

Resource Assignment Problem for Fleet Management Considering Outsourcing: Modelling and a Decomposition Approach

Filipe Monnerat[1,2], Joana Dias[1,3], and Maria João Alves[1,3(✉)]

[1] CeBER and Faculty of Economics, University of Coimbra, Av. Dias da Silva 165, 3004-512 Coimbra, Portugal
mjalves@fe.uc.pt
[2] Universidade Federal de Viçosa, Viçosa, Brasil
[3] University of Coimbra, INESC Coimbra, Rua Sílvio Lima, Pólo II, 3030-290 Coimbra, Portugal

Abstract. Many institutions have to manage internal and external resources to assure the realization of working trips. In this work we consider two different types of resources: vehicles and drivers. The assignment of these resources to trips is interlinked, since the number of drivers will depend on the number of assigned vehicles, and also on other characteristics of the trip itself. Furthermore, it is possible to improve the use of resources if they can be shared between compatible trips: trips that have compatible origin and destination pairs and are also compatible regarding the realization time periods. Instead of considering internal resources only, this work presents a new model where the possibility of outsourcing is also considered. This problem can be interpreted as an extension of the Generalized Assignment Problem, that is known to be NP-hard. An optimization approach is also proposed, based on the decomposition of the problem. Computational tests were performed to assess the solutions produced by the model and the behaviour of the decomposition approach. The computational results show that the developed model is capable of representing in a faithful way the problem, and the algorithmic approach presents a better performance when compared with a general solver applied to the whole problem.

Keywords: Combinatorial optimization · Generalized assignment problem · Fleet management · Outsourcing

1 Introduction

This work addresses a fleet management problem, where vehicles and drivers must be assigned to working trips that have to take place in predetermined time periods. These trips have also predetermined routes, so routing optimization does not need to be considered. The objective is to determine resource assignment minimizing total cost. One of the challenges of this problem is the fact that the assignment of drivers and vehicles cannot be done independently of each other. The number of vehicles needed

© Springer Nature Switzerland AG 2021
O. Gervasi et al. (Eds.): ICCSA 2021, LNCS 12953, pp. 260–273, 2021.
https://doi.org/10.1007/978-3-030-86976-2_18

depends on the vehicle availability and on the number of passengers that will travel. The number of drivers depends not only on the number of vehicles assigned but also on the trip's characteristics: a long trip may require more than one driver per vehicle in order to respect legal and safety obligations, for instance.

In a previous work, a similar fleet management resource assignment problem has already been tackled [1]. However, only internal resources were considered, which does not fully illustrate the situation in most institutions. In the present work, we consider the possibility of both internal and external resources being used, making explicit the possibility of resorting to outsourcing whenever that is needed (because no internal resources are available) or it is beneficial in terms of cost minimization. Furthermore, we also propose a decomposition method for the calculation of solutions that takes advantage of the characteristics of the problem at hand, but that can also be applied to other problems with similar structures. This decomposition method leads to an optimal solution of the problem provided that the subproblems are solved to optimality.

The motivation for this work came from the identification of this problem in a real institution (a public Brazilian university) and the need to properly address it from a resource management point of view. Several data sets were created, with different characteristics, aiming to assess the solutions calculated for the developed model and the behaviour of the decomposition optimization algorithm proposed. This approach is compared with the use of a general solver for the entire model (without decomposition).

Although there is an extensive published literature related to fleet management problems, a large part is focused on vehicle routing problems. Souza et al. [2] considered the vehicle routing problem for a patient transportation system, using a heterogeneous vehicle fleet. Lukman et al. [3] studied the fleet management optimization problem for winter services' resource assignment, having as starting point the Chinese postman problem. Hoff et al. [4] studied the industrial aspects of fleet composition combined with routing in maritime and rail transport, giving an overview of the combined problems of routing and composition of the fleet in transport. The authors show the importance of the subject and the difficulties associated with solving this type of problems, concluding that most studies discuss tactical decision-making and investigate the use of metaheuristics due to the difficulty of solving larger instances. An identification of the most relevant problems in fleet management for different means of transport was carried out in [5] referring to problems of routing and scheduling of vehicles, dynamic fleet management, private and public urban transport, dial-to-ride transport and specific problems related to air, sea, rail and intermodal transport. Loxton et al. [6] considered the problem of forming a new heterogeneous fleet of vehicles. The authors developed an algorithm based on dynamic programming to determine the fleet composition that minimized cost. A short literature review on the fleet sizing problem can be found in [7]. More recently, a heterogeneous fleet management system deployed to collect waste with each route being able to start and end in different warehouses has been developed and is described in [8]. The dynamic fleet management problem is also studied in [9], considering explicitly uncertain demand and service levels chosen by the customer. Billhardt et al. [10] address dynamic fleet management in real-time in an event-based architecture with simultaneous task allocation and vehicle allocation. Fleet management problems in the airline industry have also been studied [11, 12]. In a recent work, Zhang et al. [13] studied the fleet management

problem considering not only the determination of vehicles' utilization schedules but also decisions regarding vehicles' purchasing and retirement schedules. Dauer and Prata [14] studied the vehicle allocation and trip scheduling problem assuming the existence of multiple depots, a heterogeneous fleet and the existence of time windows.

