

"Wave-Consistent" Ray Tracing Using Chebyshev Polynomials Representation of a Model in Three Dimensions

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Abstract. We suggest solving the two-point ray tracing problem for a 3D medium using the bending method, which considers the band-limited nature of real seismic signals. It is based on a modified Fermat's principle. As a result, one may obtain more reliable ray paths and traveltimes in complex mediums. Chebyshev polynomials for model parameterization provide considerable algorithmic advantages for the two-point ray tracing based on the bending method because traveltimes and their derivatives with respect to ray parameters can be calculated analytically. Hence nonlinear conjugate gradient method can be applied efficiently for ray computation. 3D numerical experiments prove the efficiency of the proposed approach.

Keywords: Seismic exploration · Ray tracing · Ray bending · "shooting" method · Chebyshev polynomials · Fresnel volume

1 Introduction

Many technical applications require calculating traveltimes and travel paths between known locations of source and receiver of some probing signals. In seismic exploration, such a problem, known as two-point ray tracing, plays a significant role. Two-point raytracing is essential in ray-based seismic imaging techniques like Kirchhoff-type migration and seismic inverse problems such as traveltime tomography. Modern seismic exploration problems, either imaging or inversion, are often three-dimensional, where the spatial size of the computational domains may be huge. Practical experience has shown that the so-called "shooting" method for solving a two-point ray tracing problem often does not provide satisfactory results. Also, for three-dimensional cases, "shooting" requires significant computational resources. An alternative to the "shooting" method is the so-called "bending" method [1]. It is based on the modification of the initial curve that connects the source-receiver pair in such a way as to satisfy Fermat's principle, that is, to minimize the traveltime along the trajectory. In practice, it turns out that for 3D problems, the bending method is much more efficient than the shooting method.

In seismic applications, methods oriented for traveltimes and travel paths calculation usually are based on high-frequency approximations of seismic signals propagation

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(the Ray method, [2]). And they don't take into account the effects caused by the fact that (temporal) spectra of the real seismic signals are band-limited. High-frequency approximation often leads to the non-physical behavior of the constructed rays and corresponding traveltimes. After all, the ray theory has limitations on its application in complex environments [3]. Also, band-limited signals propagate not just along with rays but also within a volume surrounding the ray. The width of this zone depends on the dominant frequency of a signal. This zone is called the Fresnel volume (or zone).

Several methods are developed to go beyond the standard ray method based on the high-frequency approximation. The main idea behind these approaches is to take into account the impact of the Fresnel zone on traveltimes and ray trajectories [4–7]. In [8, 9], a promising approach called "wave-tracing" was proposed for two-dimensional situations. In this paper, we adopt "wave-tracing" for three-dimensional models using original 3D model representation via Chebyshev polynomials expansion proposed in [10, 11]. The combined approach allows, to some extent, to solve the two-point ray tracing problems taking into account the limited spectrum of the probing signals without resorting to high computational costs. It provides ray trajectories and traveltimes that give a better description of real seismic band-limited data than the standard approach.

2 Model Representation by Chebyshev Polynomials

The geological medium, a scope in seismic exploration, is usually represented as a set of sub-horizontal layers formed by sedimentary rocks. The most intensive variations of physical properties (velocity of seismic waves, density, etc.) occur in depth. Following the work [10], we assume that a 3D model of the medium (where physical parameters are dependent on three spatial coordinates x,y,z) is represented as a set of layers separated by non-planar surfaces. In each layer, a function of "slowness" $S_j(x, y)$ (the value of the reciprocal of seismic waves velocity) is specified. Slowness in each layer depends on the two lateral coordinates x, y, but does not depend on the depth, z. The function $S_j(x, y)$ is defined as two dimensional Chebyshev polynomials of 3rd order,

