Using Mathematica and 4Nec2 to Design and Simulation of an Antenna in 2100 and 3500 MHz Bands

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Abstract. The design of an efficient antenna allows the delocalization of electromagnetic energy making possible the transmission of wireless energy. In this paper, we describe the design of the model of an antenna with an efficiency of 99.06% and an SWR of 1.01 after the impedance matching process at the 3500 MHz frequency. This antenna is generated from control points on a curve lying on a lemniscatic torus and it's projected on the XY plane. To carry out the design we used the Mathematica symbolic calculation system. In addition, we propose two frequencies for simulation purposes with the assistance of the free software 4Nec2, which will allow us to see its main characteristics, one of 2100 MHz (0.1428 m wavelength) and another of 3500 MHz (0.1428 m wavelength).

Keywords: Design · Simulation · Antenna · Lemniscatic torus

1 Introduction

An antenna is a transitional structure between free space and a guiding device [1]. The importance of the design of an antenna lies in its efficiency since the design of an efficient antenna allows the delocalization of electromagnetic energy making possible the transmission of wireless energy [4]. The use of computer software to perform modeling and simulation of various objects is gaining current popularity due to the development of computers and the constant updating, as well as the development, of specialized software [9].

In this paper, we describe the design of the model of an efficient antenna generated from control points on a curve lying on lemniscatic torus [3] and its projection on the XY plane. To carry out the design we use the Wolfram Language [10] of the Mathematica v.11.0 symbolic calculation system [11]. In addition, we propose two frequencies for simulation purposes with the assistance of the free software 4Nec2 [2], which will allow us to see its main characteristics, one of 2100 MHz (0.1428 m wavelength) and another of 3500 MHz (0.1428 m wavelength).

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We believe it convenient to mention the importance of having a guide for the correct writing of research articles in disciplines related to ICT. This paper has been prepared following the recommendations proposed in [6].

The structure of this paper is as follows: Sect. 2 introduces the mathematical definition of some surfaces and curves based on which the antenna proposed in this paper will be designed. Then, Sect. 3 introduces the antenna design proposed in this paper. Here, in the Wolfram Language, the lemniscatic torus, its projection, and the curve on which are the control points for the antenna design are defined. The simulation of the designed antenna with the 4Nec2 software is performed in Sect. 4. Finally, Sect. 5 closes with the main conclusions of this paper and some further remarks.

2 Mathematical Preliminaries

2.1 Surfaces of Revolution

Let C be a curve lying on a plane $P \subset \mathbb{R}^3$, and let A be a line that also lies on P but that does not meet C. When this profile curve C is revolved around axis A, it sweeps out a surface of revolution M in $\mathbb{R}^3$ [7].

2.2 Torus of Revolution

The torus of revolution is the surface of the revolution obtained when the profile curve C is a circle, according to Subsect. 2.1 [8].

2.3 Lemniscate of Bernoulli

The lemniscate of Bernoulli is defined as the locus of points of a plane such that the product of distances from two fixed points, of the same plane, $(-A, 0)$ and $(A, 0)$, $A > 0$, is a constant A^2. The Cartesian equation obtained from this definition is $(x^2 + y^2)^2 = 2A^2(x^2 - y^2)$. The parametric equations for the lemniscate with half-width A are

$$x(t) = \frac{A \cos t}{1 + \sin^2 t};\ y(t) = \frac{A \sin t \cos t}{1 + \sin^2 t};\ 0 < t < 2\pi \quad [5].$$

2.4 Lemniscatic Torus

The parametric equations for Bernoulli's lemniscate lying on the $x = B$, $B \in \mathbb{R}$, plane are

$$x(v) = B;\ y(v) = \frac{A \cos v}{1 + \sin^2 v};\ z(v) = \frac{A \sin v \cos v}{1 + \sin^2 v};\ -\frac{\pi}{2} < v < \frac{3\pi}{2}.$$

