A Probability Estimation of Aircraft Departures and Arrivals Delays

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Abstract. Delays in air transport systems have a significant impact on the safety of aviation. We propose to use a trajectory of airplanes to estimate time of airplane delay. Surveillance data is obtained using the Automatic Dependent Surveillance-Broadcast receiver. A network of software defined radios is used to receive position reports of particular airspace user transmitted by airplane transponder of Mode 1090ES. Linear regression with a spline function is used for airplane trajectory approximation and time synchronization. Delays are estimated based on rapid changing of airplane altitude with respect to runway elevation in comparison with a scheduled time. Statistical data of airplane take-off time, landing time, and flight duration based on numerous historical flights is analyzed. Probabilities of airplane delays during take-off and landing are estimated with a help of the Kernel density function. Proposed approach for probability estimation of aircraft departures and arrivals delays can be useful in air traffic management and airline planning for efficient usage of aviation transport system.

Keywords: Transport · Aviation · Air traffic delays · Big data · Automatic dependent surveillance-broadcast · Software defined radio · Airplane trajectory · Linear regression · Spline function · Kernel density estimator · Probability density function

1 First Section

Aviation is referred to one of the most developed transport domains. Numerous advantages of air transportation lead to a continuously increasing amount of carried cargo

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and passengers. It increases the number of flights and amount of aircraft which use airspace simultaneously [1, 2]. Current air traffic system operates in highly congested conditions, due to limited capacity of runways, airports, and flight routes net-work [3]. Operation of air traffic system in highly congested conditions may be easily affected by numerous factors which can lead to delays in transport services [4]. Airplane departure and arrival delays play an important role in the safety of transport system.

Airplane delays have huge economical and safety effects. An air traffic has al-ways a highly congested schedule. Many low-cost airlines would like to minimize time spent in airport facilities as less as possible. Thus, an unexpected delay causes delays for all further scheduled flights. On the other side, unexpected delays of one connecting flight cause serious problems to continue the journey and significant losses in airline profit. Also, delays have dramatical influence on aviation safety. Each flight is preplanned by the flight plan. Operation of air traffic system within scheduled time is connected with the low-risk environment due to planned traffic capacity at different airspace volumes. For example, delayed departure of an airplane may be a reason to put an airplane at holding pattern during arrival at a busy airport due to losing a scheduled landing window. All of that significantly reduce the safety of air transportation. Thus, operation of all airport services has to be provided based on delays minimization criteria. In common case, the safety of air transportation is based on accurate following of predefined trajectory [5–7] withing the defined time.

Statistical analysis of delays helps to identify the main factors which affect air transportation service to minimize its influence into the future air traffic system [8]. Numerous studies consider data processing of delays based on departure and arrival times available in the air traffic control system [9–11]. In this case, the data processing includes histogram analysis and data fitting by Gaussian probability density function. But, numerous factors affecting departure and arrival times with different probability of occurrence taking the assumption about Gaussian noise may degrade performance of estimation.

In our study, we would like to propose an approach for airplane departure and arrival delay estimation based on trajectory data processing. Surveillance data can be easily obtained under the Automatic Dependent Surveillance-Broadcast concept. Statistical analysis of flight based on numerous trajectories helps to identify delays exactly. The probabilities of airplane delays can be estimated based on the probability density function.

2 The Main Causes of Delays in Air Transportation

A statistic of delays is dramatically increased over the European airspace [12]. A continuously increasing tendency is the same for both departures and arrivals. Statistics of average delay per departure and arrival in more than 5 min of a total amount of flights is represented in Fig. 1 [12, 13].

Based on data in Fig. 1 the amount of delayed aircraft is constantly increasing until 2018, were reached its maximal value for departure in 48.4% and 42.8% for the arrival of total flights over Europe. It means that about a half of total flights was delayed in more than 5 min. Also, based on this statistics, the amount of delayed airplanes during departures is always more than for arrivals.