$$\begin{aligned} S_j(x,y) &= C_0 + C_1 x + C_2 y + C_3 xy + C_4\left(2\mathrm{x}^2 - 1\right) + C_5\left(2\mathrm{y}^2 - 1\right) \\ &+ C_6\left(2\mathrm{x}^2 - 1\right)y + C_7\left(2\mathrm{y}^2 - 1\right)x + C_8\left(4\mathrm{x}^3 - 3\mathrm{x}\right) + C_9\left(4\mathrm{y}^3 - 3\mathrm{y}\right), \end{aligned} \tag{1}$$

where C_k are coefficients of decomposition defined for each layer. Each of the surfaces $Z_j(x, y)$ is described by Chebyshev polynomials by analogy with the expression (1) with the coefficients Z_k (see Fig. 1). This kind of model parametrization significantly reduces the number of required parameters compared to the standard scheme where parameter distribution is described by voxels grid. The chosen scheme provides significant computational advantages while solving raytracing problems [10, 11]. The travel times and their derivatives with respect to the rays' parameters may be expressed analytically. Hence nonlinear conjugate gradients approach or Newton-like methods of minimization may be applied very efficiently for ray computation. One more advantage of the proposed parametrization is for solving 3D traveltime tomography. It turns out that the Fréchet derivatives, i.e., a matrix of the traveltimes derivatives with respect to the model parameters (Chebyshev coefficients representing local slowness distribution within each layer and corresponding Chebyshev coefficients of the interfaces) may be expressed analytically, [10].

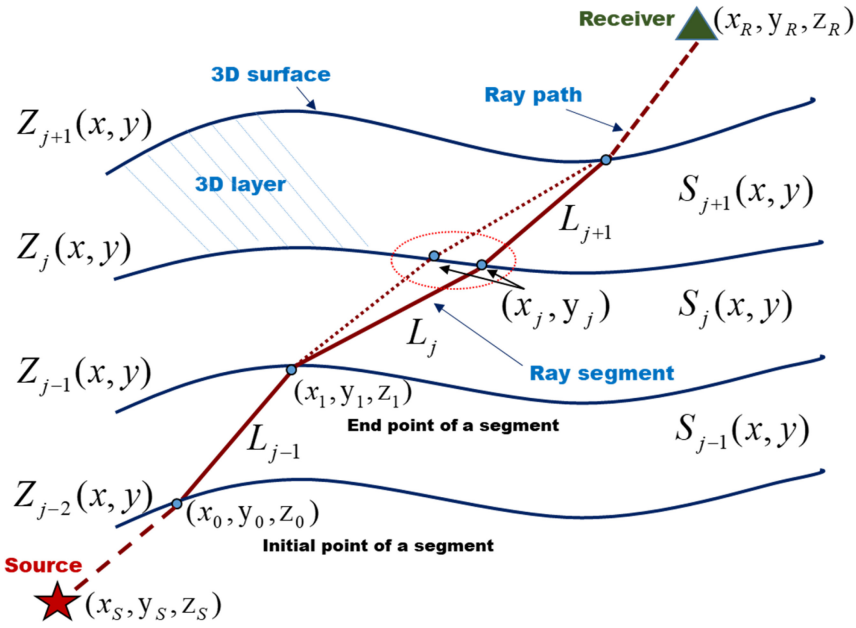

Fig. 1. The scheme of a model representation and a ray trajectory (see the text for more details).

3 Two-Points Ray Tracing Using the Bending Method

By definition, a ray is a trajectory that provides minimum traveltime of some signal between two points (Fermat's principle). Let the initial (source) and the final (receiver) points of the ray are given. Source and receiver have coordinates (x_S, y_S, z_S), (x_R, y_R, z_R) respectively. Having in mind the model parametrization described above, we assume that in each layer, the ray is a straight line segment specified by the coordinates of the intersection points (x_j, y_j, z_j) and $(x_{j+1}, y_{j+1}, z_{j+1})$ with two neighboring surfaces, j and $j+1$, $j = \overline{1, N}$. Where N is the total number of the surfaces (Fig. 1). Note that $z_j = z_j(x_j, y_j)$ because surfaces are defined analytically via Chebyshev polynomials. The ray will be fully defined if appropriate coordinates of intersection points (x_j, y_j), are known. We aim to find a proper location of $N+1$ straight line segments to minimize traveltime calculated along a trajectory. We call a set of coordinates pairs (x_j, y_j) as ray parameters. Let T_j means traveltime within j layer. Total traveltime from source to receiver points is a sum of traveltimes along with the segments, $T = \sum_{j=0}^{N} T_j$. To construct a ray, i.e., a trajectory satisfying Fermat's principle, one must minimize the total traveltime (Fermat functional) of the form,

