

Molodensky Seven Parameter Transformation for Precise Urban Mapping

Javier Urquizo[1(✉)] and Clifford Mugnier[2]

[1] Escuela Superior Politécnica del Litoral, FIEC, Campus Gustavo Galindo, Km. 30.5 vía perimetral, Guayaquil, Ecuador
jurquizo@espol.edu.ec

[2] Department of Civil and Environmental Engineering, Louisiana State University, Baton Rouge, LA 70803, USA
cjmce@lsu.edu
http://www.espol.edu.ec, https://www.lsu.edu

Abstract. The existing coordinate system for the entire country of Ecuador is the Provisional South American Datum of 1956 (PSAD56) which originated in La Canoa, Venezuela. This coordinate system is considered one of the major datums of the world and is generally associated with the countries of South America that comprise the Andes Mountain Range. The system is referenced to the International Ellipsoid and is rigorously constrained by gravimetric observations. Shortly after the fall of the Iron Curtain, the traditionally classified gravity system of the U.S. Department of Defense was offered to private geodesists on a case-by-case basis. The Guayaquil project was one of those offered access to the WGS84 geoid. The availability of cartographic data in different datums made it necessary to create a new application software. This software needs to look at the input and output of all the components and consolidate all the information into a final depository a block based Geographical Information System (adjusted to the World Geodetic System 1984 using Normal Mercator projection). This paper shows a Molodensky seven parameter transformation shift from a PSAD56 classical geodetic system to WGS84 Word Geodetic System 1984 for precise urban mapping applications. All future work within the city of Guayaquil will be able to use the GPS technology with no systematic bias. Such facility using the PSAD56 system is not possible.

Keywords: Spatial interpolation · Record imputation · Distributed target scenarios · Domestic energy consumption · Structure detection · Urban energy modelling

1 Introduction

The PSAD56 is a modern, classical geodetic datum, and has been considered a model for other continental regions within the technological constraints of

© Springer Nature Switzerland AG 2021
O. Gervasi et al. (Eds.): ICCSA 2021, LNCS 12951, pp. 567–587, 2021.
https://doi.org/10.1007/978-3-030-86970-0_40

the late 1950's to the middle of the 1960's. In 1982, a branch of the Ecuadorian Army the 'Instituto Geográfico Militar' (IGM) started a project called "Plano de Guayaquil", a series of maps at 1: 1000 scale were compiled by photogrammetric means in 1984 using a 1983 basic horizontal control, the projection used was a modified version from the Transverse Mercator projection. These maps were referenced to the PSAD56 data. As a Horizontal Datum, PSAD56 provides a frame of reference for measuring locations on the earth's surface, this defines the origin and orientation of latitude and longitude lines. All data is based on an ellipsoid, which is approximate to the shape of the earth. Since the beginning of the 18th century, scientists have estimated the largest and smallest axis on the earth's ellipsoid. The goal in all of this was to find the most convenient way for mapping a particular region or city. At the beginning (before 1960) the estimated ellipsoid was used in conjunction with an initial point on the earth's surface to produce a local horizontal datum. The starting point in PSAD56 is La Canoa – Venezuela. This point is assigned a latitude, longitude and elevation on the ellipsoid and related to a known point. All ground control measurements are then relative to this local datum (ellipsoid and starting point).

The Transverse Mercator or Gauss-Krüger projection is a conformal mapping of the earth ellipsoid where a central meridian is mapped into a straight line at constant scale. Because it cannot be expressed in terms of elementary functions, the mapping is usually computed by means of a truncated series [1]. The resulting mapping approximates the true map. Lee [2] implemented the formulas for the exact mapping and credits E. H. Thompson (1945) for their development. If the mapping is needed at greater distances from the central meridian, the use of an algorithm based on the exact mapping, an accuracy of 9 nm (1 nm = one billionth of a meter) is needed. Instead, the Krüger series, truncating equations to order n^6 with double precision gives an accuracy of 5 nm for distances up to 3,900 km from the central meridian [3]. The initial Grid established by IGM was based on a Gauss-Krüger Transverse Mercator projection. The scale factor at origin, $m_o = 1.0$, the central meridian of the belt (C.M.), $\lambda_o = 79^o$ 53' 05.8232" West of Greenwich, the False Easting at C.M. = 624 km, and the False Northing = 10,000,051.000 m.

As technology progressed in the field of global geodesy, the satellite datums that began to emerge in the late 1960's through the 1970's rapidly began to render classical geodetic datums obsolete. Those with virtually unlimited budgets experienced the luxury of applying a comparative "perfect" ruler to the old classical systems, including the PSAD56.

With the declassification of the World Geodetic Systems of 1972 by Thomas Seppelin [4] in 1976; the world was allowed a tantalizing view of the possibilities of the future. The WGS72 datum (NWL9D) was the major impact in global geodesy, but the Cold War necessitated the continued restricted access of detailed gravity data on a worldwide basis known as the Geoid [5]. The late 1980's allowed civilian use of the new NAVSTAR satellite system otherwise known as the Global Positioning System (GPS). The reference frame of GPS is the World Geodetic System 1984 (WGS84) [6]. WGS84 is the standard of the Department of Defense.

The origin is the centre of mass of the earth. The WGS-84 reference ellipsoid is a geocentric ellipsoid of revolution.

Datum shift techniques and parameters are needed to relate positions on local datums [7] as the PSAD56. The most accurate approach for obtaining WGS 84 coordinates is to acquire satellite tracking data at the site of interest and position it directly in WGS 84 using the Satellite Point Positioning technique. Section 2 presents a detailed mathematical modelling of the transformation formulae.

2 Mathematical Modelling

Let us start with the definition of the terms involved in the datum shift transformation. The US Army Corps of Engineers [8] defines the following terms. Figure 1 shows the geometric relationship between Geodetic, Geocentric and Cartesian coordinates, and Fig. 2 shows the relationship between the reference ellipsoid, the geoid and the surface of the earth.