Given

$$R(u) = \begin{pmatrix} \cos u & -\sin u & 0 \\ \sin u & \cos u & 0 \\ 0 & 0 & 1 \end{pmatrix},$$

the lemniscatic torus (Fig. 1) is defined by

$$LT(u,v) = R(u) \cdot (x(v), y(v), z(v))^{\top};\ 0 < u < 2\pi;\ -\frac{\pi}{2} < v < \frac{\pi}{2},$$

explicitly

$$LT(u,v) = \left(B\cos(u) - \frac{A\sin(u)\cos(v)}{\sin^2(v)+1}, \frac{A\cos(u)\cos(v)}{\sin^2(v)+1} + B\sin(u), \frac{A\sin(v)\cos(v)}{\sin^2(v)+1} \right);$$
$$0 < u < 2\pi;\ -\frac{\pi}{2} < v < \frac{\pi}{2}.$$

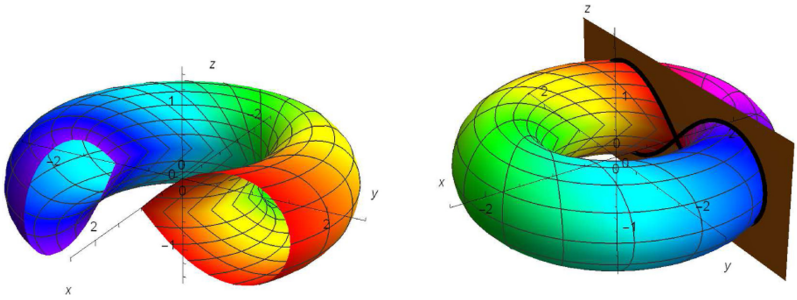


Fig. 1. Profile curve and the torus generated by a lemniscate (left), the lemniscate is also observed (right).

The families of basic curves inherent to the lemniscatic torus are the parallels, circumferences; and the meridians, lemniscate leaves.

2.5 Curves Lying on the Lemniscatic Torus

Curves lying on the lemniscatic torus are obtained by substituting the parameters u and v by differentiable functions $\alpha_1(t)$ and $\alpha_2(t)$, respectively. This is $LT(\alpha_1(t), \alpha_2(t))$.

3 Design of an Antenna Model with Mathematica

First, we define the lemniscatic torus in Mathematica:

```
LT[{u_,v_},{A_,B_}]:={B Cos[u]-(A Cos[v] Sin[u])/(1+Sin[v]^2),
  B Sin[u]+(A Cos[u] Cos[v])/(1+Sin[v]^2),
   (A Cos[v] Sin[v])/(1+Sin[v]^2)}
```

After compiling this definition it is possible to use the built-in `Parametric Plot3D` and `Show` functions to plot the lemniscatic torus and the curves lying on it (Fig. 2, left).

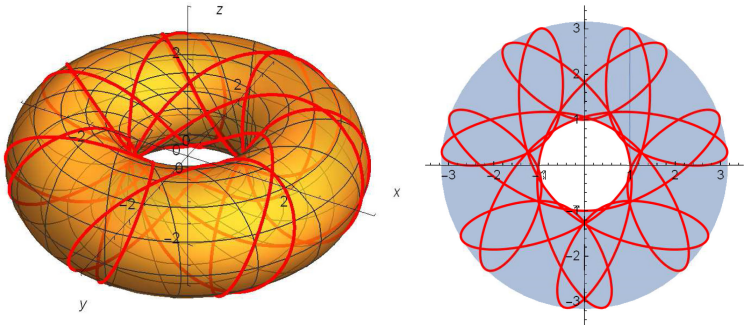


Fig. 2. A lemniscatic torus, `LT[{u,v},{3,1}]`, and a curve, `LT[{3t,7t},{3,1}]`, liying on it (left). Projections of the previous objects (right).

Next, we project the lemniscatic torus onto the XY plane. In Mathematica we have:

```
LTP[{u_,v_},{A_,B_}]:={B Cos[u]-(A Cos[v] Sin[u])/(1+Sin[v]^2),
  B Sin[u]+(A Cos[u] Cos[v])/(1+Sin[v]^2)}
```

After compiling this definition it is possible to use the built-in `ParametricPlot` and `Show` functions to plot the projections of the lemniscatic torus and the curves lying on it (Fig. 2, right).