The work presented here differs from the works cited. The set of vehicles available is known, the fleet is heterogeneous and previously determined. All the defined trips must take place, with scheduling and routing decisions already made. Each trip is a route that starts and ends at the origin (the institution) and has predetermined stops. The goal is to optimize the assignment of vehicles and drivers to the planned trips, within a defined planning horizon, allowing for outsourcing (acquisition of full transport service for a trip), so that costs are kept as low as possible. This problem can thus be interpreted as an extension of the generalized assignment problem (GAP), which is NP-hard. However, it differs from the multi-resource GAP since a task (trip) needs two separate resources (vehicles and drivers), and the requirements of one of the resources (drivers) will depend on the number of assigned units of the other resource (vehicles). Moreover, tasks do not take place all at the same time, so the time limits in which they occur must be considered when considering resource capacity limitations and sharing possibilities.

The main contributions of this work are the description of a resource assignment problem for fleet management considering internal and external resources, and the development of a decomposition approach that is able to calculate optimal solutions for this problem and can also be applied to problems with a similar structure (an implicit timeline that affects the way resource assignment can be done).

This manuscript is organized as follows. The proposed model is presented in Sect. 2. Section 3 describes the decomposition approach. Section 4 presents the computational experiments and results. Section 5 presents some conclusions.

2 The Proposed Model

The model considers the simultaneous assignment of vehicles and drivers to travel requests, as well as the possibility of having trips with no internal resources assigned and resorting to outsourcing. For these requisitions, the number of passengers, the destination and the schedule are predefined. The number of drivers that must be considered for every vehicle used is also known.

This model follows closely the one described in [1], except for the outsourcing modelling that is new in the model presented herein.

Considering the existing information about the destination and schedule of the trips, two data matrices with binary elements must be created. One is the compatibility matrix, which indicates whether two requests are compatible in time and space (they have the same destination or one of them is on the route of the other trip), and therefore can share resources. The other one is a non-overlapping matrix that indicates whether two requests have no period of time in common (i.e., do not overlap). These matrices determine whether the same resource can be assigned to two different trips: if the trips are not compatible, the same vehicle/driver cannot be assigned to both trips, unless they occur in different periods of time. If these trips do not overlap regarding their realization time periods, then the same resource can be used for both. If the trips are compatible, they can share resources (vehicles and drivers).

As compatible trips correspond to situations that share the same route, or one trip is a sub-route of another one, to simplify the constraints of the model it is assumed that all the trips are ordered by increasing costs of drivers, vehicles and outsourcing. Since these costs are mainly related to distance and travel time, it is realistic to assume that the same order applies to the different costs (drivers, vehicles and outsourcing) and even to the number of drivers required for each vehicle. Therefore, this assumption does not limit the application of the model.

The notation used is defined as follows:

$i = 1, \ldots, V$ is the index used for vehicles, with V the total number of vehicles owned by the institution.

$j = 1, \ldots, J$ is the index used for trips, with J the total number of trips to consider in the defined planning horizon.

$m = 1, \ldots, M$ is the index for existing drivers, employed by the institution, with M the total number of existing drivers.

C_{ij} = cost of assigning vehicle i to trip j, $\forall i, j$

D_{mj} = cost of assigning driver m to trip j, $\forall m, j$

F_j = cost of using outsourcing for the realization of trip j, not using any other internal resources, $\forall j$

P_j = total number of passengers associated with trip j, $\forall j$

Q_i = number of seats in vehicle i, $\forall i$

n_j = number of drivers per vehicles needed for trip j, $\forall j$

Compatibility matrix: $A = [a_{jj'}]$, $\forall j, j' : j < j'$

$$a_{jj'} = \begin{cases} 1, \text{ if trips } j \text{ and } j' \text{ are compatible, meaning they} \\ \text{can share resources} \\ 0, \text{ otherwise} \end{cases} \quad ,\forall j, j' : j < j'$$

Non-overlapping matrix: $B = [b_{jj'}]$, $\forall j, j' : j < j'$

$$b_{jj'} = \begin{cases} 1, \text{ if trips } j \text{ and } j' \text{ do not have any intersection} \\ \text{in their realization timings} \\ \\ 0, \text{otherwise} \end{cases} \quad ,\forall j, j' : j < j'$$

We should note that, by definition, $a_{jj'} + b_{jj'} \leq 1, \forall j, j'$.