$$T = \sum\nolimits_{j=0}^{N} T_j(x_j, y_j; x_{j+1}, y_{j+1}), \tag{2}$$

with respect to variables (x_j, y_j) while source and receiver positions are fixed. Any ray constructed in this way will satisfy Snell's law at the interfaces between the layers.

Minimization of the cost function (2) may be performed using the nonlinear conjugate gradient (CG) method. In this work, we use a conjugate gradient algorithm for solving an unconstrained minimization problem of the form "min $F(\vec{x})$" proposed in [12]. To apply the chosen approach, one needs to provide two functions. In each iteration of CG, one must be able to calculate the value of the cost function (traveltime along a current trajectory) and gradient of the cost function $\nabla_{\vec{x}} F$ depending on the parameters vector $\vec{x}$, i.e., the traveltime derivatives $\frac{\mathrm{dT}}{\mathrm{dx}_j}$, $\frac{\mathrm{dT}}{\mathrm{dy}_j}$ in our case.

Let us calculate analytical expressions for the traveltime and its derivatives along a single segment of the ray. It is necessary to calculate the curvilinear integral of the first kind of the polynomial function (1) along a straight line. To simplify notation, we assume that its first and last points (the intersection with the neighboring surfaces) have coordinates (x_0, y_0, z_0) and (x_1, y_1, z_1). Definition of a straight line in 3D space in the parametrical form is given as $x = x_0 + a \cdot l$, $y = y_0 + b \cdot l$, $z = z_0 + c \cdot l$, where a, b, c are cosines of directional angles; l is an independent parameter (distance along the line). Using the parametric representation of a line, one will obtain

$$T_j = \int_{\mathrm{Line}} S_j(x, y)\mathrm{d}l = \int_0^{L_j} S_j(x_0 + \mathrm{a}l, y_0 + bl)\mathrm{d}l, \tag{3}$$

where $L_j = \sqrt{(x_1 - x_0)^2 + (y_1 - y_0)^2 + (z_1 - z_0)^2}$ is a length of a segment, $a = (x_1 - x_0)/L, b = (y_1 - y_0)/L$. Thus, it is necessary to integrate the polynomial expression (1) over the variable l within the interval $[0, L]$ after the appropriate substitution $x = x_0 + a \cdot l, y = y_0 + b \cdot l$. After some manipulations, one comes to the following expression for the traveltime along the straight ray's segment as a function of coordinates (x_0, y_0) and (x_1, y_1):

$$T_j = L_j \cdot Q, \tag{4}$$

where Q is represented as

$$\begin{aligned} Q = {} & C_0 + C_1 \tfrac{1}{2}(x_1 + x_0) + C_2 \tfrac{1}{2}(y_1 + y_0) + C_3\left(\tfrac{1}{3}(x_0 y_0 + x_1 y_1) + \tfrac{1}{6}(x_0 y_1 + x_1 y_0)\right) \\ & + C_4 \tfrac{2}{3}\left(x_1^2 + x_0 x_1 + x_0^2 - 1.5\right) + C_5 \tfrac{2}{3}\left(y_1^2 + y_0 y_1 + y_0^2 - 1.5\right) \\ & + C_6 \tfrac{1}{6}\left(x_1^2(3\mathrm{y}_1 + y_0) + 2\mathrm{x}_0 x_1(y_1 + y_0) + x_0^2(y_1 + 3\mathrm{y}_0) - 3(y_1 + y_0)\right) \\ & + C_7 \tfrac{1}{6}\left(y_1^2(3\mathrm{x}_1 + x_0) + 2\mathrm{y}_0 y_1(x_1 + x_0) + y_0^2(x_1 + 3\mathrm{x}_0) - 3(x_1 + x_0)\right) \\ & + C_8(x_1 + x_0)\left(x_1^2 + x_0^2 - 1.5\right) + C_9(y_1 + y_0)\left(y_1^2 + y_0^2 - 1.5\right) \end{aligned} \tag{5}$$