Next, to facilitate later calculations, we define the curve, `cur2d`, as follows:

```
cur2d[t_]:=LTP[{3t,7t},{3,1}]
```

The domain of the curve is the interval $[0, 2\pi]$, we partition it into increments of $\frac{\pi}{35}$ and find the corresponding points on the curve:

```
m=cur2d/@Table[i,{i,0,2 Pi-Pi/35,Pi/35}];
```

We rearrange the point's list, `m`, and divide the coordinates of its elements by 100, taking into account that `m[[i]]` is the ith element of `m`.

```
data=m[[#]]&/@{46,45,63,39,60,67,18,64,12,11,66,65,13,59,
  10,17,38,14,32,31,16,15,33,9,30,37,58,34,52,51,36,35,53,
   29,50,57,8,54,2,1,56,55,3,49,70,7,28,4,22,21,6,5,23,69,20,
    27,48,24,42,41,26,25,43,19,40,47,68,44,62,61}/100;
```

See Table 1.

Table 1. Control points, stored in `data`, duly ordered and arranged in columns.

$(-0.0040, -0.031)$	$(-0.014, 0.015)$	$(0.031, 0.031)$
$(-0.0043, -0.020)$	$(-0.0077, 0.0081)$	$(0.021, 0.00026)$
$(-0.0014, -0.011)$	$(-0.0061, 0.0093)$	$(0.011, 0.0010)$
$(-0.0034, -0.011)$	$(-0.013, 0.016)$	$(0.011, -0.00095)$
$(-0.0066, -0.020)$	$(-0.017, 0.027)$	$(0.021, -0.0021)$
$(-0.011, -0.017)$	$(-0.010, 0.030)$	$(0.019, -0.0070)$
$(-0.0062, -0.0093)$	$(-0.0048, 0.020)$	$(0.010, -0.0040)$
$(-0.0077, -0.0080)$	$(-0.0035, 0.011)$	$(0.0095, -0.0057)$
$(-0.013, -0.016)$	$(-0.0015, 0.011)$	$(0.018, -0.0092)$
$(-0.022, -0.023)$	$(-0.0025, 0.020)$	$(0.027, -0.016)$
$(-0.027, -0.016)$	$(0.0025, 0.020)$	$(0.022, -0.023)$
$(-0.018, -0.0092)$	$(0.0015, 0.011)$	$(0.013, -0.016)$
$(-0.0095, -0.0057)$	$(0.0035, 0.011)$	$(0.0077, -0.0080)$
$(-0.010, -0.0040)$	$(0.0048, 0.020)$	$(0.0062, -0.0093)$
$(-0.019, -0.0070)$	$(0.010, 0.030)$	$(0.011, -0.017)$
$(-0.021, -0.0021)$	$(0.017, 0.027)$	$(0.0066, -0.020)$
$(-0.011, -0.00095)$	$(0.013, 0.016)$	$(0.0034, -0.011)$
$(-0.011, 0.0010)$	$(0.0061, 0.0093)$	$(0.0014, -0.011)$
$(-0.021, 0.00026)$	$(0.0077, 0.0081)$	$(0.0043, -0.020)$
$(-0.031, 0.0031)$	$(0.014, 0.015)$	$(0.0040, -0.031)$
$(-0.030, 0.011)$	$(0.018, 0.011)$	
$(-0.019, 0.0087)$	$(0.0096, 0.0057)$	
$(-0.010, 0.0039)$	$(0.010, 0.0039)$	
$(-0.0096, 0.0057)$	$(0.019, 0.0087)$	
$(-0.018, 0.011)$	$(0.030, 0.011)$	

The points in the *data* list are the control points for designing the antenna. Finally, we join the points of list `data` using straight lines and obtain the antenna design (Fig. 3).

```
lab=Table[Subscript[P,i],{i,Length[data]}];
g1=Graphics[{Blue,Thick,Line[data]}];
g2=Graphics[{Cyan,AbsolutePointSize[7],Point[data]}];
g3=Graphics[{MapThread[Text[#1,#2]&,{lab,data}]}];
Show[g1,g2,g3]
```

