Full Waveform Inversion in Viscoelastic Media

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Abstract. Accumulations of gas hydrates in the bottom layers pose not only a potential threat to offshore engineering structures and shipping. They can also cause the release of significant volumes of methane into the atmosphere. The most natural way to detect gas hydrates is to use seismic methods. The technology of their application on the shelf is quite well developed, and there is a wide range of instrumental and methodological solutions that ensure their practical use. Detecting gas hydrates belongs to the class of multi-parameter inverse problems. Indeed, gas hydrates in the surrounding medium lead to a change in the propagation velocity of seismic waves and an increased level of wave absorption in these areas. Thus, the correct localization of the gas hydrate accumulations requires determining those regions in space where both the propagation velocities of seismic waves and quality factor change simultaneously. This work deals with the study of the trade-off in velocities and the quality factor of seismic waves. Namely, which condition we need to separate these two parameters' perturbations by solving the dynamic inverse problem of seismic wave propagation.

Keywords: Viscoelasticity · Seismic attenuation · Generalized standard linear solid · Inverse problem · Singular value decomposition

1 Introduction

The Siberian shelf includes the Kara, Laptev, and East Siberian seas, is the most massive continental shelf on Earth. It is of strategic importance because of oil and natural gas reserves. However, it is poorly studied due to the harsh climatic conditions. State of the water and sea ice of the shelf depends on a range of climatic processes, among them are the following: the variability of atmospheric dynamics determining the processes of freezing and melting of sea ice, its drift, and water circulation in the surface layer, interaction with neighbouring regions, and the Siberian rivers. The results of observations of recent years indicate that the region has clearly expressed climatic changes that have occurred over the

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past decades. In particular, vivid evidence of the changes taking place due to warming is the intensification of the erosion of the Arctic shores.

The Arctic shelf is an area where permafrost sub-aqueous rocks can exist (see results in [29]). The existence of permafrost provides conditions for the formation of a zone of stability of gas hydrates in bottom sediments at shallow water depths. Hence there is a possibility of accumulation of the gas hydrates in the near bottom sediments caused by global warming. As a result, methane gas, trapping in gas hydrates, start moving to the surface and penetrate the atmosphere.

The history of the problem dates back to the early 1990 s, when at the bottom of the Barents Sea, clusters of craters were discovered, whose diameter reached one kilometre (http://earth-chronicles.ru/news/2017-06-05-105087). Subsequently, the Norwegian geologists put forward and confirmed by conducting a detailed study of the seabed the hypothesis that these craters appeared during the explosions of gas bubbles associated with the retreat of the glaciers. Subsequently, the so-called "bumps of heaving" were discovered, where, presumably, gas still accumulates as a result of the melting of gas hydrates. Analysis of the collected data showed that modern methane of the Barents Sea was formed more than thirty thousand years ago at a depth of about a kilometre below the seabed. At that time, a massive ice sheet with a thickness of more than two kilometres covered the North Seas. Its weight exerted tremendous pressure on the bottom layers, and as a result of which compressed natural gas into hydrates - crystalline compounds with water. Then, as the ice shield melted, the pressure decreased, and the gases began to accumulate in the cavities, forming massive domes in the bottom layers [31]. A further increase in water temperature will inevitably lead to the destruction of these domes and the release of methane to the surface and further into the atmosphere. A similar situation may develop on the shelf of the Kara Sea and other North Seas.

Of particular interest are the Siberian seas (Kara, Laptev, East Siberian, and Chukchi). Here the formation of gas hydrate deposits occurs in the ice ages under sub-aerial conditions as a result of lowering sea levels and the formation of permafrost in the bottom sediments. These relic hydrates can be especially sensitive to climate change. Margin relic gas hydrates Arctic shelf is estimated to 65 thousand Tg(CH4) per year [24].

An increase in the temperature of the bottom layer enhances the degradation of the permafrost. It can lead to destabilization of gas hydrates. Observation data for the years 1920–2009 in the shallow part of the shelf and the coastal zone of the Laptev Sea and the East Siberian Sea show a significant increase in the bottom temperature (up to 2.1°) starting the mid-1980s [11]. Several cases of unprecedented warming in the bottom waters were observed on the central shelf in winter [16,17].

Thus, the dynamics of bottom water temperatures near gas hydrates accumulations should be monitored especially carefully. When there is a lifting of a vast amount of gas, gas bubbles can achieve several hundred meters in diameter, posing a threat to engineering structures and shipping. Localization of such

clusters and control of their state is an essential task in the light of the changing Arctic climate caused by global warming, as well as the increasing activity of geological exploration and increase in the intensity of navigation and the construction of various complex engineering structures [3]. Therefore, the study of climate change and, as a result, possible changes in temperature conditions in conjunction with the localization of clusters of gas hydrates seems to be a necessary stage for reliable prediction of risks in various regions of the shelves of the North Seas.

In the case of the development of the oil and gas industry on the shelf of the Arctic Ocean, the environmental burden on the region will inevitably increase. Exploration, production, and any type of oil transportation pose a serious threat to the natural environment. Natural purification after oil spills in the Arctic may not last for years (as in temperate regions), but for decades [27]. To provide scientific and practical support for the economic development of the Arctic shelf, it is necessary to develop criteria for the vulnerability of the region under consideration, based on the knowledge of the physical mechanisms that form the current state of its natural environment.

As follows from the above, the mathematical formulation of the problem of detecting accumulations of gas hydrates by seismic methods should base on the theory of propagation of seismic waves in viscoelastic media [28,32]. This inverse problem is multi parameters because we need to recover two families of unknowns - P- and S-wave propagation velocities and P- and S-quality factors. The theoretical proof of uniqueness of the inverse problem for viscoelastic media one can find in [30]. Here we concentrate on numerical analysis of some specific feature of multi parameter inverse problem known as coupling of parameters (see [1]). This term describes the situation when perturbations of one of the parameters, for example, absorption, appears as a perturbation of another, for example, wave propagation velocity. In more details see [20,25,26].

Below we consider the inverse problem for viscoelastic media in a linearized setting. In such media, the following inhomogeneities can occur:

- in wave propagation velocities at constant quality factor;
- in quality factor at constant wave propagation speeds;
- both in P- and S-wave propagation velocities and in P- and S- quality factors.

Let us remind, that the presence of gas hydrate inclusions increases absorption of the seismic energy, which, in its turn, leads to the increase of the wave propagation velocity [2,5,7], etc. Thus, the simultaneous increase of the phase velocity and attenuation, the latter means decrease of the quality factor, may serve as a reliable characteristic of the presence of gas hydrates.

The tool used in the paper to study the coupling of perturbations of quality factors and wave propagation velocities is the singular value decomposition of the corresponding integral operators, similar to [19]. We consider the singular value decomposition of these operators and analyze the projections of the perturbations of the required parameters onto the linear span of the elder singular vectors corresponding to the estimated condition number and a given level of noise in the data.

2 Seismic Waves Propagation in Linear Viscoelastic Media

The model of an ideal elastic medium does not consider energy absorption during the propagation of seismic waves. But the possibility of localizing areas with such absorption and its quantitative assessment provides essential additional information about the properties of the studied geological objects. The generally accepted value for describing the loss of seismic energy is the quality factor of the medium:

$$Q^{-1} = \frac{1}{2\pi}\frac{\Delta E}{E}$$

where E characterizes the elastic energy per unit volume per unit time, and δ E corresponds to the energy lost due to absorption. A distinctive feature of seismic energy dissipation, which has been repeatedly confirmed as a result of field observations, is the quality factor's constancy over the entire seismic frequency range (see [22]).

In this paper, we use the τ-method to implement the Generalized Standard Linear Solid model (GSLS) to describe the propagation of seismic waves in media with absorption, see e.g. [15] and Chapter 5 of the book [14]. When using GSLS, several mechanisms are introduced that describe the relationship between deformations and stresses, which leads to the appearance of frequency dependence of the Lamé coefficients. An essential feature of GSLS is the ability to introduce the shear and longitudinal waves' quality factors. As a result, we obtain the following Lamé coefficients:

– for shear waves

$$\mu(\omega) = \mu_r \left(1 + \frac{i\omega\tau^S}{L}\sum_{l=1}^{L}\frac{\tau_l}{1+i\omega\tau_l}\right) = \mu_r\left(1+\tau^S S(\omega)\right) \tag{1}$$

– for longitudinal waves

$$\begin{aligned}\lambda(\omega)+2\mu(\omega) &= (\lambda_r+2\mu_r)\left(1+\frac{i\omega\tau^P}{L}\sum_{l=1}^{L}\frac{\tau_l}{1+i\omega\tau_l}\right)\\ &= (\lambda_r+2\mu_r)\left(1+\tau^P S(\omega)\right)\end{aligned} \tag{2}$$

This leads to the following representation of quality factors for P- and S-waves:

$$Q_{P,S} = \frac{\sum_{l=1}^{L}\left(1+\tau^{P,S}\frac{\omega^2\tau_l^2}{1+\omega^2\tau_l^2}\right)}{\omega\tau^{P,S}\sum_{l=1}^{L}\frac{\tau_l}{1+\omega^2\tau_l^2}} \tag{3}$$

The τ_l determine the relaxation times for stresses in the GSLS model we are using and have the dimension of time. They should provide the desired constancy of the quality factors over a given frequency interval. At the same time, $\tau^{P,S}$ govern the value of quality factor Q for both P- and S-waves. So there are two main steps in the development of the model of a viscoelastic medium with the use of the GSLS with a given spatial distribution of the quality factors $Q_{P,S}$:

1. The computation of relaxation times providing constancy of the quality factors Q by minimization of the following functional:

$$\tau_l = argmin\|const - Q_{P,S}(\omega)\|$$

2. The calculation of the spatial distribution of τ^P and τ^S providing desired values of Q everywhere in the target domain.

Let us suppose there is the volumetric point source within a viscoelastic medium. Then in the frequency time domain we have the following system of the second order partial differential equations describing seismic waves' propagation:

$$\begin{cases} \omega^2\rho u_x + \dfrac{\partial}{\partial x}\left(((\lambda+2\mu)(1+S^P\tau^P)div\boldsymbol{u} \right. \\ \left. -2\mu(1+S^S\tau^S)\dfrac{\partial u_z}{\partial z})\right) + \\ \dfrac{\partial}{\partial z}\left(\mu(1+S^S\tau^S)(\dfrac{\partial u_x}{\partial z}+\dfrac{\partial u_z}{\partial x})\right) = F_1(\omega)\dfrac{\partial\delta(\boldsymbol{x}-\boldsymbol{x}_0)}{\partial x} \\ \frac{\partial}{\partial z}\left(((\lambda+2\mu)(1+S^P\tau^P)divu - 2\mu(1+S^S\tau^S)\frac{\partial u_z}{\partial z})\right) + \\ \omega^2\rho u_z + \frac{\partial}{\partial x}\left(\mu(1+S^S\tau^S)(\frac{\partial u_x}{\partial z}+\frac{\partial u_z}{\partial x})\right) = \\ = F_2(\omega)\frac{\partial\delta(\boldsymbol{x}-\boldsymbol{x}_0)}{\partial z} \end{cases} \tag{4}$$

To close the obtained equations, we need to add the boundary conditions:

– on the free surface $z = 0$ let us admit the vanishing of normal components of the stress tensor;
– at infinity, we suppose vanishing of the $\boldsymbol{u}$.

3 Inverse Problem

As a result of solving the equation system (4) with boundary conditions, one has a wave field that depends on the elastic moduli λ, μ and relaxation times τ^P, τ^S. The inverse problem is to reconstruct these parameters using additional information:

$$\boldsymbol{u}(x_r, z_r; \omega) = B[\lambda, \mu, \tau^P, \tau^S] \tag{5}$$

We regard relation (5) as a nonlinear operator equation, the solution of which will be sought by the nonlinear least-squares method:

$$[\lambda, \mu, \tau^P, \tau^S] = argmin\|\boldsymbol{u}(x_r, z_r'\omega) - B[\lambda, \mu, \tau^P, \tau^S]\|^2 \tag{6}$$

The first step in implementing any process of minimizing a given functional is calculating its gradient. It is necessary to select a linear increment with a small perturbation of the parameters. Here we restrict ourselves to studying the linear operator arising in this case, considering the inverse problem's linearised formulation. In this case, we will assume that we know the reference medium, determined by the parameters $\lambda_0, \mu_0, \tau_0^P, \tau_0^S$, while the actual media is slightly different from them.

Thus, the linearized formulation of the inverse problem consists of determining the perturbations of the parameters $\delta\lambda, \delta\mu, \tau^P, \tau^S$ by the corresponding perturbations of the $\delta\boldsymbol{u}$, relating by the following system of linear differential equations:

$$\begin{cases} \omega^2\rho_0\delta u_x + (\lambda_0+2\mu_0)\frac{\partial^2\delta u_x}{\partial x^2} + (\lambda_0+\mu_0)\frac{\partial^2\delta u_z}{\partial x\partial z} + \\ +\dfrac{\partial^2\delta u_x}{\partial z^2} = f_1(x,z,\omega,u^0,\delta\rho,\delta\lambda,\delta\mu,\delta\tau^P,\delta\tau^S) \\ \omega^2\rho_0\delta u_z + (\lambda_0+2\mu_0)\dfrac{\partial^2\delta u_z}{\partial z^2} + (\lambda_0+\mu_0)\dfrac{\partial^2\delta u_x}{\partial x z} + \\ +\mu_0\dfrac{\partial^2\delta u_z}{\partial x^2} = f_2(x,z,\omega,u^0,\delta\rho,\delta\lambda,\delta\mu,\delta\tau^P,\delta\tau^S) \\ \omega^2\rho_0 u_x^0 + (\lambda_0+2\mu_0)\frac{\partial^2 u_x^0}{\partial x^2} + (\lambda_0+\mu_0)\frac{\partial^2 u_z^0}{\partial x\partial z} + \mu_0\frac{\partial^2 u_x^0}{\partial z^2} = \\ = F_1(\omega)\dfrac{\partial\delta(\boldsymbol{x}-\boldsymbol{x}_0)}{\partial x} \\ \omega^2\rho_0 u_z^0 + (\lambda_0+2\mu_0)\frac{\partial^2 u_z^0}{\partial z} + (\lambda_0+\mu_0)\frac{\partial^2 u_x^0}{\partial x\partial z} + \mu_0\frac{\partial^2 u_z^0}{\partial x^2} = \\ = F_2(\omega)\dfrac{\partial\delta(\boldsymbol{x}-\boldsymbol{x}_0)}{\partial z}. \end{cases} \tag{7}$$

Right-hand sides f_i are linear functions with respect to $\delta\rho$, $\delta\lambda$, $\delta\mu$, τ^P, τ^S.Thus the formulation (5) of the inverse problem takes the following form:

$$\boldsymbol{u}_0 + \delta\boldsymbol{u} = B[\lambda_0,\mu_0,0,0] + DB[\lambda_0,\mu_0,0,0]\begin{pmatrix}\delta\rho\\ \delta\lambda\\ \delta\mu\\ \tau^P\\ \tau^S\end{pmatrix} \tag{8}$$

We assume that the parameters of the reference medium are known, which allows us to treat function $\boldsymbol{u}_0$ as known as well, and the linearized inverse problem looks as:

$$\delta\boldsymbol{u} = DB[\lambda_0,\mu_0,0,0]\begin{pmatrix}\delta\rho\\ \delta\lambda\\ \delta\mu\\ \tau^P\\ \tau^S\end{pmatrix} \tag{9}$$

3.1 Numerical Experiments

We will assume that the perturbations of the viscoelastic medium's parameters are within the given rectangle

$$\Pi = \left[(x,z) \in R^2 : (-L \le x \le L; h \le z \le H)\right].$$

To reduce system (9) to a system of linear algebraic equations, we introduce a basis of functions of two variables $\phi_{lk}(x,z) = \psi_l(x)\theta_k(z)$ in this rectangle, where

$$\psi_l(x) = \frac{1}{\sqrt{2L}} sin\frac{\pi x}{2L}$$

and the elementary steps $h_k(z)$. We will look for the perturbations in the form of a finite series of the functions from this basis:

$$\delta m(x,z) = \sum_{l=1}^{N_x} \phi_l(x) \sum_{k=1}^{N_z} c_{kl}\theta_k(z). \tag{10}$$

As a result, the integral Eq. (9) transforms into the system of linear algebraic equations for finding unknown coefficients. This system approximates an integral equation of the first kind with a smooth kernel; therefore, when solving it numerically, one should keep in mind that the condition number of the resulting matrix will be the higher, the more accurately the approximation is performed. Hence, we come to the situation well-known in the numerical solution of ill-posed problems: the solution becomes more unstable as approximation improves.

Therefore, a straightforward approach to this problem's numerical solution is unlikely to lead to a correct result. Consequently, we apply a regularization of the original problem by using a truncated singular value decomposition (see [19]).

3.2 Regularization by Truncated Singular Value Decomposition

Any $M \times N$ matrix possesses Singular Value Decomposition (SVD), which is a basis of right $\boldsymbol{v}_j$, left $\boldsymbol{u}_j$ singular vectors, and singular numbers σ_j, satisfying the following relations:

$$A\boldsymbol{v}_j = \sigma_j \boldsymbol{u}_j, \quad A^*\boldsymbol{u}_j = \sigma_j \boldsymbol{v}_j,$$

where A^* is the Hermitian conjugate matrix to A. Singular vectors form orthonormal bases in their spaces:

$$(\boldsymbol{u}_i, \boldsymbol{u}_k) = \delta_{ik}, \quad (\boldsymbol{v}_i, \boldsymbol{v}_k) = \delta_{ik}.$$

This opens up the possibility of looking for a solution to a system of linear algebraic equations

$$\mathcal{A}\boldsymbol{x} = \boldsymbol{f}$$

in the form of a linear combination of right singular vectors:

$$\boldsymbol{x} = \sum_{k=1}^{N} \alpha_k \boldsymbol{v}_k$$

with coefficients

$$\alpha_k = (\boldsymbol{f}, \boldsymbol{v}_k).$$

The length N of the series depends on the value of the condition number $\frac{\sigma_1}{\sigma_N}$, which in turns, depends of the noise level in the right-hand side $\boldsymbol{f}$ and the accuracy of the matrix approximation of the integral operator.

Calculation of singular value decomposition for an arbitrary medium is computationally complicated and expensive problem. At first we use matrix representation of the operator, for these purpose used 1m-mesh for target area $z \in (1000, 2000)$m, $x \in (-700, 700)$m, the characteristic functions are used as a basis (that are equal to one inside cell, and zero outside). And frequency range (20−100 Hz) can be divided by finite intervals with the same basis functions. Such discretisation provides with matrix approximation of the operator, by replacing the integrals by finite sums.

Then we explore coupling of parametrization of the medium. Simultaneous recovery of the parameters when the perturbation of one of the parameters falsely determines as the inhomogeneity of the others is called coupling of the parameters, and it indicates the error in the solution [1]. The ambiguity arising in the solution of the inverse problem for media with attenuation is investigated in a few papers [13,26]. To understand whether our parametrization is coupled, heterogeneity of each parameter is located in separated subregion in target area: $\delta\lambda \neq 0$ in $z \in [1080, 1230]m$, $\delta\mu \neq 0$ in $z \in [1310, 1460]m$, $\delta\tau^P \neq 0$ in $z \in [1540, 1690]m$, $\delta\tau^S \neq 0$ in $z \in [1770, 1920]m$ (Fig. 2).

The convergence of the singular numbers σ_i to zero (Fig. 1) indicates that the approximated operator is compact. Study of advantage of truncated SVD was made for different values of the parameter N, exactly for those that correspond to the condition numbers $10^4, 10^8$ (Fig. 1) and in total the number of singular vectors does not exceed 1250.

Despite the perturbation of parameter λ was only in $z \in [1080 - 1230]m$ for the condition number 10^4, we see heterogeneities outside this interval (Fig. 3), it means that the r solutions reflect the perturbations of other parameters. A similar effect occurs when a parameter is defined. Moreover, when the problem 10^8 is conditioned (Fig. 4), the boundaries of the subregions become more clear, and the coupling of the parameters substantially decreases.

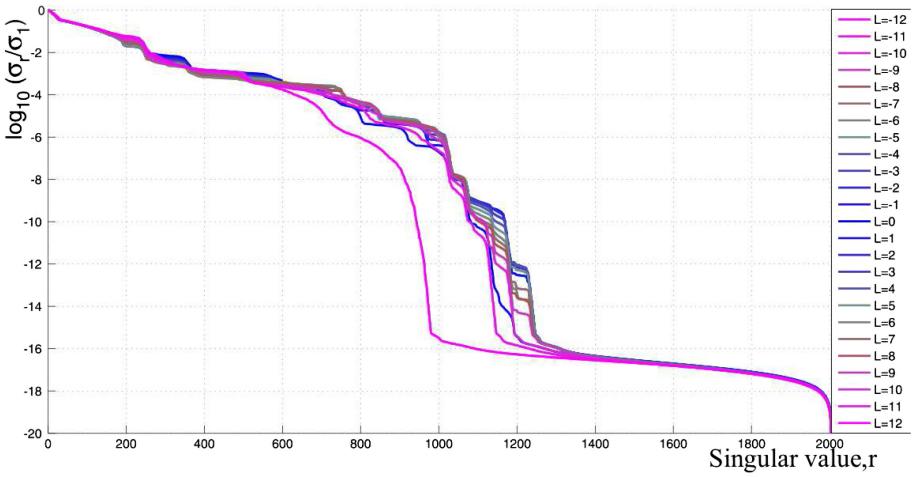

Fig. 1. Singular values in logarithmic scale.

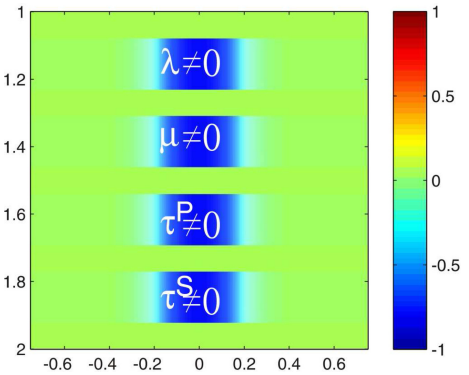

Fig. 2. Sought solution. Heterogeneities of the parameters are located in separated rectangles: $\delta\lambda$ in $z \in [1080, 1230]m$, $\delta\mu$ in $z \in [1310, 1460]m$, $\delta\tau^P$ in $z \in [1540, 1690]m$, $\delta\tau^S$ in $z \in [1770, 1920]m$.

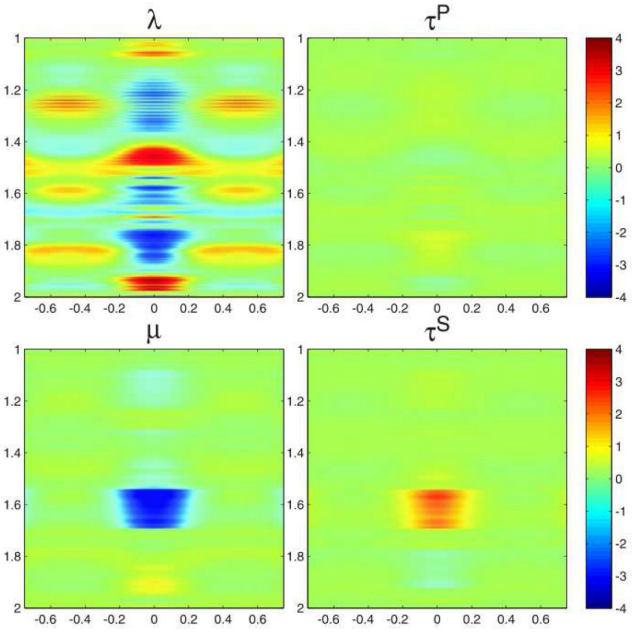

Fig. 3. Truncated SVD-solution for condition number 10^4.

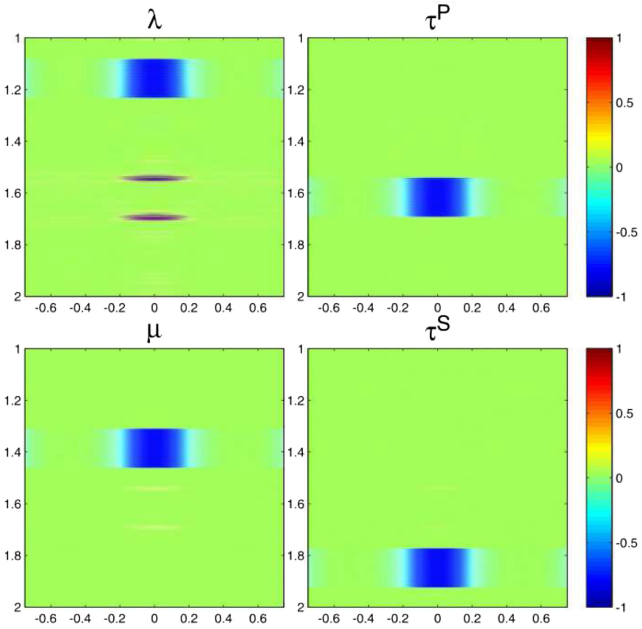

Fig. 4. Truncated SVD-solution for condition number 10^8.

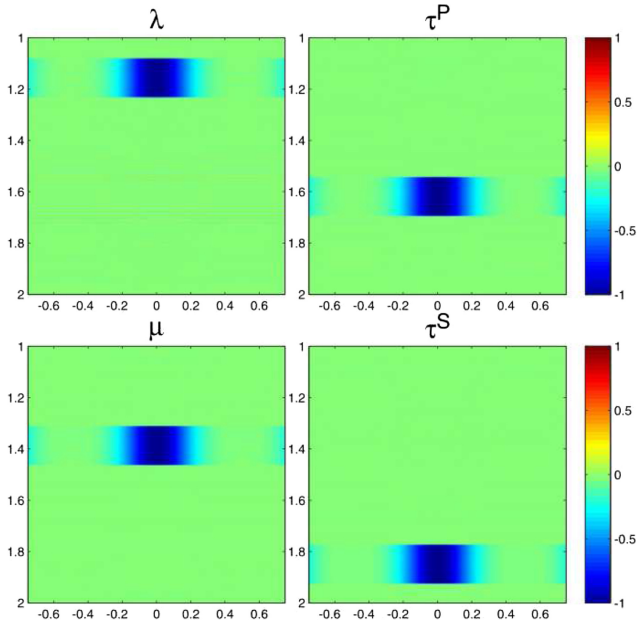

Fig. 5. Truncated SVD-solution for condition number 10^{14}.

4 Conclusions

Thus, analyzing the results obtained in the course of numerical experiments, we can formulate the following conclusions.

1. Straightforward implementation of full-waveform inversion in a viscoelastic medium inevitably leads to the coupling of parameters like the quality factors and the wave propagation velocities of seismic waves. Namely, we come to the impossibility of separating the perturbations of these parameters and their correct spatial localization.
2. To separate these perturbations, it is necessary to perform a singular value decomposition (SVD) of the linear integral operators arising in this case and to determine the subspaces corresponding to the velocities and quality factors perturbations.
3. Based on the given error in the data, determine the opening angle between the sub spaces corresponding to the perturbations of the velocities and quality factors, which ensures the correct separation of the sought parameters.
4. Using the found opening angle, determine the condition number providing correct separation of the velocity and Q-factor disturbances.
5. Based on the condition number, estimate the required accuracy of the matrix approximation of linear integral equations and the permissible error in the data.

We can describe the technique used for the solution of the problem as the follows:

- We search for the attenuation parameters of the viscoelastic medium using the Generalized Standard Linear Solid, the τ-method and the Born approximation.
- The problem of recovery the parameters of a viscoelastic medium is reduced to a system of integral equations using the Fourier transform with respect to time and the coordinates of the sources and receivers.
- We search for the characteristics of the medium as a finite Fourier series. To study the coupling between the parameters λ, τ^P, μ, τ^S we use an intermediate parametrization (that allows to construct linearized operators) and the method based on the truncated singular value decomposition.

5 Author Contributions

V.C. proposes the concept of SVD-analysis in application to the study of the coupling in multiparameters inverse problems.

E.E. derived equations connecting data variability with velocity and quality perturbations of the reference media and performed SVD computations.

G.R. has paid the main efforts to get estimations connecting data perturbations with variability of desired parameters via cond number of the matrix approximation of the operator and its apriori accuracy.

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References

1. Assous, F., Collino, F.: A numerical method for the explanation of sensitivity: the case of the identification of the 2D stratified elastic medium. Inverse Prob. **6**(4), 487–514 (1990)
2. Asvadurov, S., Knizhnerman, L., Pabon, J.: Finite-difference modeling of viscoelastic materials with quality factors of arbitrary magnitude. Geophysics **69**(3), 817–824 (2004)
3. Bogoyavlensky V. