that the transmission of fake news is faster when the stiffness index of the model is larger, i.e., the higher the stiffness ratio, the faster the re-establishment of the truth after the diffusion of fake information.

Future developments of our work include the dynamical analysis of the SEIR model presented in this paper, recasted as a tool to understand the dynamics of fake information, as well as the stiffness analysis of more complex models which include other populations and parameters. Other challenging features of this problem are the study of sentimental analysis, social network analysis and applications of machine learning techniques; see [11,13].

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