See Fig. 3 *(left).*

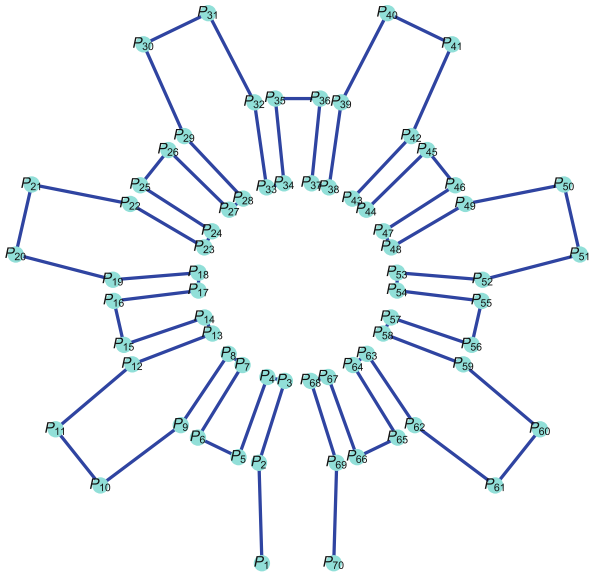


Fig. 3. Control points and antenna design.

4 Simulation of an Antenna Model with 4Nec2

The control points (Table 1) have been entered into the 4Nec2 software. With these points and in free space, the modeled antenna will have the appearance shown in Fig. 4.

4.1 Standing Wave Ratio (SWR) at 2100 MHz

Having the variables initialized, according to the points in Table 1, at a frequency of 2100 MHz the antenna presents an SWR = 13.2 (standing wave ratio) and an impedance of 52.3 Ω.

Figure 5 shows the simulation result for the 2000 MHz–2200 MHz range, with the same previous parameters. It is observed that, in a large part of the simulated range, the antenna could be adapted using some impedance matching technique; since the SWR (Standing Wave Ratio) increases with increasing frequency.

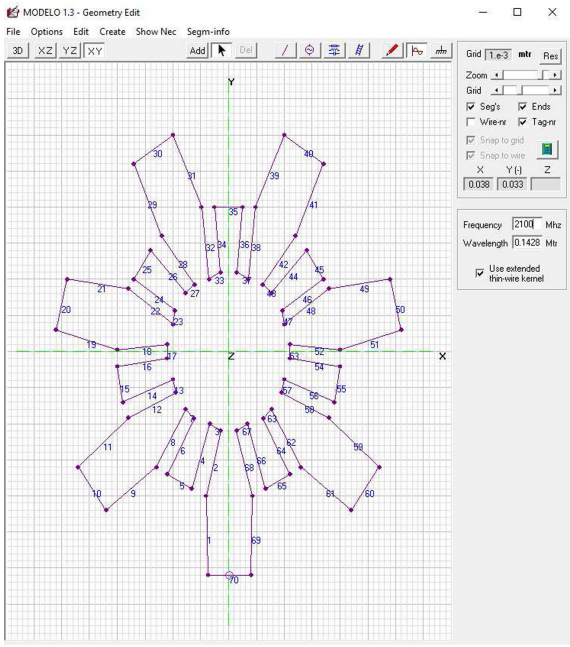


Fig. 4. Antenna modeled on 4Nec2.

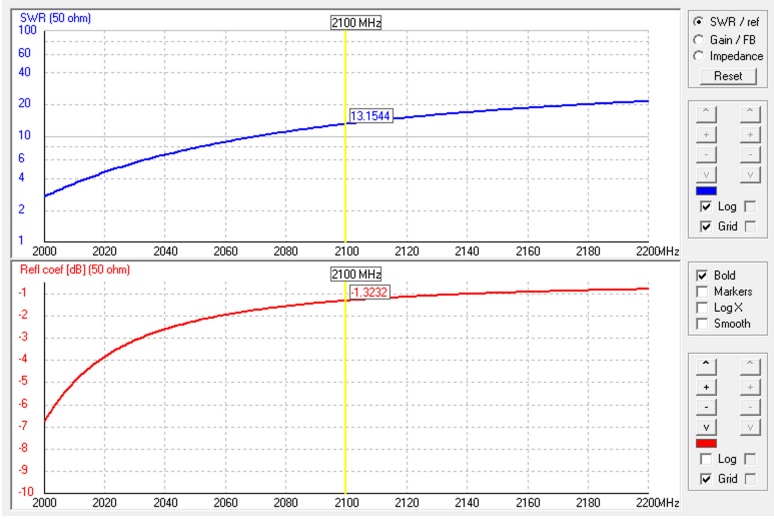


Fig. 5. Standing Wave Ratio at 2100 MHz in free space.