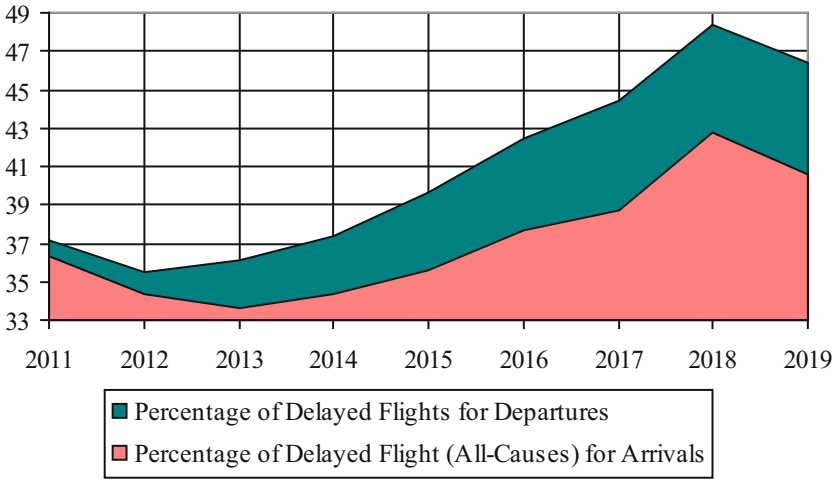

Fig. 1. Statistics of delayed flights for departures and arrivals.

According to the International Air Transport Association (IATA) regulation, it is possible to classify the causes of delays into the main seven groups [12]:

- Airline. Consists of delays caused by passenger and baggage carrying side; cargo loading; airplane and ramp handling; technical problems and aircraft equipment failure; damage of airplane part; flight operations and human factor.
- Airport. Includes delays initiated by airport facilities; restrictions at departure and arrival airports.

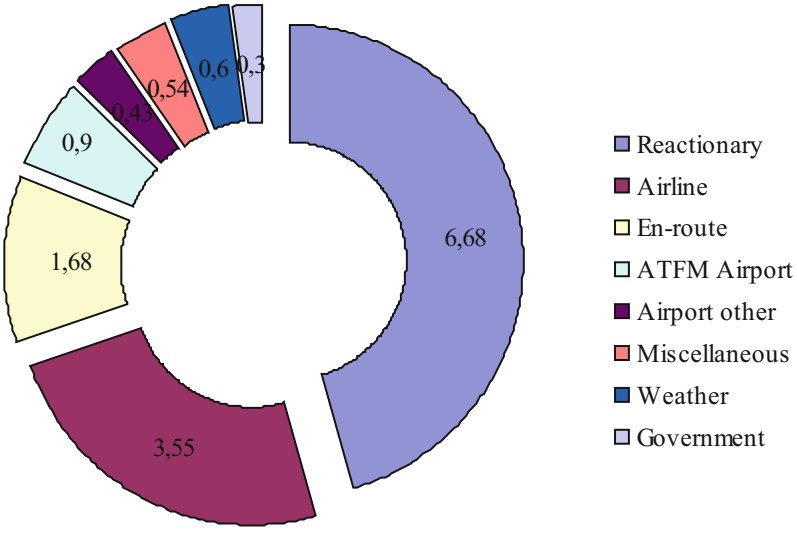

Fig. 2. An average minutes of delay per flight per categories for 2018.

– En-route. Air traffic flow management (ATFM) due to Air Traffic Control (ATC) en-route demand; ATC staff and ATFM equipment.
– Governmental. Includes delays from security and immigration reasons.
– Weather.
– Miscellaneous.
– Reactionary. Is a result of the late arrival of aircraft, passengers, cargo, or crew.

Due to airplane operation in quite a busy environment, any significant primary delay causes a reactionary one. For example, an airplane arrival delay automatically cases delays for all of the further flights of the same airplane during the scheduled time. Average time of delay per flight caused by different factors is represented in Fig. 2 [12].