Datum is any numerical o geometrical quantity or set of such quantities specifying the reference coordinate system used for geodetic control in the calculation of coordinates of points on the earth. **Horizontal datum** specifies the coordinate system in which latitude and longitude of points are located. **Vertical datum** is the surface to which elevation are referred. **Elevation** is the vertical distance measured along the local plumb line from a vertical datum, usually mean sea level or the geoid, to a point on the earth. **Ellipsoid** is the surface generated by an ellipse rotating about in its axes. Also called ellipsoid of revolution. **Equator** is the line of zero geodetic latitude; the great circle described by the semi-major axis of the reference ellipsoid as it is rotated about the semi-minor axis. **Geocentric Cartesian coordinates** are the coordinates (X, Y, Z) that define the position of a point with respect to the centre of mass of the earth. **Geodetic coordinates** (geodetic position) are the quantities of latitude, longitude, and geodetic height (ϕ, λ, h) that define the position of a point on the surface of the earth with respect to the reference ellipsoid. **Geodetic height** (ellipsoidal height, h) is the height above the reference ellipsoid, measured along the ellipsoidal normal through the point in question. **Geodetic latitude** (ϕ) is the angle between the plane of the Equator and the normal to the ellipsoid through the computation point. Geodetic latitude is positive north of the equator and negative south of Ecuador. **Geodetic longitude** (λ) is the angle between the plane of a meridian and the plane of the prime meridian. **Geographic coordinates** are the quantities of latitude and longitude that define the position of a point on the surface of the earth. **Geoid** is the equipotential surface of the earth's gravity field approximated by undisturbed mean sea level of the oceans. **Grid reference system** is a plane-rectangular coordinate system usually based on, and mathematically adjusted to, a map projection in order that geodetic positions (latitudes and longitudes) may be readily transformed into plane coordinates.

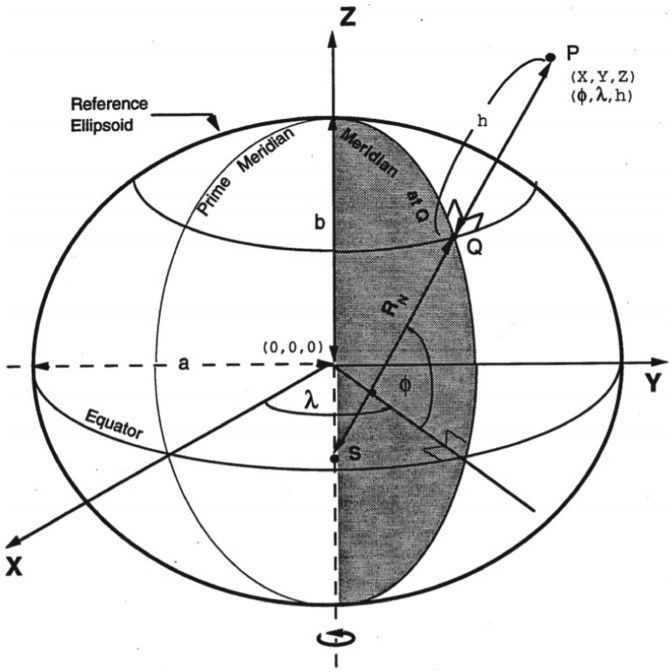

Fig. 1. The geometric relationship between Geodetic and Cartesian coordinates.

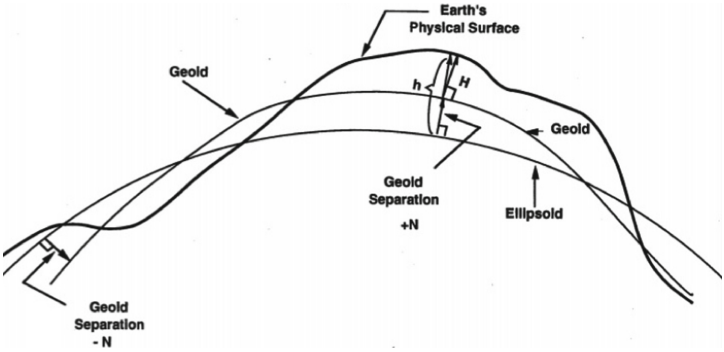

Fig. 2. Relationship between the reference ellipsoid, the geoid and the surface of the earth.

Figure 2 shows that the Z axis, the semi-minor axis is nearly parallel to the axis of rotation of the earth. The equatorial radius a is equal to 6,378,137 m at the equator, flattening f equals to 1/298.257223563, the first eccentricity squared is equal to 0.00669437999013 and the value of b equals to 6,356,752.3142 m.

2.1 Geodetic to Cartesian Coordinate Conversion

If the geodetic coordinates (ϕ, λ, h) are known, then the Cartesian coordinates (X, Y, Z) are given in Eq. 1, Eq. 2, and Eq. 3.

$$X = (R_N + h)cos\phi cos\lambda, \tag{1}$$

$$Y = (R_N + h)cos\phi sin\lambda, \tag{2}$$

$$Z = (R_N b^2/a^2 + h)sin\phi, \tag{3}$$

Where R_N, the radius of curvature in the prime vertical is given by Eq. 4.

$$R_N = a^2/\sqrt{a^2 cos^2\phi + b^2 sin^2\phi} \tag{4}$$

2.2 Cartesian to Geodetic Coordinate Conversion

In the reverse case, If the Cartesian coordinates (X, Y, Z) are known, the geodetic coordinates (ϕ, λ, h) are given in Eq. 5, Eq. 7, and Eq. 11. Computing λ for values of X different from zero, all values of Y is in Eq. 5.

$$\lambda = \arctan Y/X \tag{5}$$

Computing ϕ is an iterative procedure of Bowring [14]. Following Rapp [15] an initial approximation to a variable β is given by Eq. 6

$$\tan\beta_0 = aZ/b\sqrt{X^2 + Y^2} \tag{6}$$

Once β_0 is obtained, use Eq. 7 to substitute it of for β.

$$\tan\phi = (Z + \epsilon'^2 b\sin^3\beta)/\sqrt{X^2 + Y^2} - a\epsilon^2 cos^3\beta \tag{7}$$

Where the values of ϵ^2 and ϵ'^2 are given by Eqs. 8 and 9.

$$\epsilon^2 = (a^2 - b^2)/a^2 = 2f - f^2 \tag{8}$$

$$\epsilon'^2 = (a^2 - b^2)/b^2 = \epsilon^2/(1 - \epsilon^2) \tag{9}$$

This approximation of ϕ is substituted into Eq. 10

$$\tan\beta = (1 - f)\tan\phi \tag{10}$$

Equation 10 gives an updated approximation to β. This procedure of using the latest approximation of β to produce an updated approximation of ϕ (Eq. 7) can be continued until the updated value of ϕ is close enough to the previous value.

Once ϕ is computed, the value of h is in Eq. 11

$$h = (\sqrt{X^2 + Y^2}/\cos\phi) - R_N \tag{11}$$

Where R_N is in Eq. 4.