The decision variables represent the assignment of resources to trips.

$$x_{ij} = \begin{cases} 1, \text{ if vehicle } i \text{ is assigned to trip } j \\ 0, \text{ otherwise} \end{cases}, \forall i, j$$

$$z_{ijj'} = \begin{cases} 1, \text{if trips } j \text{ and } j' \text{ share vehicle } i \\ 0, \text{ otherwise} \end{cases} \forall i, j, j' : j < j'$$

$$Z'_{jj'} = \begin{cases} 1, \text{ if trips } j \text{ and } j' \text{ share any vehicle} \\ 0, \text{ otherwise} \end{cases} \forall j, j' : j < j'$$

$$X'_{ij} = \begin{cases} 1, \text{if vehicle } i \text{ is assigned to trip } j \text{ and to other compatible trips} \\ \text{sharing resources with } j \\ 0, \text{otherwise} \end{cases} , \forall i, j$$

$$Y'_{mj} = \begin{cases} 1, \text{if driver } m \text{ is assigned to trip } j \text{ and to compatible} \\ \text{trips that use the same vehicle} \\ 0, \text{otherwise} \end{cases}, \forall m, j$$

$$O_j = \begin{cases} 1, \text{if trip } j \text{ does not use internal resources and its realization} \\ \text{is guaranteed by outsourcing} \\ 0, \text{otherwise} \end{cases}, \forall j$$

The model can be formulated as follows:

Objective function (1) guarantees the minimization of total costs. Taking advantage of the ordering of the trips by increasing cost it is possible to assure an adequate accounting of the proper costs to consider whenever there are shared resources between different trips. The first three terms of (1) concern, respectively, vehicle, driver and outsourcing costs. The last term is a perturbation term (with ε a positive very small number), which aims to avoid unnecessary sharing in case of alternative optimal solutions.

$$\begin{aligned} Min \sum_{i=1}^{V} \sum_{j=1}^{J} C_{ij} X'_{ij} + \sum_{m=1}^{M} \sum_{j=1}^{J} D_{mj} Y'_{mj} + \sum_{j=1}^{J} F_j O_j \\ + \varepsilon \sum_{i=1}^{V} \sum_{j=1}^{J} \sum_{j':j<j'}^{J} z_{ijj'} \end{aligned} \quad (1)$$

Subject to:

Constraints (2) guarantee the coherence between decision variables X'_{ij} and x_{ij} : a vehicle i can be assigned to trip j and other compatible trips that share the same vehicle ($X'_{ij} = 1$) only if vehicle i is assigned to trip j ($x_{ij} = 1$). Constraints (3) and (4) assure the coherence between decision variables $Z'_{jj'}$ and $z_{ijj'}$: $Z'_{jj'}$ must be equal to 1 if trips j and j' share at least one vehicle, but it must be equal to 0 if j and j' do not share any vehicle.

$$X'_{ij} \leq x_{ij}, \quad \forall i, j \quad (2)$$

$$Z'_{jj'} \geq \frac{1}{V} \sum_{i=1}^{V} z_{ijj'}, \quad \forall j, j' : j < j' \quad (3)$$

$$Z'_{jj'} \leq \sum_{i=1}^{V} z_{ijj'}, \quad \forall j, j' : j < j' \quad (4)$$

Constraints (5) guarantee that every passenger in every trip has an available seat in one of the vehicles assigned to the respective trip. Passengers can be divided by different vehicles, and one vehicle can transport passengers from different trips. Guaranteeing that enough seats do exist is only necessary if outsourcing is not used. If outsourcing is used, it is implicitly guaranteed that all the necessary resources will be available. $\mathbb{M}$ represents a sufficiently large positive number.

$$\sum_{i=1}^{V} Q_i x_{ij} + \mathbb{M} O_j \geq P_j + \sum_{j':j<j'}^{J} P_{j'} Z'_{jj'}, \quad \forall j \quad (5)$$

Constraints (6) establish the coherence in the assignment of each vehicle to different trips: a vehicle i can only be assigned to two different trips j and j' if they are compatible

($a_{jj'} = 1$) or if they do not have any overlap in their realization time periods ($b_{jj'} = 1$); otherwise, the vehicle i can only be assigned to one of the trips.

$$x_{ij} + x_{ij'} \leq \left(a_{jj'} + b_{jj'}\right) + 1, \forall i, j, j^{'} : j < j^{'} \tag{6}$$

Constraints (7) state that two trips j and j' cannot share a vehicle i ($i.e., z_{ijj'} = 0$) if they are not compatible ($a_{jj'} = 0$) or they do not have any time overlapping ($b_{jj'} = 1$); otherwise, these trips can share vehicle i provided that this vehicle is assigned to both trips.

$$z_{ijj'} \leq \left(1 - b_{jj'}\right) a_{jj'} \frac{x_{ij} + x_{ij'}}{2}, \forall i, j, j^{'} : j < j^{'} \tag{7}$$

Constraints (8) ensure that $z_{ijj'} = 1$ (the vehicle i is shared by j and j') for trips compatible and using the same vehicle.

$$z_{ijj'} \geq a_{jj'} + x_{ij} + x_{ij'} - 2, \forall i, j, j^{'} : j < j^{'} \tag{8}$$

Constraints (9) guarantee that the necessary number of drivers are assigned to each trip, whenever outsourcing is not used.