3 Trajectory Data Processing

Nowadays, the Automatic Dependent Surveillance-Broadcast (ADS-B) concept is considered as the most prospective surveillance technology in the future. According to ADS-B, each airspace user has to be equipped with equipment for transmitting a data message continuously. ADS-B message includes user position report which includes actual coordinates in Lattitude, Longitude, and barometric Altitude (LLA) measured by on-board sensor [14]. The most frequently used on-board equipment of ADS-B is the Air Traffic Control Radar Beacon System (ATCRBS) or airplane transponder of Mode 1090ES [15]. An ADS-B message can be easily received by multiple Software Defined Radios (SDRs) and decoded to obtain exact airplane coordinates [16–18]. There are multiple networks of ADS-B receivers over the globe including national networks of Air Navigation Service Providers (ANSP). A radio waves of Very High-Frequency (VHF) band attenuate in airspace. The service area of each ADS-B ground station depends on transmitting power, a gain function of receiving antenna system, SDR performance, attenuation in the troposphere, multipath from high-altitude construction, and relief. Thus, maximum operational range of ADS-B communication line is limited to 400 NM, approximately. A network of ground stations is used to get stable airplane tracking over the whole flight [19]. Received and decoded ADS-B messages are sent to the data collecting and processing unit automatically. Also, some ground receivers are accompanied by a receiver of Global Navigation Satellite System (GNSS) for time synchronization (see Fig. 3). Received messages from one airplane by different ground stations, which are synchronized by time, allow measuring of airplane coordinates by Time Difference of Arrival (TDOA) method [20, 21]. Thus, some networks of ADS-B data do not provide receiving data from on-board sensor, but provide airplane localization.

According to airplane operation rules, an airplane transponder is active immediately after undocking the airplane. According to that, the ADS-B data sequence includes airplane path during taxing, take-off, en-route, landing, and taxing in destination airport up to airplane docking to particular gate. Usage of ADS-B transponder for ground operations helps the airport system to provide an exact tracking and traffic control within ground facilities [22].

The whole sequence of traffic data (ground and air) is useful to detect the exact departure, take-off, landing, and arrival phases of an airplane [19]. Departure time can

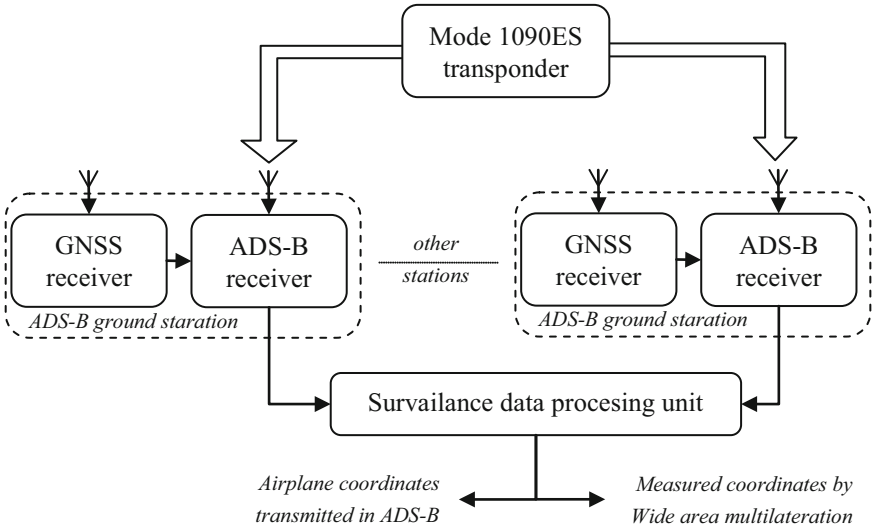

Fig. 3. A ground network of ADS-B receivers.

be obtained from the moment of initial airplane coordinates changing, based on starting moving from the gate. Time of take-off and landing can be estimated based on criteria of equal altitudes of airplane and runway. Arrival time is a time of stopping movement after landing.

In order to estimate the probability of airplane delays, we have to use various realizations of a particular flight. Regular airline connections are scheduled and usually are performed by the same flight-plan and a particular time of a day. Thus, statistical data processing of airplane trajectory from different realizations of particular flight connection creates statistics of airplane departures and arrivals. Obtained statistics can be used to estimate delays based on scheduled time. Also, the results of the statistical analysis give a possibility for efficient planning to reduce delays in future flights.

Transmitted message from on-board equipment is not synchronized in time and each user can use different settings for a time interval between transmissions. The majority of ATCRBS transponders uses reporting frequency in 1 Hz, but some of them can vary the time between transmission in the range from 2 s to 17 s. Also, some messages may be corrupted and lost during transmission. Thus, finally obtained sequence of coordinates is not synchronized. Interpolation of data can be used at the stage of the post-flight data processing to obtain a synchronized sequence of trajectory data [23].