2.3 Seven Parameter Geometric Transformation

The seven parameter transformation is the most general transformation between local and global Cartesian coordinates. The assumption is that the local (PSAD56) and WGS84 Cartesian axes differ by seven parameters: three rotational parameters (ε, φ, ω), a scale change δS, and three origin shift parameters (δX, δY, δZ). Their relations are in Fig. 3, and Eqs. 12, 13 and 14.

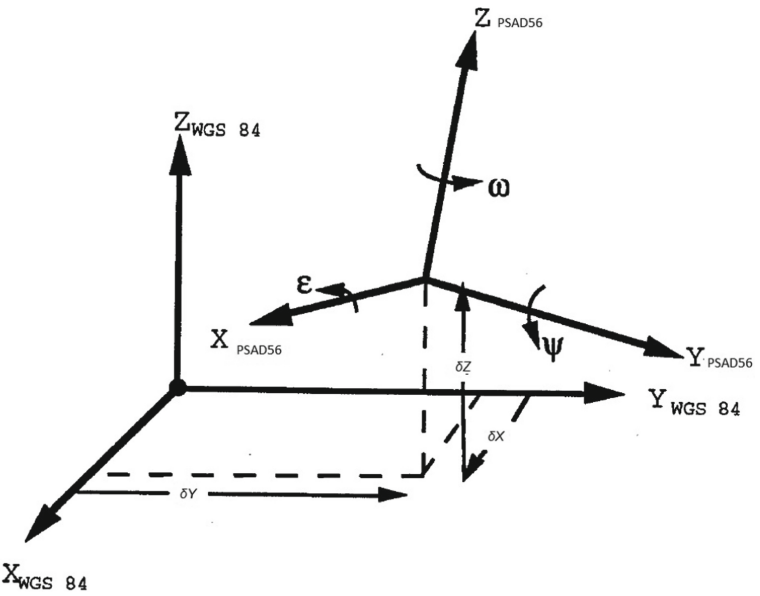

Fig. 3. Relationship between WGS84 and PSAD56 coordinate axis in a seven parameter geometric transformation.

$$X_{WGS84} = X_{PSAD56} + \delta X + \omega Y_{PSAD56} - \varphi Z_{PSAD56} + \delta S X_{PSAD56} \tag{12}$$

$$Y_{WGS84} = Y_{PSAD56} + \delta Y - \omega X_{PSAD56} + \varepsilon Z_{PSAD56} + \delta S Y_{PSAD56} \tag{13}$$

$$Z_{WGS84} = Z_{PSAD56} + \delta Z + \varphi X_{PSAD56} - \varepsilon Y_{PSAD56} + \delta S Z_{PSAD56} \tag{14}$$

2.4 Three Parameter Geometric Transformation

In certain cases, the three parameter geometric transformation can be applied directy to convert local geocentric coordinates to WGS84 geocentric coordinates.

The resulting Eqs. 15 are a subset of Eqs. 12, 13 and 14 with ε, φ, ω, and δS equal to zero.

$$X_{WGS84} = X_{PSAD56} + \delta X; Y_{WGS84} = Y_{PSAD56} + \delta Y; Z_{WGS84} = Z_{PSAD56} + \delta Z \tag{15}$$

2.5 Seven Parameter Bursa Wolf Model

We are given a set of rectangular coordinates, (X_{PSAD56}, Y_{PSAD56}, Z_{PSAD56}), in an "PSAD56" system and we want to transform these coordinates into the "WGS84" system to obtain (X_{WGS84}, Y_{WGS84}, Z_{WGS84}) $\hat{X}_{WGS84}$. The general geometry of the transformation has the translation parameters δX, δY, δZ, which will be designated T in vector form. The three rotation angles ε, φ, ω. A positive rotation is a counter-clockwise rotation about an axis when viewed from the end of the positive axis in right-handed coordinate systems. The mathematical relationship between coordinates in the seven parameter Bursa Wolf model can be written in the form of a vector Eq. 16 assuming the rotation angles are small (a few seconds of arc) [9]. Malys [10] has studied the numerical impact of the small angle approximation in obtaining Eq. 16. He found that the disagreement between an element was at the level of 0.5×10^{-11} when the rotation angles were on the order of 1"; on the order of 0.5×10^{-10} when the angles were on the order of 3"; and on the order of 0.5×10^{-9} when the angles were on the order of 9". An error of 0.5×10^{-9} propagates into a coordinate error on the order of 3 mm. We should note that the order of rotation is not important when the angles are small, as Eq. 16 is independent of the order of rotation.

$$\hat{X}_{WGS84} = (1 + \delta S) \begin{bmatrix} 1 & \omega & -\varphi \\ -\omega & 1 & \varepsilon \\ \varphi & -\varepsilon & 1 \end{bmatrix} \quad \hat{X}_{PSAD56} + \begin{bmatrix} \delta X \\ \delta Y \\ \delta Z \end{bmatrix} \tag{16}$$

2.6 Seven Parameter Molodensky Model

Various interpretations and application of the Molodensky transformation have been given in Badekas [11], Leick and van Gelder [12], Soler [13] and others. Our research follows the Molodensky-Badekas transformation: a seven-parameter conformal transformation (or similarity transformation) linking rotations and translations between the X,Y,Z Cartesian axes and a scale factor to changes in the Cartesian coordinates.

Figure 4 shows the geometry of the Molodensky transformation that makes use of a centroid. The X,Y,Z axes of system 1 are rotated by very small angles ε, φ, ω, from the X,Y,Z axes of system 2, and the origins of the two systems are displaced. The $\overline{X_1}$, $\overline{Y_1}$, $\overline{Z_1}$ system is a centroidal system whose origin is at a centroid G of a set of points in system 1 and whose axes are parallel to the X,Y,Z axes of system 1. The mathematical relationship between coordinates in the seven parameter Bursa Wolf model can be written in the form of a vector Eq. 17 assuming the rotation angles are small.