$$\sum\nolimits_{m=1}^{M} Y'_{mj} \geq n_j \sum\nolimits_{i=1}^{V} X'_{ij} - \mathbb{M} O_j, \forall j \tag{9}$$

Constraints (10) assure that a given driver m can only be assigned to two trips j and j' if they share some vehicle(s) or those trips do not have any temporal intersection ($b_{jj'} = 1$).

$$Y'_{mj} + Y'_{mj'} \leq \sum\nolimits_{i=1}^{V} z_{ijj'} + b_{jj'} + 1, \forall m, j, j^{'} : j < j^{'} \tag{10}$$

Constraints (11) and (12) guarantee the correct consideration of resource assignment costs. For all the trips that share the same vehicle i, variable $X_{ij}^{'}$ is equal to one only if j represents the most expensive trip (it corresponds to the larger index j). Constraints (12) together with (9) ensure the proper assignment of Y'_{mj} (drivers) whether the trips share the same vehicle or not. In the case of sharing, the driver is assigned to the trip with the longest journey. If outsourcing is used to realize a given trip, then no internal resources need to be assigned. This is ensured by constraints (13).

$$X_{ij}^{'} \geq x_{ij} - \sum\nolimits_{j^{'}: j < j^{'}}^{J} z_{ijj'}, \forall i, j \tag{11}$$

$$Y'_{mj} \leq 2 - z_{ijj'} - Y^{'}_{mj'}, \forall i, m, j, j^{'} : j < j^{'} \tag{12}$$

$$x_{ij} + O_j \leq 1, \forall i, j \tag{13}$$

All variables are binary.

$$x_{ij}, X_{ij}^{'} \in \{0, 1\}, \forall i, j$$

$$Y'_{mj} \in \{0, 1\}, \forall m, j$$

$$z_{ijj'} \in \{0, 1\}, \forall i, j, j'$$

$$Z'_{jj'} \in \{0, 1\}, \forall j, j'$$

$$O_j \in \{0, 1\}, \forall j$$

3 Problem Decomposition

The first attempt made to calculate the optimal solution for instances considering the described model used a general solver, Cplex. Whilst Cplex is capable of significantly reducing the number of variables in the pre-processing stage, by a priori fixing some of these variables, the optimality gaps obtained where large and the computational times were also significant, as it will be shown in Sect. 4. The difficulties experienced by the general solver motivated the development of a dedicated optimization procedure.

The main idea was to take advantage of the timeline that implicitly exists in the data, and that allows us to understand whether trips can be compatible and can share resources or they do not have any intersections considering their realization time periods. Actually, if it is possible to determine disjoint sets such that no trip in one set has any time intersection with any other trip in the other set, then it is possible to decompose the original instance into two smaller instances that can be easier to solve using a general solver.

This decomposition is made in an automatic and iterative way, trying to balance as much as possible the number of trips that are placed in each set. This can be done resorting to the resolution of an auxiliary linear integer programming problem that takes only seconds to solve. The procedure is repeated for every subset found, until it is not possible to subdivide these sets any more, or the number of trips in each set is already very small. It is possible to end up with a decomposition tree of the problem instance. Figure 1 depicts an example of a decomposition tree for one of the generated instances. The instance has a total of 122 trips. This initial set was divided in two smaller sets, both with 61 instances. The subset on the left was further divided in two smaller sets, one with 47 trips (that was not possible to subdivide any more) and another with 14. The procedure continues until the leaves in the decomposition tree represent sets that should not or cannot be further subdivided.

Let us consider R as representing the set of all the trips of the original instance. The objective is to decompose R into two subsets $R1$ and $R2$. According to the non-overlapping matrix B of the original instance, $b_{jj'} = 1$ means that there is no time overlapping between trips $j, j' \in R$, and $b_{jj'} = 0$ means that some time overlapping does exist between the two trips. If there is time overlapping, then the trips have to be placed in the same set (independently of having or not shared resources assigned to them later on). This is the only way to assure that the global solution that will come up from the resolution of all these subproblems can indeed be the optimal solution for the original

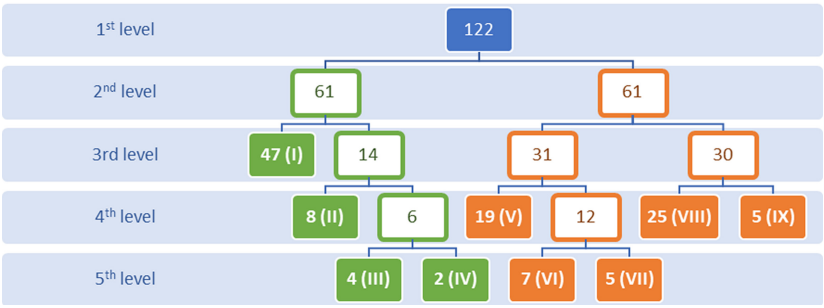

Fig. 1. Example of a decomposition tree

instance. Then, $b_{jj'}$ has to be equal to one for every trip pair (j, j') such that $j \in R1$ and $j' \in R2$. If $b_{jj'} = 0$, then trips j and j' will have to be placed together either in $R1$ or $R2$.