For trajectory data processing we propose to use linear regression with a spline function. Spline interpolation is currently one of the most accurate methods of approximation. The widespread use of splines is due to their high approximation accuracy, which is very important factor in solving practical problems [24, 25].

The spline function is a piece-wise smooth cubic polynomial function, the first and second derivatives of which are continuous in different parts of the curve. Splines are functions that are “glued” from “pieces” of polynomials. The spline curve is given by a set of coordinates of points called control points and indicate the general shape of

the curve. According to these points, a piecewise continuous parametric polynomial function is selected. An interpolation function is a curve created by joining all control points with polynomial parts.

B-spline is the most commonly used type of spline functions. They are formed by approximating a set of control points. The degree of the B-spline polynomial can be set regardless of the number of control points, and they also allow local control over the shape of the curve. In addition to local control, splines allow you to change the number of control points used to construct the curve without changing the degree of the polynomial.

Cubic B-splines, i.e. splines of the third order, were selected as the most effective taking into account the following factors:

1. Polynomials of lower degrees give very low flexibility in controlling the shape of the curve. First-order B-splines do not give a satisfactory smoothness of the approximating curve. Second-order B-splines give a smooth curve, but there is a problem at the points of connection of the curve segments. Second-order B-splines do not guarantee satisfactory continuity at the points of union of the curve segments. The use of cubic B-splines, which are C^0, C^1, and C^2 continuous can solve this problem.
2. Higher polynomials require more time in the computational process and can lead to unwanted approximation jumps.
3. Cubic B-splines allow obtaining sufficient continuity required for processing experimental data.

Therefore, despite of some difficulties in the calculations, in comparison with other methods for application in approximation problems, the choice of cubic B splines is explained by the higher accuracy of its results.

Spline function for interpolation can be represented in the next form:

$$S(t) = \sum_{j=1}^{N+3} B_{j,m}(t) P_j,\ 0 \le t \le T, \tag{1}$$

where $B_{j,m}(t)$ is a basis function of B-spline; P_j matrix of control points.

A Cox-Debour relation of different m order is used as a basis function of spline [23, 26]:

In case $m = 1$, then

$$B_{j,1}(t) = \begin{cases} 1,\ \tau_j \le t \le \tau_{j+1} \\ 0,\ \tau_j > t > \tau_{j+1} \end{cases}$$

In case $m \ge 2$, then

$$B_{j,1}(t) = \frac{t - \tau_j}{\tau_{j+m-1} - \tau_j} B_{j,\mathrm{m}-1}(t) + \frac{\tau_{j+m} - t}{\tau_{j+m} - \tau_{j+1}} B_{j+1,\mathrm{m}-1}(t), \tag{2}$$

where τ is a node matrix.

Control points are calculated based on input statistics, which includes results of not-synchronized measurements of airplane coordinates. The trajectory of an airplane

is transformed from LLA to a local Cartesian reference frame. We use the Noth-East-Down (NED) local frame with a reference point in departure airport coordinates. The total matrix of airplane trajectory includes a correspondent time of measurements and can be represented in the following form:

$$D = \left[X_{NED}^T,\ Y_{NED}^T,\ Z_{NED}^T,\ T^T\right], \tag{3}$$

where X_{NED}, Y_{NED}, Z_{NED} are matrices of airplane coordinates, T is a matrix of time of receiving ADS-B message.

Then, Eq. (1) can be represented in the following matrix form:

$$D = BP, \tag{4}$$

$$B = \begin{bmatrix} B_{1,\mathrm{m}}(t_1) & B_{2,\mathrm{m}}(t_1) & \cdots & B_{n,\mathrm{m}}(t_1) \\ B_{1,\mathrm{m}}(t_2) & B_{2,\mathrm{m}}(t_2) & \cdots & B_{n,\mathrm{m}}(t_2) \\ \vdots & \vdots & & \vdots \\ B_{1,\mathrm{m}}(t_n) & B_{2,\mathrm{m}}(t_n) & \cdots & B_{n,\mathrm{m}}(t_n) \end{bmatrix}$$

where P is a matrix of control points, B is a matrix of basis functions.