Antenna Gain. In Fig. 6 it is observed that the antenna presents a gain of 3.09 dBi and a relatively omnidirectional radiation pattern.

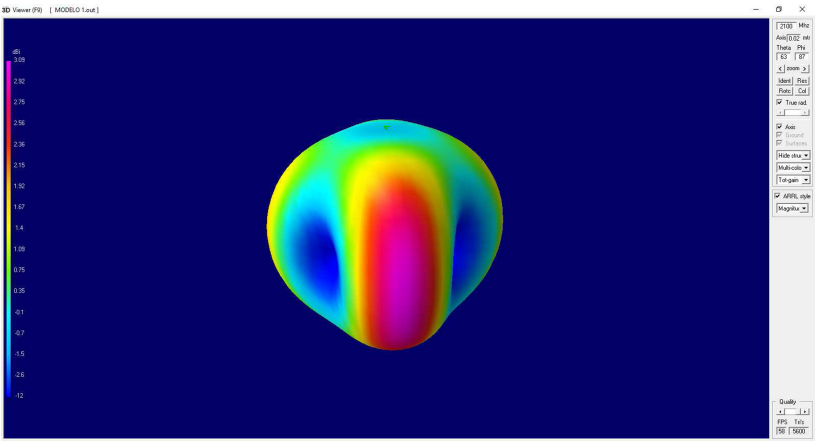


Fig. 6. Antenna gain at 2100 MHz.

Antenna Radiation Diagrams. Fig. 7 shows the radiation patterns of the antenna at 2100 MHz and Figs. 8 and 9 show the radiation patterns of the same antenna under free-space conditions.

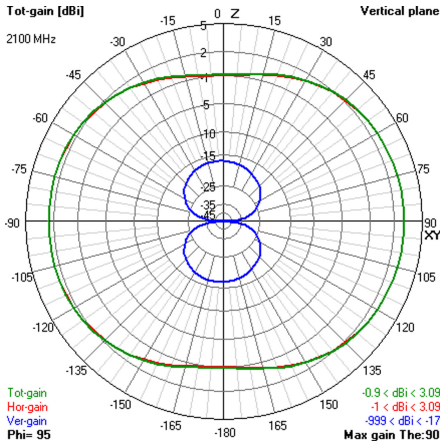


Fig. 7. Antenna radiation patterns at 2100 MHz.

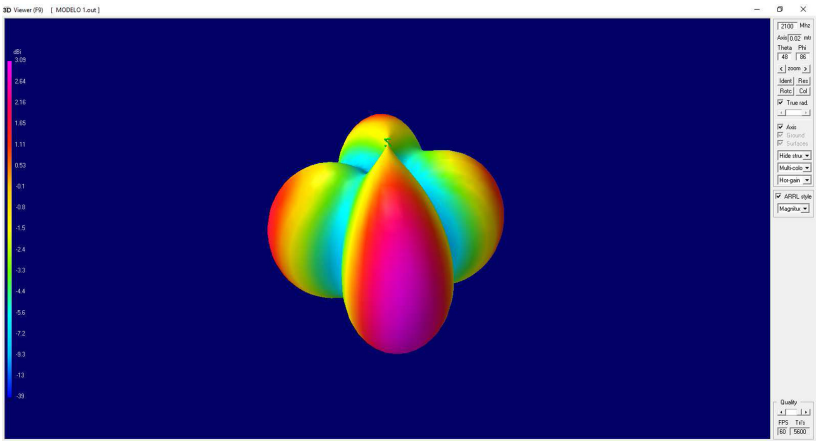


Fig. 8. 3D radiation pattern, of the antenna at 2100 MHz, in the H plane.