Let $|R1| = \alpha$ and $|R2| = \beta$, such that $\alpha + \beta$ is the total number of trips in R. Let x_j be a binary variable that is equal to 1 if $j \in R1$ and is equal to 0 if $j \in R2$.

The Decomposition Problem (DP) can be defined as follows:

$$Min(\alpha - \beta) \tag{14}$$

subject to:

$$x_j \leq x_{j'} + b_{jj'} \quad \forall j < j' \tag{15}$$

$$x_j \geq x_{j'} - b_{jj'} \quad \forall j < j' \tag{16}$$

$$\sum_{j \in R} x_j = \alpha \tag{17}$$

$$\sum_{j \in R} (1 - x_j) = \beta \tag{18}$$

$$\alpha \geq \beta \tag{19}$$

$$x_j \in \{0, 1\} \quad \forall j \in R$$

$$\alpha, \beta \geq 0$$

The objective function (14) minimizes the difference $(\alpha - \beta)$ and constraint (19) defines $\alpha \geq \beta$, but the objective and the constraint could also be defined as the opposite: min $(\beta - \alpha)$ with $\alpha \leq \beta$. The aim is to keep the subsets as balanced as possible, so the objective function is defined by the minimization of the non-negative difference of the sizes of the subsets, α and β, with α and β values defined in (17) and (18).

Constraints (15) and (16) guarantee the non-overlapping character of all the trips belonging to different sets. Thus, $b_{jj'} = 0 \Rightarrow x_j = x_{j'}$. If there is some overlap between

trips j and j', then these trips will belong to the same set: either they are both placed in $R1$ ($x_i = x_j = 1$), or in $R2$ ($x_i = x_j = 0$). If $b_{jj'} = 0$, constraints (15) and (16) can be interpreted as $x_j \leq x_{j'}$ and $x_j \geq x_{j'}$; therefore $x_j = x_{j'}$. On the other hand, if $b_{jj'} = 1$, trips j and j' can either be placed in the same set or in different sets. If this is the case, these constraints are redundant since $x_j \leq x_{j'} + 1$ and $x_j \geq x_{j'} - 1$ are always true.

The subproblems that correspond to the leaves of the decomposition tree can be solved using a general solver, e.g. the Cplex. These subproblems are defined by the same model presented in the previous section, but considering the set of trips that belong to the corresponding subset. It is possible to calculate a solution to the original instance, and the corresponding value for the objective function, by considering all the optimal values for the decision variables (the union of all the decision variables in all the instances contains all the original decision variables). The global objective function value can be calculated by the sum of all the subproblems' objective function values. If some or all of the subproblems are not solved to optimality, it is possible to calculate the maximum optimality gap. Let K be the number of solved subproblems, and $Cost_k$ the objective function value obtained for subproblem $k = 1, \ldots, K$ to which corresponds a gap equal to gap_k. Knowing that the relative gap given by the solver is calculated as $\frac{|bestbound - bestinteger|}{bestinteger}$, then $gap_k = \frac{Cost_k - Bestbound_k}{Cost_k} \iff Bestbound_k = (1 - gap_k)Cost_k$. The total cost of the solution for the original instance is $Total\ Cost = \sum_{k=1}^{K} Cost_k$, with a maximum gap equal to $gap = \frac{TotalCost - \sum_{k=1}^{K} Bestbound_k}{TotalCost}$.

Summing up, the computational times associated with the resolution of each one of the subproblems leads to the total computational time of this approach (the computational times of the auxiliary decomposition problems DP are negligible).

4 Computational Experiments

4.1 Experimental Setting

The instances were generated having as starting point the case study described in [1]. Considering the real data of a public Brazilian university, a total of 122 trips was considered (corresponding to a normal working month), with the following available internal resources: 28 vehicles with 9 different passenger capacities; 24 drivers, each one belonging to one of two different sets, according to the corresponding working contracts and the impact of these contracts on the assignment cost of a driver to a trip. The trips are all taking place within a radius of 600 km from the origin (the institution itself). The average total distance travelled is equal to 440 kms (with trips ranging from 97 to 1067 kms). The trips have different number of passengers associated, between 1 and 52. The number of drivers per vehicle that has to be considered is either 1 or 2, depending on the travelled distance. From this original set, two groups with a total of 40 instances each were then created. The first group contains 20 instances (A-T) and considers a vehicle fleet equal to the set of the base instance. The second group also contains 20 instances (A-T) but considers a vehicle fleet composed of 16 vehicles of lower capacity only (up to 5 passengers). Within each group, some of the data were randomly generated (namely the time periods associated with the realization of each trip) to test the DP with different

trips' compatibility and non-overlapping characteristics, but always guaranteeing the coherence of the data. Instances within each group (1A to 1T; 2A to 2T) were built with an increased number of trips' overlapping. This means that instances identified as A present less time overlapping than the other instances of the same group, and this overlapping increases from A to T. The compatibility between two trips depends both on the time compatibility and on the spatial compatibility (i.e., destinations). Increasing time overlapping from A to T increases the possibility of compatibility between trips but decreases de degree of decomposition by the DP approach. Table 1 summarizes this information.