The elements of the regression matrix $B_{n,m}$ depend on the time variable, called the regressor or predictor variable. In the general case, the elements of $B_{n,m}$ form linearly independent columns. Then, for known basic functions (2), control points can be obtained from the solution of this equation by least squares method in matrix form:

$$P = \left(B^T B\right)^{-1} B^T D. \tag{5}$$

The Least-Squares Method minimizes the sum of standard deviations relative to the values of the matrix of control points P. After obtaining the control points (5) and the basic functions of Eq. (2), to extrapolate the navigation parameter for the required time, we use Eq. (1) in matrix form:

$$S = BP \tag{6}$$

Interpolated airplane trajectory is used to detect the time of airplane departure and arrival.

4 Probability Density Function Fitting

Numerous flight realizations of some particular connection, based on the scheduled time of gate departure and arrival makes possible to estimate statistics of delays. Factors that take action into delay within the same connection and scheduled time have the same nature in the most cases. Thus, probabilities of delays are estimated based on statistical data of departure and arrival times for particular flight connection separately. Also, a few factors action depends on airport capacity which varies in time of the day, day of the week or holidays.

Each of the factors has the probabilistic nature of the action. In common case, it is possible to assume the Normal (Gaussian) distribution of summarized action of all of these factors. Normal Probability Density Function (PDF) can be represented in the following form:

$$\rho(y) = \frac{1}{\sigma\sqrt{2\pi}} exp\left(-\frac{(y-\mu)^2}{2\sigma^2}\right), \tag{7}$$

where σ is a mean squared deviation, μ is a mean value.

Normal PDF (NPDF) gives a rough estimation due to the high rate of rare factors occurrence which shifts scheduled time much far than mean value. In some cases, a Double or Triple Univariate Generalized Error Distribution (DUGED, TUGED) can be used to fit a PDF [27, 28]. TUGED takes into account the appearance of rare delays for a long time. In general, TUGED can be represented by the following form:

$$\rho(y) = \frac{\alpha}{2a_1 b_1 \Gamma(b_1)} exp\left(-\left|\frac{y-b_1}{a_1}\right|^{b_1^{-1}}\right) + \frac{\beta}{2a_2 b_2 \Gamma(b_2)} exp\left(-\left|\frac{y-b_2}{a_2}\right|^{b_2^{-1}}\right)$$
$$+ \frac{1-\alpha-\beta}{2a_3 b_3 \Gamma(b_3)} exp\left(-\left|\frac{y-b_3}{a_3}\right|^{b_3^{-1}}\right), \tag{8}$$

where a_1, a_2, a_3 are scale factors; b_1, b_2, b_3 are shape coefficients; μ_1, μ_2, μ_3 are mean values; α and β are weight coefficients.

However, the best fitting property of statistics dataset with unknown factors action can be obtained by Kernel Density Estimation (KDE) [11, 29, 30]. KDE is a PDF estimator that does not require an assumption about a family of the underlying function. In common case, KDE of an identically distributed random sample of X data can be represented in the following form [31]:

$$\rho_{KDE}(x) = \frac{1}{nh^d} \sum_{i=1}^{n} K\left(\frac{x - X_i}{h}\right) \tag{9}$$

where *K(y)* is a kernel function; h is a smoothing bandwidth ($h > 0$); X is a statistical data set.

Based on an assumption of the normal action of each factor, we use the Gaussian kernel function:

$$K(y) = \frac{exp\left(-0.5|y|^2\right)}{\int exp\left(-0.5|y|^2\right)dy}, \tag{10}$$

A special attention should be taken to choose a smoothing bandwidth. Incorrect values of h can lead to under or over smoothing. Both of them lead to significant errors in PDF. Thus, h should be chosen based on some criteria of optimality. Basically, h can be chosen based on minimization of mean square error of KDE [31]:

$$h_{AMISE} = \left(\frac{4\mu_K}{n\sigma_K^4 \int \left|\nabla^2 p(x)\right|^2 dx}\right)^{\frac{1}{d+4}}, \tag{11}$$

where μ_K and σ_K are mean and mean squared deviation of kernel function; $\nabla^2 p(x)$ is a Laplacian of the function $p(x)$.