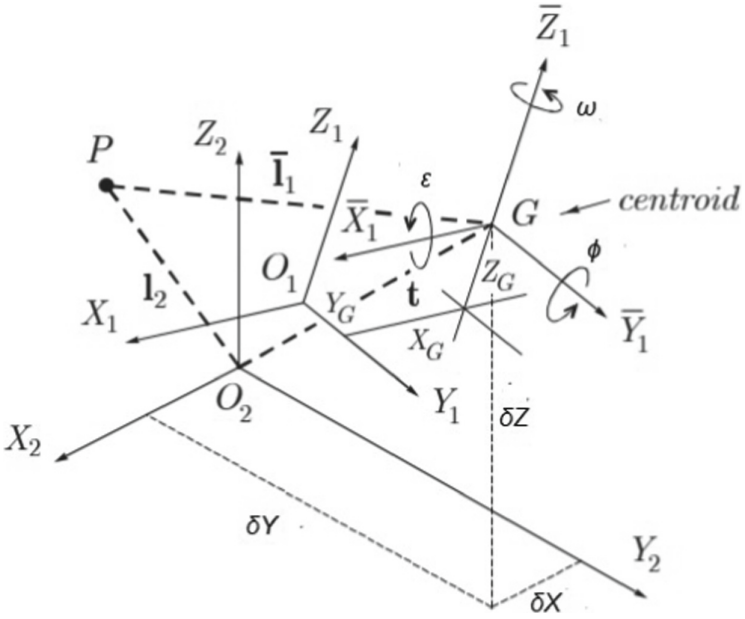

Fig. 4. Relationship between WGS84 and PSAD56 coordinate axis in a seven parameter Molodensky geometric transformation.

$$\hat{X}_{WGS84} = (1 + \delta S) \begin{bmatrix} 1 & \omega & -\varphi \\ -\omega & 1 & \varepsilon \\ \varphi & -\varepsilon & 1 \end{bmatrix} \quad \widehat{(X - X_G)}_{PSAD56} + \begin{bmatrix} \delta X \\ \delta Y \\ \delta Z \end{bmatrix} \tag{17}$$

In summary, the magnitude of the shift parameters is minimized in the Molodensky model by the scalar because the geocentric coordinates of the origin point, La Canoa, is subtracted from each geocentric coordinate of the data points. That is the primary difference between Bursa-Wolf and Molodensky. The scalar reduces the translations. We prefer to use the Molodensky model to the Bursa-Wolf model in cases like this when the local area is a great distance from the datum origin. Note that the net computational results (of shift transformations) are identical, but the shift parameters in this case are less correlated and thus they appear more meaningful with the Molodensky model.

2.7 The Mercator Projection

As a projection falls into the cylindrical group (it is the cylindrical orthomorphic), it has several characteristics which are common to all cylindrical projections. First, meridians and parallels are straight lines set at right angles. Secondly, the equator is the correct length, but as all other parallels are the same length as the equator, they are progressively exaggerated in length pole wards. Clearly, the pole cannot be shown on this projection. On the globe, the pole is a point, but on any cylindrical projection, it would be represented as a straight line of the same length as the equator and would, therefore, be infinitely exaggerated. To maintain the property of orthomorphism, the scale along the meridian at the pole would also have to be infinitely exaggerated and so the pole would be infinitely distant from the equator.

The new Normal Mercator parameters of the City of Guayaquil are COUNTRY: Republic of Ecuador, GRID: Guayaquil/ESPOL La Rotonda, DATUM: WGS84, ELLIPSOID: WGS84, Semi-major axis = 6,378,137.000, 1/flattening = 298.25722356300, Units: 1 m = 1.000000000 Meters, Projection type = 41 Normal Mercator, Central Meridian = -79° 52' 45.160", Latitude at origin = -2° 11' 33.0900", Scale factor at origin = 0.999272829, False easting = 500,000.0000 m and False northing = 2,242,320.5100 m.

3 Data Available from Field Survey

The object control for the photos was obtained from two sources. The Horizontal Control was from the GPS survey made by Offshore Navigation Inc (see Fig. 5) in Guayaquil and the Vertical Control was from the Monographs from the IGM. For a full breakdown see Table 1 and Table 2 below. Table 1 shows the grid coordinates (Easting and Northing) and the a priori standard deviation and Table 2 shows the elevation and a priori standard deviation. The geodetic survey of Guayaquil produced thirty points plus fifteen kinematic points. From those the points Jordan (GPS6), Antena C-10 (GPS15) and Faro (GPS11) were sent to Defense Mapping Agency for its adjustment to the ephemeris and referencing to WGS84.

There were three objectives to the field survey.

1. The first objective was to establish absolute WGS84 control points on IGM second order monuments. The data for these points were collected for four hours. During this time, both code and carrier data on L1 and L2 were collected in accordance to DMA specifications. Data from these stations were processed by DMA using precise ephemeris and orbit smoothing to obtain absolute WGS84 coordinates.