Table 1. Percentage of compatible trips (*CT*) in the different generated instances

Instance	A	B	C	D	E	F	G	H	I	J
% CT	2.40	2.70	3.30	3.80	3.90	5.70	6.40	7.10	7.10	7.90
Instance	K	L	M	N	O	P	Q	R	S	T
% CT	9.60	9.80	10.90	10.90	11.30	14.30	14.70	16.70	19.90	22.10

Instances built in this way allow to assess the impact of trip compatibility in the sharing of resources and the use of outsourcing. Random data generation associated with each instance considered the following:

- Each vehicle has an associated cost per km travelled, randomly calculated considering the size of the vehicle and considering as base the data obtained from a specialized company [15] from the city of São Paulo.
- For group 1, the number of seats of each vehicle was randomly generated, considering a variation of + 1 or −1 from the original data. For group 2, only the smallest vehicles were considered.
- The number of passengers associated with each trip is generated from the original dataset, considering random changes between + 1 or −1.
- The outsourcing costs were calculated based on the mean cost of the vehicles with smaller capacity ($\overline{c}_{vs}$), plus the average cost of drivers ($\overline{D}_m$) multiplied by the needed number of drivers for each trip (n_j) and divided by the average number of available seats in the smaller vehicles (Q_{vs}). This reference value is then multiplied by a number randomly generated (σ) within the interval [–0,10; +0,30]. The obtained result is then multiplied by the number of passengers associated with each trip (P_j). This means that $F_j = \left(\frac{\overline{C}_{vs} + \overline{D}_m \times n_j}{Q_{vs}} \right) \times \sigma \times P_j, \forall j.$

Complete data of all the problems are available at:
https://www.uc.pt/en/feuc/mjalves/DataFleetManagement/data_40inst

The computational experiments were run in an Intel Xeon Silver 4116, 2.1 GHz, 12-core CPU, 128 GB RAM. The general solver used was Cplex, with a time limit of 18000 s for all the instances to be solved. To be able to compare the behaviour of the

general solver applied to the whole problem and the decomposition method, whenever the decomposition approach needed more than 18000 s in total, Cplex was run again for the whole problem considering exactly the same total computational time.

4.2 Computational Results

We will first compare the times and objective function values (costs and gaps) obtained with the decomposition method and with the general solver. Then, the characteristics of the solutions will also be analysed.

Considering the instances from group 1, both Cplex and the decomposition approach were able to find the optimal solution in only 3 instances out of 20. It is possible to conclude that both methods have increased difficulties in solving the problems as the time overlapping increases, presenting successively larger gaps (the gap is greater than 80% for 1T). It was not possible to find the optimal solution for instances belonging to group 2, within the computational time limits defined. The same pattern of having an increased gap with an increased time overlapping of the trips is also observed, but the gaps obtained are lower than the ones obtained for group 1. For both groups, the decomposition method was able to find better solutions than Cplex in most of the instances, and it was able to produce better bounds for solutions with gap different from 0. However, the decomposition approach still leads to solutions with a significant optimality gap in instances where trips have a large overlapping. Tables 2 and 3 summarize these results.

Analysing the obtained solutions, it is possible to classify the assignment of resources to trips into three different classes: trips that share resources, trips that do not share resources and the ones that are outsourced. In group 1, on average, 83.44% of the trips shared resources, with this value varying between 69.67% and 94.26%. Outsourcing was used on average in 5.7% of the trips, reaching a maximum of 28.69% in one of the instances. Outsourcing shows a trend of increasing with the increase of the compatibility and time overlapping between trips. Actually, this increase in the compatibility and time overlapping can have two distinct consequences: until a certain point it will promote resource sharing; however, beyond a certain level, there are too many trips taking place at the same time and there are not enough resources to be able to cope with all the trips.

Instances in group 2 are characterized by having a homogeneous fleet of small vehicles. In this setting, it is necessary to resort to outsourcing much more frequently: on average 28.20% of the trips are outsourced, being the largest percentage equal to 47.54% for instance 2T. As the number of available seats per vehicle is small, the use of shared resources is limited even when there are compatible trips. It is more often used when all of the compatible trips have a low number of passengers. If this number increases, then the use of more own vehicles to assure the needed capacity turns out to be more expensive than outsourcing the trip. Whilst it is not possible to compare these solutions with solutions manually defined by the people in charge of this assignment, since some of the data have been randomly generated, we have already concluded that the use of this type of models leads to better solutions than the ones that are manually created for cases in which outsourcing was not considered [1]. Adding an extra degree of freedom (outsourcing) makes this problem even harder to solve manually, so we believe this is a valuable tool for organizations that have to deal with similar situations.

Table 2. Computational results obtained for Group 1 instances.