Note that the segment length L_j is a function of the two lateral coordinates of the intersection points $L_j = L_j(x_0, y_0, x_1, y_1)$. Chebyshev coefficients of the surfaces, Z_k, are hidden within L_j. Time derivatives (and second derivatives) are calculated analytically by differentiating the resulting expression (4). The most laborious moment is the differentiation of the factor L_j due to nonlinearity and cumbersome dependence on the parameters (Sect. 4 provides the derivation of these formulas). The total travel time along the desired ray is represented as the sum of each segment's traveltimes. The derivative of the total traveltime along the ray with respect to variable x_j(or y_j) is a sum of two travetime derivatives in two adjacent layers, T_j and T_{j+1} (see Fig. 1):

$$\frac{\mathrm{dT}}{\mathrm{dx}_j} = \frac{\mathrm{dT}_{j+1}}{\mathrm{dx}_j} + \frac{\mathrm{dT}_j}{\mathrm{dx}_j} \tag{6}$$

Having analytical expressions for the derivatives of traveltimes with respect to intersection points' coordinates with the layers' surfaces, we use the nonlinear conjugate gradient method to minimize the Fermat cost functional. Nonlinear Newton method may be used as well. (It will require analytical expressions for the Hessian (matrix of the second derivatives, [10]).

The number of parameters that need to be determined for each ray is equal to the number of layers intersected by the ray multiplied by two. Ray is defined by the minimization of traveltime functional, which is carried out in two stages:

1) For a given source-receiver pair, a ray is constructed in a simplified medium model. As a "simplified" model, a model consisting of homogeneous layers separated by flat surfaces is used. It means that only the Chebyshev polynomials expansion's first coefficients differ from zero both for the slowness within the layers and for the interfaces. This ray serves as an initial guess for the next step.
2) A ray is constructed in a real model, where ten coefficients of the Chebyshev polynomials describe the slowness in the layers and the interfaces between them.

4 Traveltime Derivatives with Respect to Ray Parameters

This section shows how the first derivatives of the traveltime along a ray segment are calculated. Consider the expression (4) where Q denotes a polynomial multiplier given by the formula (5). It consists of 10 terms: the coefficients of Chebyshev polynomials of the slowness expansion within the layer. Note that in the expression L_j the depth coordinates of the intersection points $z_{0,1}$ are expressed in terms of lateral coordinates $x_{0,1}$,$y_{0,1}$ using the representations of surfaces by Chebyshev polynomials similar to (1). Let $Z_k^{(0)}$ and $Z_k^{(1)}$ be the coefficients of Chebyshev polynomials expansion of two adjacent surfaces (upper index correspond to surface, lower index means expansion coefficient). Using notation p = 0, 1 one will have,

$$\begin{aligned} z_p = Z_0^{(p)} + Z_1^{(p)} x_p + Z_2^{(p)} y_p + Z_3^{(p)} x_p y_p + Z_4^{(p)} \left(2\mathrm{x}_p^2 - 1\right) + Z_5^{(p)} \left(2\mathrm{y}_p^2 - 1\right) \\ + Z_6^{(p)} \left(2\mathrm{x}_p^2 - 1\right) y_p^2 + Z_7^{(p)} \left(2\mathrm{y}_p^2 - 1\right) x_p^2 + Z_8^{(p)} \left(4\mathrm{x}_p^3 - 3\mathrm{x}_p\right) + Z_9^{(p)} \left(4\mathrm{y}_p^3 - 3\mathrm{y}_p\right) \end{aligned} \tag{7}$$

Derivatives are expressed as.