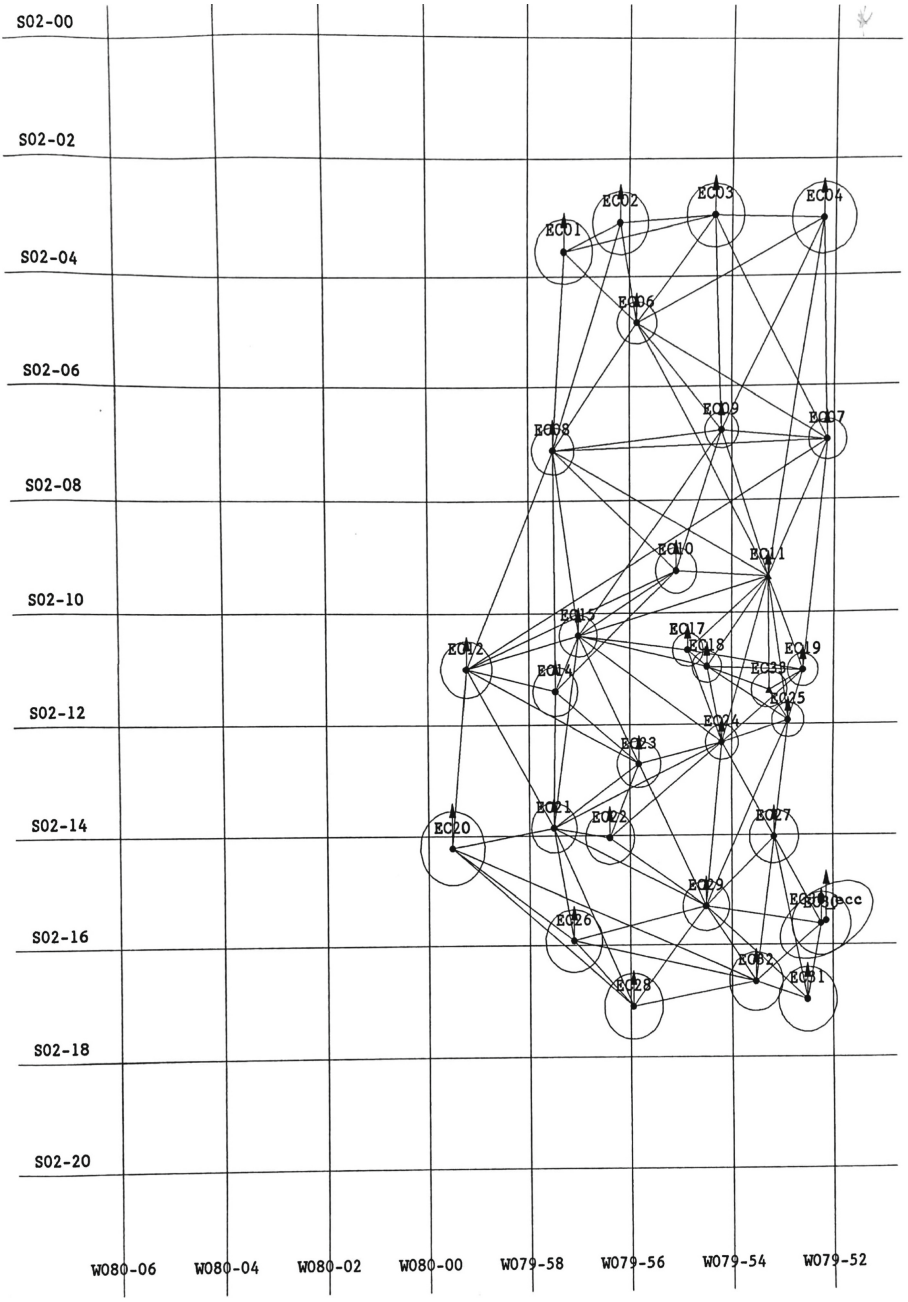

Fig. 5. GPS Ecuador (EC) data available from field survey.

Table 1. Horizontal Control Points (ONI), all units are space units (meters).

Control point	Easting	Northing	Elevation	Easting	Northing	Elevation
ID	Position			A priori standard deviation		
GPS1	491,599.246	2,014,685.053	23.278	0.007	0.007	0.010
GPS2	493,654.311	2,015,585.286	5.542	0.006	0.006	0.010
GPS3	497,076.149	2,015,812.419	14.360	0.007	0.007	0.010
GPS4	500,967.861	2,015,705.493	7.233	0.007	0.007	0.010
GPS6	494,254.413	2,012,380.494	94.438	0.005	0.005	0.008
GPS7	501,136.259	2,008,458.029	4.733	0.004	0.004	0.008
GPS8	491,183.733	2,008,145.422	52.971	0.005	0.005	0.008
GPS9	497,318.375	2,008,785.593	5.290	0.004	0.004	0.008
GPS10	495,660.659	2,004,257.312	100.301	0.005	0.005	0.008
GPS12	487,980.567	2,001,021.131	6.802	0.006	0.006	0.009
GPS14	491,246.484	2,000,275.383	11.910	0.005	0.005	0.008
GPS15	492,093.995	2,002,121.963	316.442	0.004	0.004	0.007
GPS17	496,091.295	2,001,637.492	76.487	0.004	0.004	0.007
GPS18	496,788.643	2,001,077.778	27.077	0.003	0.003	0.007
GPS19	500,278.543	2,000,948.621	3.781	0.003	0.004	0.007
GPS21	491,182.747	1,995,750.284	3.599	0.005	0.005	0.008
GPS23	494,306.867	1,997,848.204	34.653	0.005	0.005	0.008
GPS26	491,896.507	1,991,959.302	2.066	0.007	0.007	0.009
GPS27	499,239.497	1,995430.020	42.698	0.006	0.006	0.008
GPS28	494,078.448	1,989,778.300	3.509	0.007	0.007	0.009
GPS29	496,751.453	1,993,113.360	26.717	0.005	0.005	0.008
GPS31	500,460.263	1,989,991.929	2.101	0.007	0.007	0.009
GPS32	498,574.265	1,990,580.463	3.060	0.006	0.006	0.009
4N	493,592,961	2,012,306.257	11.030	0.036	0.043	0.094
6N	495,137.181	2,013,490.837	10.141	0.050	0.048	0.120
7N	494,823.537	2,014,589.045	8.369	0.051	0.056	0.123
2S	492,268.085	2,006,812.702	42.235	0.027	0.028	0.066
3S	492,581.441	2,005,593.927	44.126	0.032	0.027	0.078
5S	491,946.455	2,000,275.306	13.255	0.059	0.066	0.174
6S	491,483.872	1,998,369.640	4.467	0.069	0.076	0.213
7S	491,399.715	1,996,350.465	5.165	0.080	0.088	0.249

2. The second objective was to survey approximately thirty points that were identified in the new aerial photos. Some of these points were also second order stations established in Guayaquil by the IGM. For these points GPS receivers were configured for synchronous data acquisition on five survey points for each session. There were three sessions each day and each session lasted forty-five minutes. For each session data was collected from a five or six satellite constellation.
3. The third objective was to run a kinematic survey where possible. Lines were run alongside the free-way that runs around Guayaquil. The control station for this phase was EC-08.

Table 2. Vertical Control Points (ONI), all units are space units (meters).

Control point ID	Elevation Position	Elevation A priori standard deviation
PV-2	10.305	0.005
PV-3	6.537	0.005
PV-4	4.072	0.005
PV-14	5.306	0.005
PV-15	6.057	0.005
PV-27	13.531	0.005
PV-29	8.559	0.005
PV11-A	3.974	0.005
PV12-A	3.501	0.005
PV-16A	25.039	0.005
PV22-A	4.051	0.005
PV25-A	3.037	0.005
PV26-A	2.538	0.005
PB-B2	3.326	0.005
PV-B4	3.584	0.005
PV-B7L3	3.220	0.005
PV-B7L4	3.220	0.005
PV-B10	3.206	0.005
PV-B11	3.808	0.005
PV-B12	3.734	0.005
PV-B13	3.121	0.005
PV-B15	2.985	0.005
PV-B16	3.244	0.005
PV-C4	3.210	0.005
PV-C7	1.604	0.005
PV-C9	3.180	0.005
PV-C10	3.413	0.005

4 The Datum Shift Parameter Transformation Results

Local PSAD56 geodetic coordinates can be shifted to WGS84 coordinates in three steps. Fist, convert local geodetic PSAD56 to local geocentric Cartesian coordinates using Eqs. 1, 2 and 3. Second, shift the local geocentric Cartesian coordinates to WGS84 geocentric Cartesian coordinates using Eqs. 12, 13, and 14. Finally, convert WGS84 geocentric coordinates to WGS84 geodetic coordinates using Eqs. 5, 7, and 11.