	Resolution of the whole problem by CPLEX				Decomposition method			
	Total time	Gap	Best solution	Best bound	Total time	Gap	Best solution	Best bound
1A	18000	0.66%	**31976.48**	31764.85	141	0.00%	**31976.48**	-
1B	276	0.01%	**30391.12**	30388.09	63	0.00%	**30391.12**	-
1C	18000	2.29%	**27894.15**	27256.47	15083	0.00%	**27894.15**	-
1D	25522	8.20%	**27803.4**	25524	25519	4.35%	27898.92	26686.07
1E	20021	4.30%	**29064.64**	27813.72	20017	1.76%	31192.36	30643.5
1F	54233	14.53%	25893.51	22130.57	54225	10.32%	**25417.85**	22795.84
1G	72707	24.43%	27115.35	20491.62	72695	18.53%	**26186.32**	21335.22
1H	54011	22.98%	27291.7	21021.09	54005	16.52%	**25867.43**	21594.41
1I	55315	27.97%	25802.17	18585.39	55314	19.86%	**24632.85**	19740.05
1J	37909	41.87%	26409.92	15352.56	37907	36.94%	**25667.31**	16186.59
1K	48158	40.09%	25699.41	15396.74	48156	31.64%	**23786.58**	16259.87
1L	67915	46.71%	28061.11	14953.62	67913	41.13%	**27567.88**	16229.4
1M	54025	47.45%	**27329.03**	14361.62	54010	44.18%	27461.56	15329.47
1N	83545	51.03%	23328.49	11424.56	83534	42.76%	**22378.8**	12810.28
1O	36038	55.02%	31715.1	14265.6	36036	51.14%	**29391.53**	14361.17
1P	36048	66.94%	37803.61	12497.98	36048	62.37%	**35402.92**	13320.81
1Q	36038	55.29%	29231.47	13069.63	36035	54.10%	**28746.54**	13194.22
1R	36007	70.27%	36607.29	10884.84	36007	68.49%	**34716.5**	10939.08
1S	36007	80.72%	**38410.77**	7403.76	36002	83.82%	44618.06	7217.68
1T	36005	82.53%	**44704.04**	7811.1	36004	82.37%	45873.14	8089.7

Finally, we would like to emphasise that these problems are difficult to solve but the use of a general solver has revealed to be more efficient than dedicated metaheuristics. The development of a metaheuristic for this problem is a highly complex task, due to the difficulty of guaranteeing solution feasibility. We have extended to the proposed model a *matheuristic* previously developed for the model without outsourcing [1]. However, the solutions obtained were worse than those obtained by Cplex. This observation motivated the development of the deterministic decomposition approach proposed in this paper.

Table 3. Computational results obtained for Group 2 instances.

	Resolution of the whole problem by CPLEX				Decomposition method			
	Total time	Gap	Best solution	Best bound	Total time	Gap	Best solution	Best bound
2A	16482	4.43%	67568.92	64575	18766	0.35%	**67465**	67232.1
2B	15582	6.76%	66131.35	61663.83	37200	0.66%	**66085.9**	65646.57
2C	37474	7.77%	65707.65	60603.58	37443	4.06%	**65622.18**	62959.46
2D	30898	16.95%	70947.14	58924.48	73070	8.20%	**70162.15**	64408.76
2E	52159	13.97%	70250.19	60436.07	52140	8.73%	**70097.85**	63980.04
2F	108093	26.66%	71599.11	52508.05	108076	16.26%	**70797.88**	59283.14
2G	72484	31.29%	68156.21	46829.34	72468	20.18%	**67327.06**	53742.24
2H	54035	24.77%	69505.31	52291.42	54033	22.50%	**68654.86**	53205.79
2I	90530	28.29%	79655.33	57124.12	90505	20.03%	**76243.93**	60970.8
2J	72304	34.65%	80162.45	52384.68	72302	26.36%	**78503.18**	57809.16
2K	36772	38.70%	75930.69	46542.57	36767	30.06%	**74588.05**	52165.52
2L	72122	43.72%	74247.11	41783.63	72120	33.39%	**71520.26**	47641.03
2M	54056	45.73%	78290.2	42488.9	54055	39.52%	**74851.17**	45273.35
2N	72038	45.07%	79071.17	43436.64	72035	32.01%	**76591.46**	52074.12
2O	36168	41.87%	80247.75	46644.7	36166	34.39%	**76516.2**	50204.98
2P	36093	46.92%	80112.62	42521.84	36092	41.61%	**77427.41**	45207.15
2Q	37002	51.29%	69498.76	33854.7	37002	43.52%	**68482.77**	38681.25
2R	36030	50.89%	80475.72	39520.84	36005	46.58%	**78894.72**	42146.03
2S	36006	63.21%	79986.84	29423.82	36002	60.26%	**79859.43**	31738.97
2T	36043	53.86%	88228.4	40712.68	36001	47.49%	**87748.68**	46080.04

5 Conclusions

This work presents a model and algorithmic approach for an extension of the GAP: the problem of assigning two distinct resources (drivers and vehicles) to a set of tasks (trips), where the assignment of one resource is dependent on the assignment of the other. If the tasks are compatible, they can share resources. The model also includes the possibility of outsourcing. This work was motivated by the identification of an existing decision-making problem in a public Brazilian University.