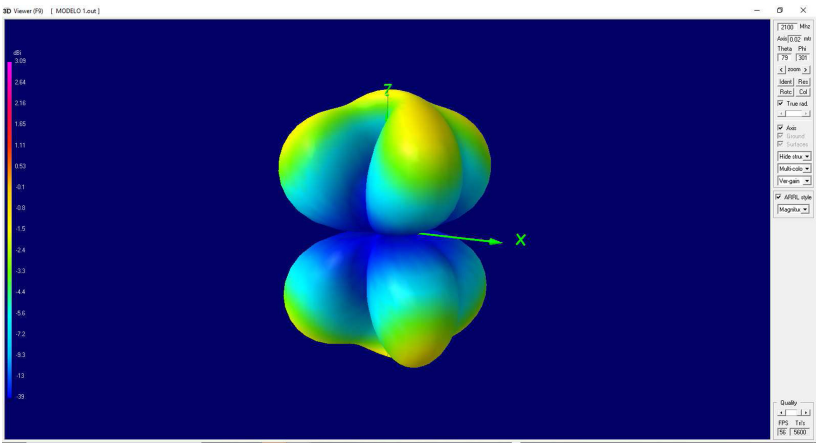


Fig. 9. 3D radiation pattern, of the antenna at 2100 MHz, in the V plane.

4.2 Standing Wave Ratio (SWR) at 3500 MHz

Having the variables initialized, according to the points in Table 1, at a frequency of 3500 MHz the antenna presents an SWR = 10.81 with an impedance of 136 Ω.

Figure 10 shows the simulation result for the 3400 MHz–3600 MHz range, with the same previous parameters. It is observed that, in a large part of the simulated range, the antenna could be adapted using some impedance matching technique, since it has a fairly high SWR, ideally being the SWR closer to unity to avoid losses due to reflected power.

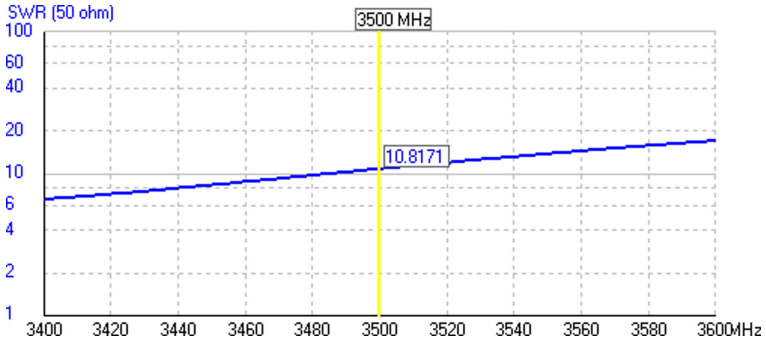


Fig. 10. Standing Wave Ratio at 3500 MHz in free space.

Antenna Gain. In Fig. 11 it is observed that the antenna presents a gain of 5.85 dBi.

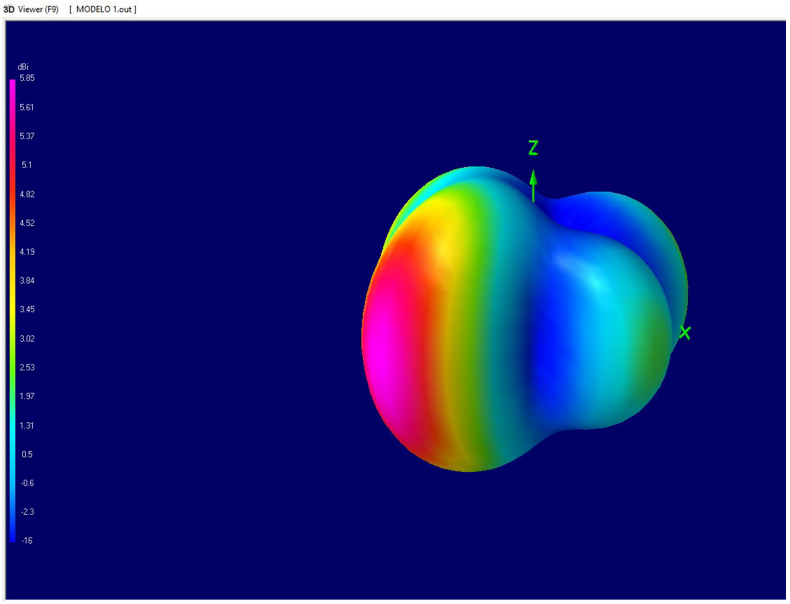


Fig. 11. Antenna gain at 3500 MHz.

Antenna Radiation Diagrams. Figure 12 shows the radiation patterns of the antenna at 3500 MHz and Figs. 13 and 14 show the radiation patterns of the same antenna under free-space conditions.