In order to transform between datums several methods have been offered [16]. From those, the methods based upon Molodensky and Bursa-Wolfe transformations, referred to as three- and seven-parameter transformation, have received great acceptance [17,18]. The three-parameter transformation accounts for shifts in the origin of the X, Y, and Z axes at the centre of the earth. This in essence, is like sliding the latitude/longitude graticule across the surface of the earth. The seven-parameter transformation accounts for the shifts in origin, as well as rotation about each for the three axes and a difference in scale. Section 4.1 uses the three-parameter model. Data from one or more positions are required to derive a three-parameter geometric transformation. Section 4.2 uses the generalized geometric transformation model. This model assumes the origins of the two coordinate systems are offset from the other, the axes are not parallel and there is a scale difference between the two datums. This seven-parameter model needs at least three well-spaced positions. Section 2.5 and Sect. 2.6 show the Bursa Wolf and Molodensky seven parameter transformation formulae, respectively.

4.1 Provisional Three Transformation Parameters

To confirm the common points in the two datums, the geodetic coordinates were transformed to geocentric coordinates independently, in each datum. These coordinates were used to calculate three transformation parameters from WGS84 to PSAD56, which are the translations in X, Y, Z. Note that these parameters are provisional parameters because it is an initial transformation of WGS84 to PSAD56, just to confirm the common points. The three parameters were used to transform the geocentric coordinates of the GPS points in WGS844 to PSAD56. The geocentric coordinates of the GPS points, already in PSAD56, were transformed to geodetic coordinates. The WGS84 data is shown in Table 3 and the PSAD56 data is shown in Table 4.

Table 3. Geodetic coordinates of GPS (WGS84) – data provided by Offshore Navigation.

	Latitude	Longitude	Geodetic height
JORDAN (GPS 6)	-2° 04' 50.97029"	-79° 57' 17.03262"	94.438 m
FARO (GPS 11)	-2° 09' 20.74185"	-79° 53' 17.29297"	22.220 m
ANTENA C-10 (GPS 15)	-2° 10' 24.00518"	-79° 57' 01.02111"	316.442 m
SUBURBIO (GPS 23)	-2° 12' 43.14517"	-79° 55' 49.40620"	34.653 m
PANORAMICO (GPS 25)	-2° 11' 55.83892"	-79° 52' 53.39967"	68.560 m
FERTISA (GPS 29)	-2° 15' 17.29233"	-79° 54' 30.29234"	26.717 m

Table 4. Geodetic coordinates of PSAD-56 – data provided by IGM.

	Latitude	Longitude	Geodetic height
JORDAN (GPS 6)	-2° 04' 38.07738"	-79° 55' 43.14658"	92.740 m
FARO (GPS 11)	-2° 09' 08.81595"	-79° 53' 09.35307"	21.765 m
ANTENA C-10 (GPS 15)	-2° 10' 12.07286"	-79° 56' 53.07339"	315.856 m
SUBURBIO (GPS 23)	-2° 12' 31.21684"	-79° 55' 41.45763"	35.241 m
PANORAMICO (GPS 25)	-2° 11' 43.90930"	-79° 52' 45.46013"	68.614 m
FERTISA (GPS 29)	-2° 15' 05.36320"	-79° 54' 22.34870"	27.209 m

The geodetic coordinates of WGS-84 points are transformed to geocentric coordinates as shown in Table 5 and the geodetic coordinates of PSAD-56 points area transformed to geocentric coordinates as shown in Table 6.

Table 5. Geocentric coordinates – converted from Geodetic WGS-84.

	X	Y	Z
JORDAN (GPS 6)	1,114,418.813	–6,275,878.449	–230,009.979
FARO (GPS 11)	1,119,029.765	–6,274,672.056	–238,317.507
ANTENA C-10 (GPS 15)	1,112,261.692	6,276,099.370	–240,270.451
SUBURBIO (GPS 23)	1,114,362.856	–6,275,274.572	–244,530.417
PANORAMICO (GPS 25)	1,119,732.941	–6,274,409.799	–243,079.750
FERTISA (GPS 29)	1,116,735.965	–6,274,657.224	–249,261.248

Table 6. Geocentric coordinates – converted from Geodetic PSAD-56.

	X	Y	Z
JORDAN (GPS 6)	1,114,704.833	–6,276,093.956	–229,646.278
FARO (GPS 11)	1,119,317.707	–6,274,889.135	–237,954.029
ANTENA C-10 (GPS 15)	1,112,549.642	6,276,316.706	–239,906.779
SUBURBIO (GPS 23)	1,114,651.138	–6,275,493.197	–244,166.985
PANORAMICO (GPS 25)	1,120,021.030	–6,274,627.621	–242,716.236
FERTISA (GPS 29)	1,117,024.205	–6,274,875.941	–248,897.851

The initial (approximate) three parameter transformation are in Table 7.

Table 7. Initial three parameters geometric transformation.

	δX	δY	δZ
Three parameter transformation	288.21935 m	–217.08152 m	363.68780 m

The PSAD56 geodetic coordinates using the initial parameter transformation are shown in Table 8 and the modified Transverse Mercator map coordinates are shown in Table 9.

Table 8. Geodetic coordinates of PSAD-56 – using initial three parameter transformation.