Also, h can be chosen to get a balance between bias and variance in the under smoothing approach [31]:

$$\mathrm{h} \approx \mathrm{n}^{-\frac{1}{d+4}}. \tag{12}$$

Different confidence bands can be easily estimated based on the obtained PDF. A probability of delay at some particular time (t_x) is an important value in air traffic scheduling:

$$\mathrm{P}[x \geq t_x] = \int_{t_x}^{+\infty} \rho(x)dx. \tag{13}$$

5 Numerical Demonstration

Numerical demonstration of statistical analysis of delays is based on ADS-B data. We use ADS-B trajectory data obtained from network of SDRs. The data includes only ADS-B transmitted airplane position reports. We use a dataset of connection between Kyiv Borispol International (UKBB) departure airport and Frankfurt International (EDDF) arrival airport. The dataset includes 107 unique flights between Sep 12, 2020 and Jan 23, 2021. Take-off is scheduled at 4:30 UTC and landing at 7:20 UTC. Flights are mostly performed by A319 and A321 airplanes. Results of statistical analysis of input statistics are represented in Fig. 4 and Fig. 5 for a frequency of take-off and landing, correspondently. We use a 5-min interval for histogram bins construction.

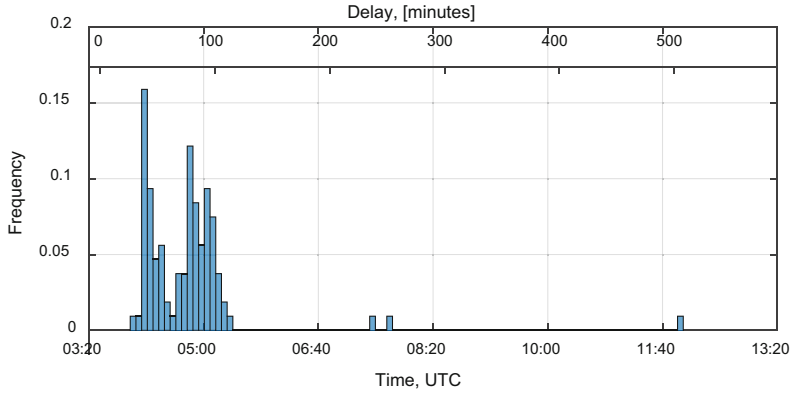

Fig. 4. Frequency of take-off in Borispol departure airport.

Based on the scheduled time and available statistics, we estimate delays for aircraft departure and arrivals. The histogram of flight duration, represented in Fig. 6, includes the time between aircraft take-off and landing.

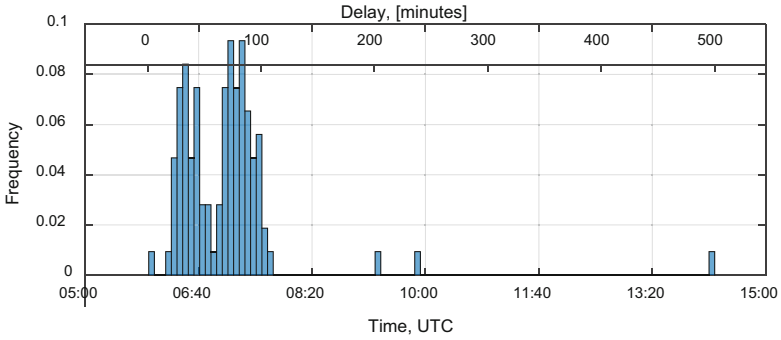

Fig. 5. Frequency of landing in Frankfurt arrival airport.

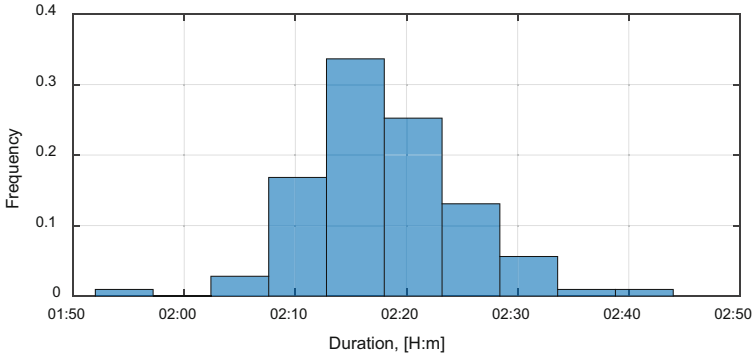

Fig. 6. Statistics of airplane flight duration.

Results of PDF estimation for flight duration, take-off, and landing time are presented in Fig. 7, Fig. 8, Fig. 9. Obtained results for KDE fit input statistics more accurately than NPDF.

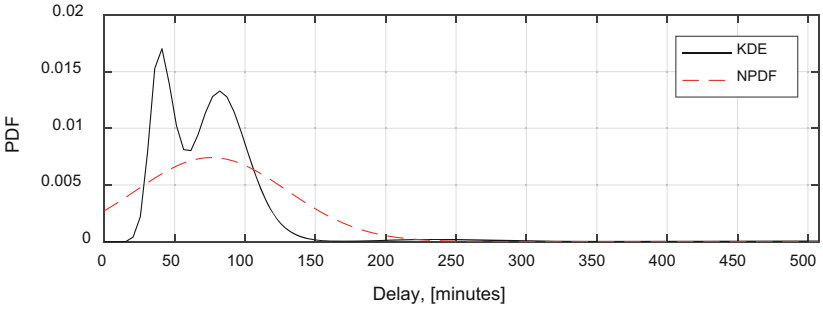

Fig. 7. PDF of take-off in Borispol airport.

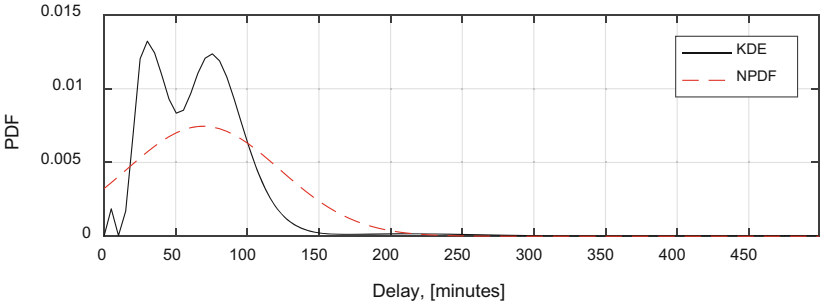

Fig. 8. PDF of landing in Frankfurt airport.

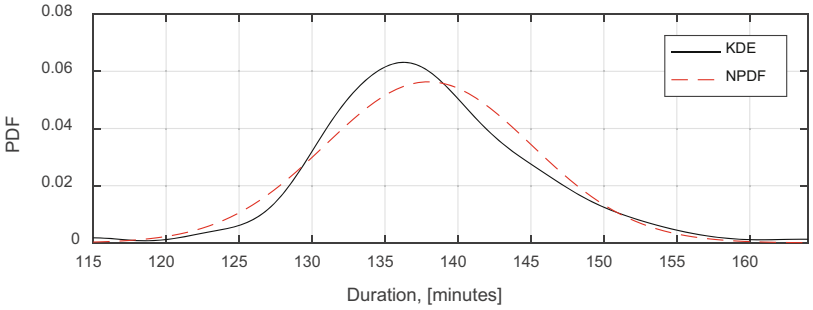

Fig. 9. PDF of flight duration.

Also, a Cumulative Distribution Function (CDF) has been estimated. Results of probability to be late at defined time t_x are estimated by (13) and represented in Fig. 10, Fig. 11, Fig. 12.

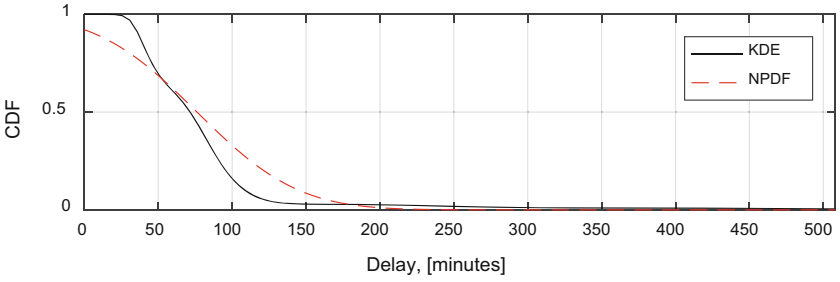

Fig. 10. CDF of take-off in Borispol airport.

Correct choosing of a bandwidth is an important parameter for KDE value. KDE estimated based on different bandwidth values including optimal one, based on a minimal error of approximation (11) is represented in Fig. 13.