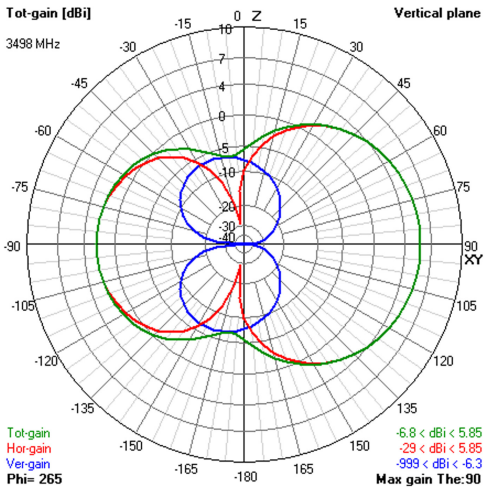


Fig. 12. Antenna radiation patterns at 3500 MHz.

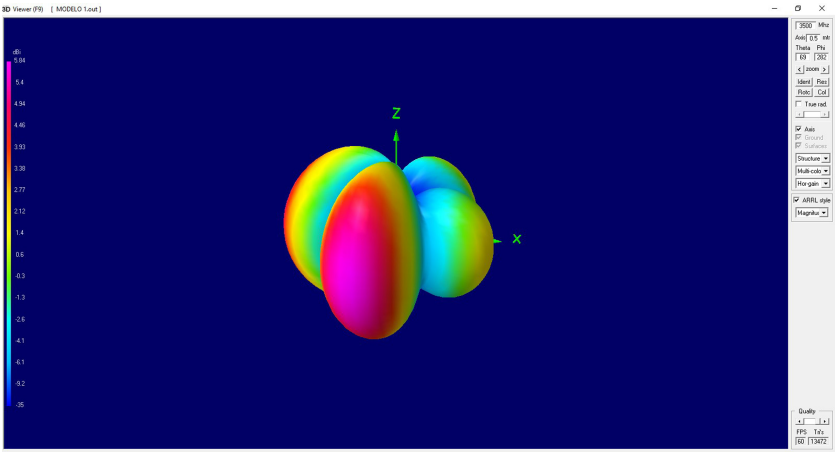


Fig. 13. 3D radiation pattern, of the antenna at 3500 MHz, in the H plane.

Antenna Impedance Matching. Because the impedance found in the previous simulation is 136 Ω, it is necessary to perform the impedance matching simulation between the transmission line, which we assume 50 Ω, and the experimental antenna tested.

In Fig. 15 we can see the adaptation using 4Nnec2.

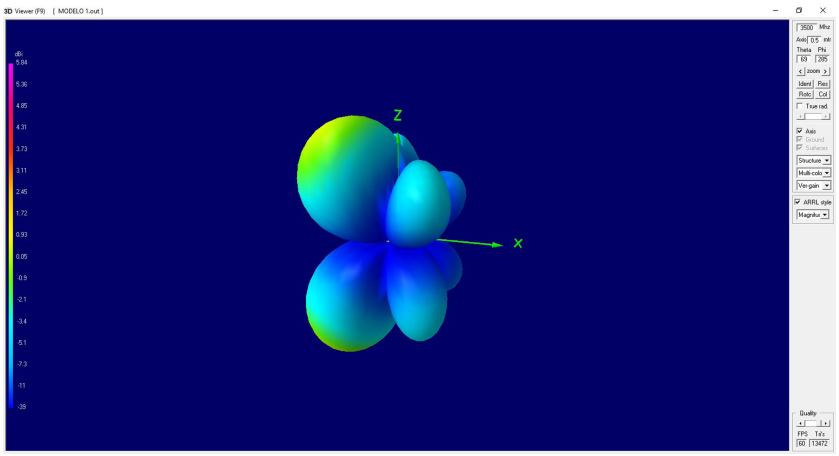


Fig. 14. 3D radiation pattern, of the antenna at 3500 MHz, in the V plane.