$$\frac{\partial T_j}{\partial \zeta_p} = \frac{\partial L_j}{\partial \zeta_p} \cdot Q + L_j \frac{\partial Q}{\partial \zeta_p}, \tag{8}$$

where we use the notation $\zeta_p = x_p$ or $\zeta_p = y_p$.
Performing an explicit differentiation of each term, one obtains

$$\frac{\partial L_j}{\partial x_p} = \pm \frac{1}{L_j} \{x_0 - x_1 + \Psi_p \cdot (z_0 - z_1)\}, \tag{9}$$

Where

$$\begin{aligned} \Psi_p &= Z_1^{(p)} + Z_3^{(p)} y_p + 4 \cdot Z_6^{(p)} x_p y_p + Z_7^{(p)} \left(2y_P^2 - 1\right) + Z_8^{(p)} \left(12x_p^2 - 3\right) \\ \frac{\partial L_j}{\partial y_p} &= \pm \frac{1}{L_j} \{y_0 - y_1 + \Phi_p \cdot (z_0 - z_1)\}, \end{aligned} \tag{10}$$

Where

$$\Phi_p = Z_2^{(p)} + Z_3^{(p)} x_p + 4 \cdot Z_5^{(p)} y_p + Z_6^{(p)} \left(2x_p^2 - 1\right) + 4 \cdot Z_7^{(p)} x_p y_p + Z_9^{(p)} \left(12 y_p^2 - 3\right).$$

The sign "+" is used for $p = 0$, and the sign "–" is used for $p = 1$.

Derivatives $\frac{\partial Q}{\partial \zeta_p}$ are calculated in an obvious way as derivatives of polynomials in expression (5). Thus, all the necessary components needed for the explicit calculation of the gradient are obtained. Explicit (or implicit) gradient expression is required to minimize the target functional (2) by any gradient method. Note that it is easy to obtain terms for the second derivatives, which are needed to calculate the Hessian matrix. So Newton's method of minimization may be applied straightforwardly [10].

5 Modification of Fermat Cost Function and 3D "Wave-Consistent" Ray Tracing

To consider the band-limited nature of real seismic signals, we adapt for three-dimensional cases the approach proposed in the works [8, 9] for two-dimensional two-point raytracing. The essence of the approach is that instead of the classical Fermat functional (2), a modified cost function is considered. One more term is added which characterizes the length of a ray:

$$F = \left[\sum_{j=0}^{N} \frac{T_j}{T_{\mathrm{SR}}} + \frac{\alpha}{2} \sum_{j=1}^{N} \left(\frac{L_j}{L_{\mathrm{SR}}} \right)^2 \right], \tag{11}$$

where T_j, L_j are traveltime and a ray segment length within the jth layer; T_{SR}, L_{SR} are auxiliary parameters used for normalization within the cost function. In our case, L_{SR} is the distance between source and receiver, T_{SR} is traveltime along this line. By minimization of (4) "wave-consistent" ray trajectory is found. The corresponding traveltime is calculated after that using explicit formulas (4).

Regularization parameter $\alpha > 0$ depends on the dominant frequency of the signal, υ_0. As one can see, if $\alpha = 0$ minimization of the cost function (11) gives a "standard" ray (i.e., a ray which satisfies Fermat's principle (2)). Suppose $\alpha \to \infty$ minimization gives a straight line.

We use the following expression, which is an adaptation of formula (18) form [8] for the 3D case,

$$\alpha = \frac{1}{\sigma} \frac{Z_{SR}}{D_{SR}} \cdot \frac{1}{\sqrt{\upsilon_0 T_{SR}}}, \tag{12}$$

where $Z_{SR} = |z_S - z_R|$, $D_{SR} = \sqrt{(x_R - x_S)^2 + (y_R - y_S)^2}$, $(x_S, \mathrm{y}_S, \mathrm{z}_S)$, $(x_R, \mathrm{y}_R, \mathrm{z}_R)$ are the source and receiver coordinates; $\sigma = \sigma_r / \bar{r}$ is the relative standard deviation of horizontal components $r_j = \sqrt{(x_j - x_{j+1})^2 + (y_j - y_{j+1})^2}$ of the minimum traveltime ("standard") ray,

$$\bar{r} = \tfrac{1}{N+1} \textstyle\sum_j r_j,\ \sigma_r = \sqrt{\tfrac{1}{N+1} \textstyle\sum_j (r_j - \bar{r})^2}, \tag{13}$$

As it was proven in [8], parameter α is defined to obtain a ray with the minimum length among all rays belonging to the Fresnel volume corresponding to the "standard" ray.