	Latitude	Longitude
GPS 1	−2° 03' 23.0419"	−79° 57' 09.0744"
GPS 2	−2° 02' 53.7308"	−79° 56' 02.5694"
GPS 3	−2° 02' 46.3353"	−79° 54' 11.8334"
GPS 4	−2° 02' 49.8166"	−79° 52' 05.8916"
GPS 6 (JORDAN)	−2° 04' 38.0762"	−79° 55' 43.1490"
GPS 7	−2° 06' 45.7854"	−79° 52' 00.4417"
GPS 8	−2° 06' 55.9638"	−79° 57' 22.5207"
GPS 9	−2° 06' 35.1207"	−79° 54' 03.9943"
GPS 10	−2° 09' 02.5515"	−79° 54' 57.6403"
GPS 11 (FARO)	−2° 09' 08.8094"	−79° 53' 09.3442"
GPS 12	−2° 00' 47.9121"	−79° 59' 06.1798"
GPS 14	−2° 11' 12.1910"	−79° 57' 20.4896"
GPS 15 (ANTENA C-10)	−2° 10' 12.0724"	−79° 56' 53.0630"
GPS 17	−2° 10' 27.8451"	−79° 54' 43.7042"
GPS 18	−2° 10' 46.0675"	−79° 54' 21.1369"
GPS 19	−2° 10' 50.2722"	−79° 52' 28.1983"
GPS 20	−2° 14' 01.0303"	−79° 59' 23.8659"
GPS 21	−2° 13' 39.5105"	−79° 57' 22.5521"
GPS 22	−2° 13' 50.4848"	−79° 56' 16.0648"
GPS 23 (SUBURBIO)	−2° 12' 31.2108"	−79° 55' 41.4508"
GPS 24	−2° 12' 07.8808"	−79° 54' 03.3481"
GPS 25 (PANORAMICO)	−2° 11' 43.9048"	−79° 52' 45.4517"
GPS 26	−2° 15' 42.9268"	−79° 56' 59.4535"
GPS 27	−2° 13' 49.9365"	−79° 53' 01.8232"
GPS 28	−2° 16' 53.9284"	−79° 55' 48.8424"
GPS 29 (FERTISA)	−2° 15' 05.3562"	−79° 54' 22.3400"
GPS 30	−2° 15' 25.0890"	−79° 52' 06.0540"
GPS 30-	−2° 15' 21.5880"	−79° 52' 00.1016"
GPS 31	−2° 16' 46.9735"	−79° 52' 22.3171"
GPS 32	−2° 16' 27.8141"	−79° 53' 23.3509"
GPS 33	−2° 11' 11.2206"	−79° 53' 07.7171"

Table 9. Modified Transverse Mercator map coordinates of PSAD-56 – using initial three parameter transformation.

	Easting (meters)	Northing (meters)
GPS 1	616,482.681	9,772,662.071
GPS 2	618,537.894	9,773,562.462
GPS 3	621,960.039	9,773,789.692
GPS 4	625,892.107	9,773,682.764
GPS 6	619,138.143	9,770,357.426
GPS 7	626,020.448	9,766,434.786
GPS 8	616,067.437	9,766,121.978
GPS 9	622,202.369	9,766,762.364
GPS 10	620,544.665	9,762,233.879
GPS 11	623,891.196	9,762,041.695
GPS 12	612,864.591	9,758,997.301
GPS 14	616,130.568	9,758,233.879
GPS 15	616,978.001	9,760,098.373
GPS 17	620,975.363	9,759,614.021
GPS 18	621,672.726	9,759,054.315
GPS 19	625,162.648	9,758,925.174
GPS 20	612,318.489	9,753,065.461
GPS 21	616,067.054	9,753,726.686
GPS 22	618,121.530	9,753,389.688
GPS 23	619,191.032	9,755,824.693
GPS 24	622,222.446	9,756,541.356
GPS 25	624,629.496	9,757,277.807
GPS 26	616,780.971	9,749,935.877
GPS 27	624,123.600	9,753,406.635
GPS 28	618,962.877	9,747,755.081
GPS 29	621,635.662	9,751,090.035
GPS 30	625,846.836	9,750,483.930
GPS 30-	626,030.764	9,750,591.465
GPS 31	625,344.296	9,747,968.781
GPS 32	623,458.410	9,748,557.281
GPS 33	623,941.476	9,758,281.732

After this Section the GPS 10 might be Mapasingue Auxiliar, which is a third order station in the IGM monographs. Therefore, our research has an additional point (a total of seven points) to compute the datum shift seven parameter transformation.

4.2 Final Seven Transformation Parameters

Dr Munendra Kumar from the Defense Mapping Agency established the parameters of the Bursa-Wolfe Model and the parameters of the Molodensky Model. The parameters of the Molodensky model are shown in Table 10.

Table 10. Parameters of the Molodensky model.

δ x	δ y	δ z	Rx	Ry	Rz	δs
–263.91 m	–25.05 m	–285.81 m	+3.54”	–3.42”	–36.88”	–0.00000361

A computational test point for instance, is station Panoramico where the PSAD 56 coordinates are: $\phi = 02^o$ 11’ 43.9093” South, $\lambda = 79^o$ 52’ 45.4601” West, and h = 68.614 m. The Panoramico WGS 84 coordinates are: $\phi = 02^o$ 11’ 55.8406” South, $\lambda = 79°$ 52’ 53.4010” West, and h = 68.530 m.

Each PSAD56 city block were shifted from the PSAD56 classical system to the new WGS84 datum using custom software development. As the city grew mainly to the west and north, it was necessary to have additional sources of data for these new developments in the city. This research also build a block based Geographic Information System from compilation out of stereoscopic models produced from new controlled photos directly in WGS1984 for the new developments. Figure 6 shows the map4439 in the Guayaquil Block based Geographical Information System.

Figure 6 shows the coincidence of the digitized Mylar map information shifted from PSAD-56 Modified Transverse Mercator to WGS84 Normal Mercator Projection, and the compiled photo information directly in WGS84 Normal Mercator Projection.



Fig. 6. Digitized Mylar map information (yellow) and the complied photo information (red). (Color figure online)

5 Discussion

The datum shift approach done for Guayaquil, although a proper approach, was also constrained by budget. The solution for a least square fit to the Geocentric Reference System for the Americas (SIRGAS) is substantially better for the country as a whole, but it must be realized that for a larger area there must be a compromise that will result in some error and compromise for any given region.

Dr. Kumar was involved in the Guayaquil project because the geoid was still a U.S. military classified Secret thing back then. Since then, the Earth Gravity Model 1996 (EGM96) is now completely unclassified and is available over the internet. That will allow one to convert between WGS84 ellipsoid heights and Mean Sea Level elevations to a world-wide accuracy of about one meter – for elevations. The best for Ecuador would be to obtain a least square fit for all (100%) of the IGM points in and surrounding the Guayaquil new developments if the fit to IGM existing control is the objective. With 100% of the points, however many that might be, that will give the best compromise specifically related to the project at hand. Looking ahead for Guayaquil, the mathematical model for the Datum Shift chosen depends on just how close one wish to find a fit to existing IGM control. Visiting 100% of the points in specific area might be expensive for 4–8 h' observation at each point, that is something that might be done in order to match existing IGM maps and IGM control.

The vertical datums currently used in Latin America refer to different tide gauges and, therefore, at different sea levels and at different times. These do not take into account the variations of the heights and the reference level with time, and in general, the extension of the vertical control by means of levelling nets does not include the reductions due to the effects of gravity. Consequently, the heights associated with them present considerable discrepancies between neighbouring countries, do not allow the exchange of vertical information either on a continental or global scale and are not able to support the practical determination of heights from the Global navigation satellite system (GNSS) technique. SIRGAS is working on a new vertical reference system defined in terms of potential quantities (W_0 as the reference level and geopotential numbers or heights as fundamental coordinates). In accordance with this, each country will be able to introduce the type of physical heights that it prefers, along with the corresponding reference surface: geoid for orthometric heights or quasi-geoid for normal heights. The definition of the new vertical reference system for SIRGAS is identical to the definition of the International Height Reference System (IHRS) described in Resolution No. 1, 2015, of the International Geodesy Association (IAG). The realization of the new vertical reference system for SIRGAS should be a regional densification of the International Height Reference Frame (IHRF).

Current technology has placed an emphasis on Continuously Operating Reference Stations (CORS) for GPS-derived national control. That is what it has done in the state of Louisiana. Louisiana State University (LSU) now has 50 CORS sites throughout the entire state. That is 50 GPS dual frequency geodetic receivers permanently bolted to buildings in Louisiana, and all simultaneously working as a Real Time Network. LSU is now selling subscriptions to Surveyors so that they can connect to its GPS RTN and achieve zero-length baseline positions via cell phone data links. The new realization of the North American Datum of 1983 for the United States is now based exclusively on CORS - nationwide. As time goes on, each country will eventually do the same, including Ecuador. CORS networks are a reality in Guatemala, Honduras, Argentina, and Brazil.

The earthquake that occurred in April 2016 on the Northwest coast of Ecuador severely affected the Geodetic Reference Frame of the country, physically reflected in the damage produced in the stations of (GNSS) Network and landmarks of the GPS National Network; and in a non-tangible way in the deformation of its coordinates, invalidating the georeferencing of any cartographic product in the country. Before 2016, Ecuador maintained the GNSS Network for Continuous Monitoring of Ecuador (REGME) and the Nation GPS Network of Ecuador (RENAGE) referring to the International Terrestrial Reference (ITRF) Frame. The earthquake damage of the Geodetic Frame leaving the country without having a guaranteed and quality reference.

6 Conclusions

A new Grid was devised for the City of Guayaquil based on the Normal Mercator projection, WGS 84 Datum. The origin is at the Rotonda of Simón Bolívar in downtown Guayaquil. We occupied a number of existing IGM triangulation stations as well as new photo-identifiable points based on recent IGM aerial photography. Co-located fiducial points were observed according to DMA/NIMA specifications, and NIMA performed the subsequent fiducial point adjustment to the precise ephemeris. The solution from PSAD56 to WGS 84 for a 7–Parameter Molodensky model (using the PSAD56 origin at La Canoa with Northern latitude), yielded: DX = –263.91 m, DY = –25.05 m, DZ = –285.81 m, scale = -3.61×10^{-6}, Rz = –36.88", Ry = –3.42", Rx = +3.54". This research solve for the 7-Parameter Moledensky model using rigorous least squares techniques.

The current Geographical Information System being established is based on the WGS84 Datum using the U.S. Department of Defense satellites. This basic geometrical foundation will serve all practical applications of municipal works for decades.

References

1. Krüger, L.: Konforme Abbildung des Erdellipsoids in der Ebene (Conformal mapping of the ellipsoidal earth to the plane). Royal Prussian Geodetic Institute 1912, New Series, vol. 52, pp. 1–172 (1912)
2. Lee, T.: Conformal projections based on Jacobian elliptic functions. Cartographica **13**, 67–101 (1976)
3. Karney, C.F.: Transverse Mercator with an accuracy of a few nanometers. J. Geod. **8513**, 475–485 (2011)
4. Seppelin, T.: The department of defense world geodetic system 1972. Can. Surv. **28**, 496–506 (1974)
5. Fisher, I.: The role of the geoid in datum transformation. Can. Surv. **28**, 507–513 (1974)
6. Decker, B.L.: World geodetic system 1984. In: Proceedings of the Fourth International Geodetic Sysmposium on Satellite Positioning, The University of Texas at Austin, United States, 28 April–2 May 1986

7. Defense Mapping Agency: Department of Defense World Geodetic System 1984. Its definition and relationships with local Geodetic Systems. In DMA TR 8350.2; Publishing House: Fairfax, USA (1991)
8. U.S. Army Corps of Engineers: Topographic Engineering Center. Handbook for Transformation of Datums, Projections, Grids, and Common Coordinate Systems. In TEC-SR-7; Publishing House: Alexandria, USA (1966)
9. Rapp, R.: Geometric Geodesy Part II. Ohio State University Department of Geodetic Science and Surveying, Columbus (1993)
10. Malys, S.: Dispersion and Correlation Among Transformation Parameters Relating Two Satellite Reference Frames. Ohio State University Department of Geodetic Science and Surveying, Columbus (1988)
11. Badekas, J.: Investigations Related to the Establishment of a World Geodetic System Ohio State University Department of Geodetic Science and Surveying (1969)
12. Leick, A., van Gelder, B.: On Similarity Transformations and Geodetic Network Distortions Based on Doppler Satellite Ohio State University Department of Geodetic Science and Surveying (1975)
13. Soler, T.: On Differential Transformations Between Cartesian and Curvilinear (Geodetic) Coordinates Ohio State University Department of Geodetic Science and Surveying (1975)
14. Bowring, B.: Transformation from spatial to geographical coordinates. Surv. Rev. **181**, 323–327 (2011)
15. Rapp, R.: Geometric geodesy part I. Ohio State University Department of Geodetic Science and Surveying, vol. 28, pp. 123–124 (1984)
16. Mueller, I.: Review of problems associated with conventional geodetic datums. Can. Surv. **28**, 514–523 (1974)
17. Janssen, V.: Understanding coordinate reference systems, datums and transformations. Int. J. Geoinform. **5**, 41–53 (2009)
18. Abbey, D.A., Featherstone, W.E.: Comparative review of Molodensky-Badekas and Burša-Wolf methods for coordinate transformation. J. Surv. Eng. **146**(3), 04020010 (2020)