This problem is NP-hard and it has proved to be very difficult to solve by using a general solver (Cplex). This motivated the development of a dedicated approach based on the decomposition of the problem. Trips in the original instance can be iteratively divided into smaller disjoint sets, guaranteeing that there is no time overlapping between trips that are in distinct sets. The decomposition approach was able to calculate, for most of

the instances, better solutions with lower gaps. Although the gaps obtained for instances with high overlapping between trips is still considerable, it is possible to conclude that the use of this model and the algorithmic approach can efficiently automate the decision-making process. It enables to reach solutions that are better than the solutions calculated manually promoting greater sharing of resources.

Acknowledgements. This work has been funded by national funds through FCT – Fundação para a Ciência e a Tecnologia, I.P., Projects UIBD/00308/2020 and UIDB/05037/2020. The first author has been supported by CAPES Proc. 009345/2013–02 BEX.

References

1. Monnerat, F., Dias, J., Alves, M.J.: Fleet management: a vehicle and driver assignment model. Eur. J. Oper. Res. **278**(1), 64–75 (2019). https://doi.org/10.1016/j.ejor.2019.03.021
2. Souza, A.L.S., Bernardo, M., Penna, P.H.V., Pannek, J., Souza, M.J.F.: Bi-objective optimization model for the heterogeneous dynamic dial-a-ride problem with no rejects. Optim. Lett. **5**, 1–20 (2021). https://doi.org/10.1007/s11590-020-01698-6
3. Lukman, R.K., Cerinšek, M., Virtič, P., Horvat, B.: Improving efficient resource usage and reducing carbon dioxide emissions by optimizing fleet management for winter services. J. Clean. Prod. **177**, 1–11 (2018). https://doi.org/10.1016/j.jclepro.2017.12.142
4. Hoff, A., Andersson, H., Christiansen, M., Hasle, G., Løkketangen, A.: Industrial aspects and literature survey: fleet composition and routing. Comput. Oper. Res. **37**, 2041–2061 (2010). https://doi.org/10.1016/j.cor.2010.03.015
5. Bielli, M., Bielli, A., Rossi, R.: Trends in models and algorithms for fleet management. Procedia - Soc. Behav. Sci. **20**, 4–18 (2011). https://doi.org/10.1016/j.sbspro.2011.08.004
6. Loxton, R., Lin, Q., Teo, K.L.: A stochastic fleet composition problem. Comput. Oper. Res. **39**, 3177–3184 (2012). https://doi.org/10.1016/j.cor.2012.04.004
7. Ertogral, K., Akbalik, A., González, S.: Modelling and analysis of a strategic fleet sizing problem for a furniture distributor. Eur. J. Ind. Eng. **11**, 49–77 (2017). https://doi.org/10.1504/EJIE.2017.081428
8. Markov, I., Varone, S., Bierlaire, M.: Integrating a heterogeneous fixed fleet and a flexible assignment of destination depots in the waste collection VRP with intermediate facilities. Transp. Res. Part B Methodol. **84**, 256–273 (2016). https://doi.org/10.1016/j.trb.2015.12.004
9. Shi, N., Song, H., Powell, W.B.: The dynamic fleet management problem with uncertain demand and customer chosen service level. Int. J. Prod. Econ. **148**, 110–121 (2014). https://doi.org/10.1016/j.ijpe.2013.09.010
10. Billhardt, H., et al.: Dynamic coordination in fleet management systems: toward smart cyber fleets. IEEE Intell. Syst. **20**, 70–76 (2014). https://doi.org/10.1109/MIS.2014.41
11. Hane, C.A., Barnhart, C., Johnson, E.L., Marsten, R.E., Nemhauser, G.L., Sigismondi, G.: The fleet assignment problem: Solving a large-scale integer program. Math. Program. **70**, 211–232 (1995). https://doi.org/10.1007/BF01585938
12. Abara, J.: Applying integer linear programming to the fleet assignment problem. Interfaces (Providence) **19**, 20–28 (1989). https://doi.org/10.1287/inte.19.4.20
13. Zhang, L., Gu, W., Fu, L., Mei, Y., Hu, Y.: A two-stage heuristic approach for fleet management optimization under time-varying demand. Transp. Res. Part E Logist. Transp. Rev. **147**, 102268 (2021). https://doi.org/10.1016/j.tre.2021.102268
14. Dauer, A.T., Prata, B.: Variable fixing heuristics for solving multiple depot vehicle scheduling problem with heterogeneous fleet and time windows. Optim. Lett. **15**(1), 153–170 (2020). https://doi.org/10.1007/s11590-020-01577-0
15. Tabelas de Frete Company: Jardim Paulista, São Paulo-SP, Brasil (2019). https://www.tabelasdefrete.com.br. Accessed 08 Aug 2019