Minimization procedure used for modified functional (11) consists of three stages:

1) A straight ray is constructed for a given source-receiver pair. Auxiliary parameters T_{SR}, L_{SR} are defined.
2) A "standard" ray is constructed using the procedure described above. Regularization parameter α is defined. This ray is used as an initial guess for the last stage.
3) Minimization of the modified cost function (11) is performed. The procedure is almost the same as minimization used for a standard functional (2), with the exception that the term $\frac{\alpha}{2}\left(\frac{L_j}{L_{SR}}\right)^2$ is added for each ray segment.

6 Numerical Example

The proposed approach's advantages are illustrated here using a realistic numerical example where a situation typical for 3D crosswell traveltime tomography is modeled. The problems of crosswell traveltime tomography are essentially three-dimensional. The reason is that the boreholes in which the stimulation and registration of seismic signals occur are not strictly vertical but are significantly deviated in 3D space. Also,

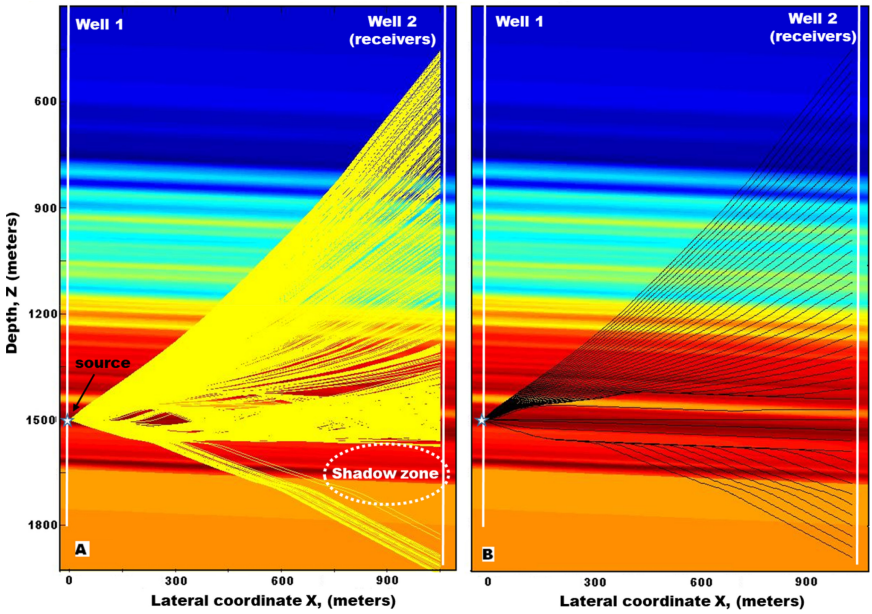

Fig. 2. Rays constructed in the realistic 3D model using two methods overlaid over a 2D section of the seismic velocities distribution: (A) "shooting" method (B) ray "bending" method. In both cases, Chebyshev polynomial representation of the model was used. (Rays projections on the plane between two wells are presented.)