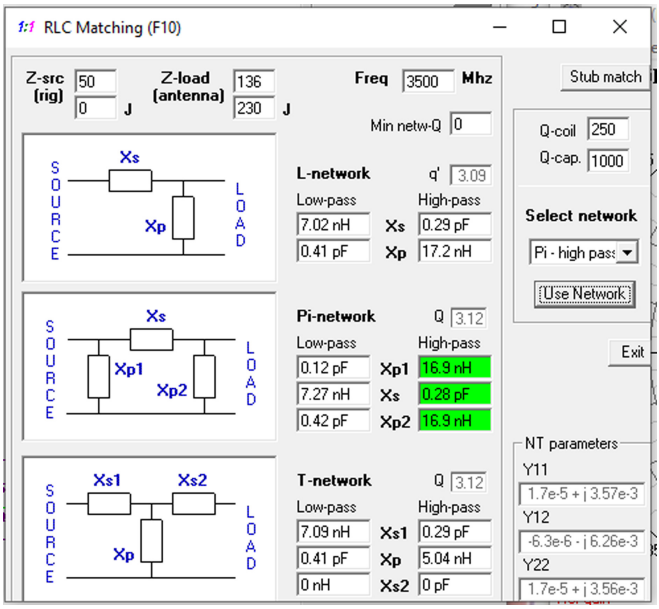


Fig. 15. Impedance matching with RLC Matching.

Figures 16 and 17 show the results after adapting the antenna to a frequency of 3500 MHz. It can be seen that the impedance of the antenna almost equals the impedance of the line, achieving an SWR of 1.01. This indicates that this antenna has high efficiency of 99.06%.

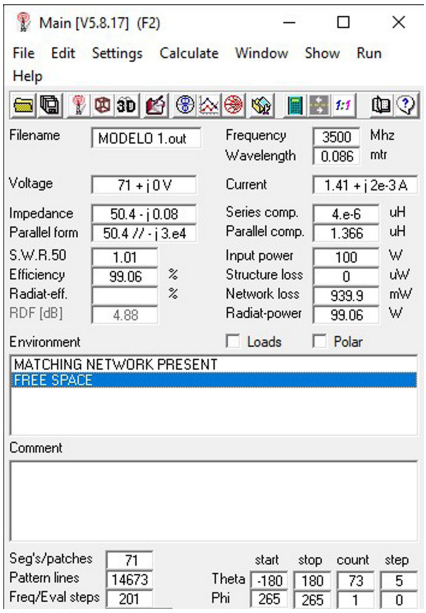


Fig. 16. Antenna parameters after matching with RLC Matching.

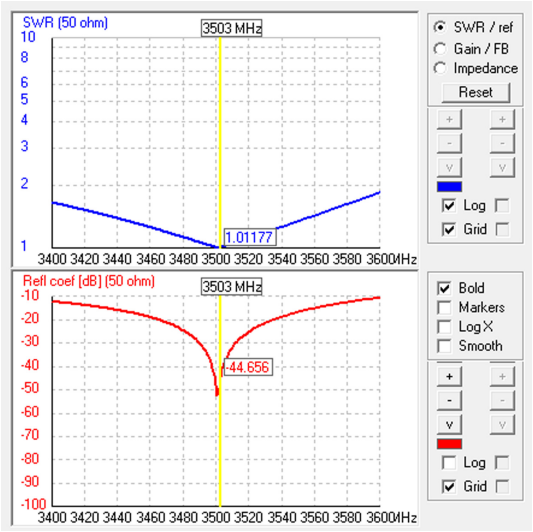


Fig. 17. SWR after adaptation with RLC Matching.

5 Conclusions and Further Remarks

This document describes the design of the model of an antenna, with an efficiency of 99.06% and a SWR of 1.01 after the impedance matching process at the 3500 MHz frequency, generated from control points on a curve lying on a lemniscatic torus and it's projected on the XY plane. The Mathematica v.11.0 symbolic calculation system was used to carry out the design. In addition, two frequencies were proposed for simulation purposes with the assistance of the free software 4Nec2, one of 2100 MHz (0.1428 m wavelength) and the other of 3500 MHz (0.1428 m wavelength). The designed antenna has qualities to be used in fourth-generation 4G mobile communication systems, and especially in fifth-generation 5G, being in Peru a possible frequency for its implementation. The antenna presents excellent qualities of gain (5.85 dBi), radiation pattern, adaptation, and efficiency; which promises its use in technologies in full development such as 4G and future 5G. Since there is an infinite range of possibilities to obtain more control points, we are currently working on the design of other antenna models; as well as, in the test of its operation. The obtained results will be reported elsewhere.

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