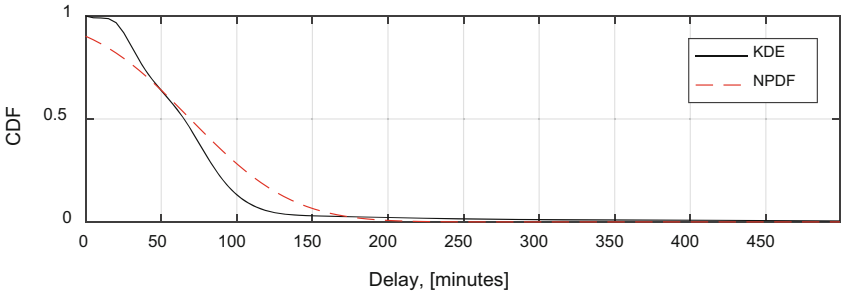

Fig. 11. CDF of landing in Frankfurt airport.

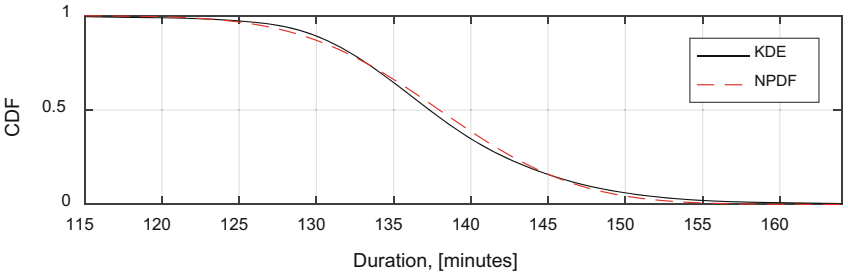

Fig. 12. CDF of flight duration.

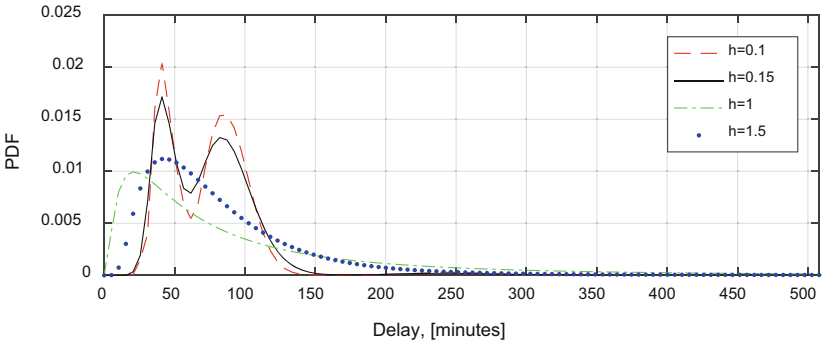

Fig. 13. KDE with different bandwidth values for take-off data in Borispol airport.

Results of mean and mean standard deviation values for each data set, together with optimal bandwidth are presented in Table 1.

Represented values of CDFs have a significant practical application. It shows probability/risk to be delayed for various time intervals. Obtained probabilities are useful in air traffic planning for ATC and airline representatives. Moreover, only one flight connection should be used for statistics collection in order to minimize ATC error influence.

Table 1. Mean and mean standard deviation of delay and flight duration

Time	Mean value	Mean Standard Deviation	Bandwidth
Delay of take-off	76	53	0.15
Delay of landing	69	53	0.19
Duration of flight	137	7	0.02

6 Conclusion

Airplane delay is an important problem of current and future air traffic system. Delays are caused by multiple factors, some of which are quite difficult for simulation due to the low probability of their occurrence. The obtained results show a practical side of delays estimation based on statistical data processing of airplane trajectory obtained by collecting position reports from on-board equipment by the ADS-B concept. Also, linear regression with a B-spline function is used for trajectory fitting to provide data time synchronization. Accumulated statistics of numerous trajectories for unique flight connection have been used in statistical analysis.

Usage of some assumptions about PDF of delays does not fit statistical data very well, due to influence of numerous factors and different forms of PDF. Normalization in this case leads to significant mistakes. Estimation of PFD with a kernel density function gives more accurate fitting in comparison with NPDF and TUGED. Obtained PDFs for flight duration, take-off, and landing delays are used for estimation of airplane risk to be delayed. Probability estimation of an airplane delay is an important step for efficient traffic configuration and safety of aviation.

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