when a detailed model in the vicinity of a borehole is constructed, data obtained in several wells located at a considerable distance from each other are often used. Thus, it becomes necessary to perform two-point ray tracing for large 3D models. Sources and receivers are located at various depths, so rays' trajectories become subhorizontal, i.e., their directions of propagation become close to the boundaries of geological layers. It makes considerable difficulties for two points ray tracing. The "wave-consistent" approach, which is discussed in the paper, may successfully resolve these problems. The three-dimensional model of the medium used here has $1100 \times 1000 \times 2000$ m in X, Y, and Z (depth) directions. It consists of about 1400 thin layers. Layers have a variable thickness (decreasing down to 2 m) but do not intersect with each other anywhere. All 10 Chebyshev coefficients describe each surface between the layers. Velocities in the layers vary along the lateral coordinates X, Y. Velocity variations in different layers are within the range 1200 - 5500 m/s. Two vertical boreholes are located at a distance of 1050 m from each other. It is presumed that sources of the seismic signals are located within the left well. In the right borehole, 50 receivers with a step of 30 m in depth are placed. Figure 2 shows a cross-section of the 3D velocity model in the plane of the wells. There are thin layers with rapid variations of velocity distribution. It is an unfortunate case for standard ray tracing methods, especially for the "shooting" method. Figure 2 compares the two-point ray tracing results obtained by the shooting method and the "standard" bending method. For demonstration purposes, we present the rays for one source position only. As one can see, the calculated rays' behavior differs significantly

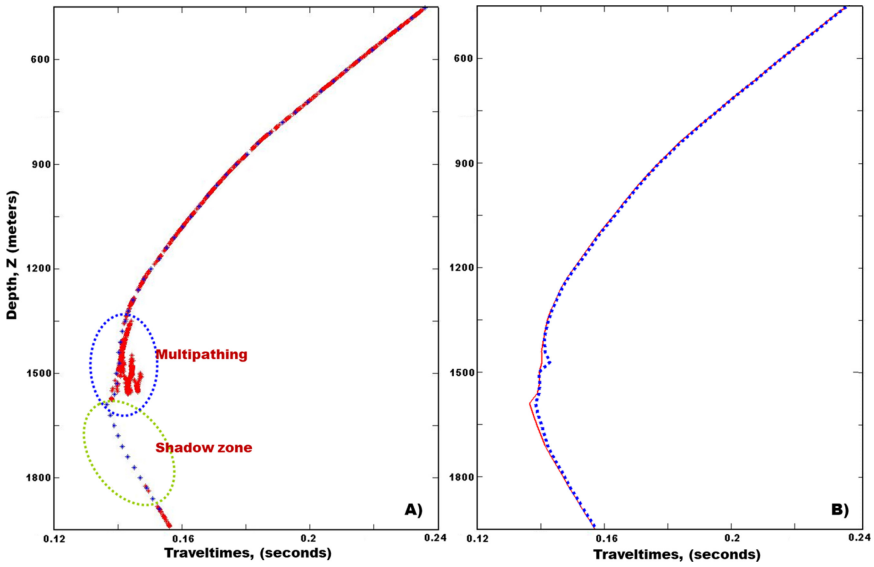

Fig. 3. A) Comparison of traveltimes calculated by the "shooting" method (shown in red) with traveltimes computed using the "standard" bending method (shown in blue); B) Comparison of traveltimes obtained by the standard bending method (red) with "wave-consistent" bending method (blue). Note that the blue line is very close to the red line. It means that the "wave-consistent" rays belong to Fresnel volumes of the "standard" rays. (Color figure online)

in the area where they are sub-horizontal. In the shooting method, many rays had to be used to illuminate the borehole where receivers are located. Rays are very irregular, which contradicts the physical nature of wave propagation in a real geological medium. There is a wide shadow zone where no rays exist. It will cause unavoidable problems during traveltime inversion.

In Fig. 3a comparison of calculated traveltimes is presented. As one can see, the rays obtained by the shooting method form "holes" while going through the high-velocity pack of layers (one of them is marked with a green oval). There are areas with multiple arrivals (marked by a blue oval), which is unfavorable since we are only interested in minimum traveltime arrivals. At the same time, the bending method gives quite adequate results if one considers traveltimes only. However, suppose one takes a closer look at the ray distribution obtained by the "standard" bending method. In that case, one will notice that the ray distribution is also quite irregular in the lower part of the figure. Rays "stick together" bend in the direction opposite the propagation direction when they go along the boundary with a sharp jump of velocities (see Fig. 4). This problem is completely resolved by "wave-consistent" ray tracing, where we presume that the dominant frequency of the signal used to define regularization parameter, is $\upsilon_0 = 100$ Hz. In Fig. 4, a comparison of "standard" rays and "wave-consistent" rays are presented. The latter demonstrates much more physical behavior from the point of view of real band limited signal propagation. It is interesting to emphasize that despite ray trajectories are pretty different, the traveltimes

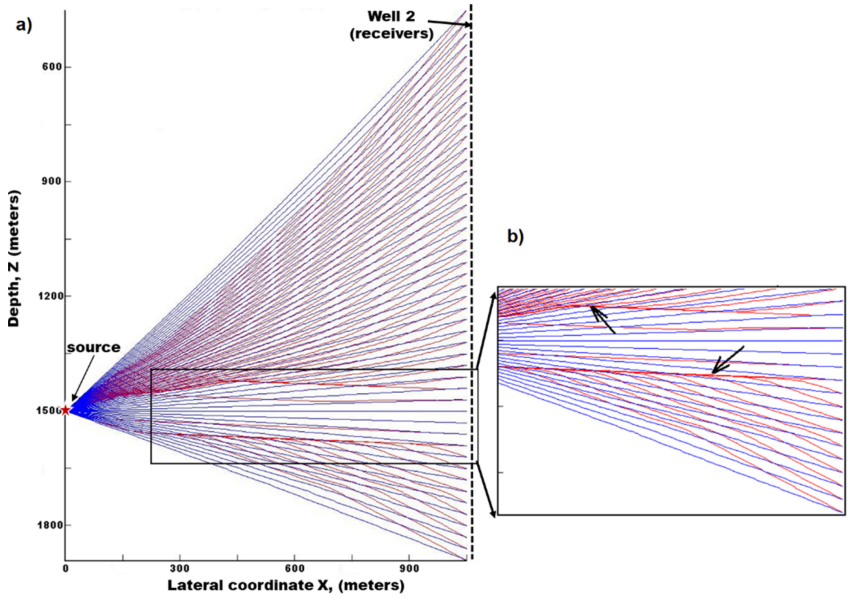

Fig. 4. a) Rays calculated within the realistic 3D model by the use of the "standard" bending method (shown in red) and "wave-consistent" bending method (shown in blue); b) Zoom of Fig. 4a. In this figure, the non-physical behavior of rays constructed by the classical bending method compared to rays obtained by "wave-consistent" bending is shown. (Projections on the plane between two wells are presented.). (Color figure online)

obtained by these two methods are very close (see Fig. 3b). The maximum difference is less than 0.002 s. This fact proves the statement we have been made above that the "wave-consistent" rays belong to Fresnel volumes of the "standard" rays. In this example, the number of parameters used in the CG –based minimization procedure is approximate ~ 1400*2. The maximum number of CG iterations does not exceed 200 and corresponds to the longest rays. CG convergence for wave-consistent ray bending is the same as for "standard" ray bending approach. Thus, the calculation for the entire set of source-receiver pairs (where the number of sources is more than 200) in a given three-dimensional model takes several minutes using a personal computer.

7 Conclusions

An algorithm for fast 3D two-point ray tracing based on the bending method is proposed, considering the effects of the band-limited spectrum of real acoustic signals. It is based on the use of a modified Fermat functional. The essence of modification is that an additional term is introduced into the standard Fermat functional that controls the ray length. As a result, the "wave-consistent" ray belongs to the Fresnel volume of the minimum traveltime ("standard") ray. A pack of layers represents the 3D model of the medium where the rays should be constructed. Physical parameters distributions within the layers and geometry of the layers' boundaries are described using laterally varying 2D Chebyshev polynomials. It gives noticeable computational advantages while solving two-point raytracing problems in three dimensions since travel times and travel-times derivatives are calculated analytically. Modified Fermat functional responsible for "wave-consistent" ray tracing is minimized using nonlinear conjugate gradients method. "Wave-consistent" ray bending provides much more physical ray's distribution within complex 3D model than "standard" ray bending approach. Numerical examples prove the presented algorithm's high efficiency while dealing with sufficiently large and complex three-dimensional models.

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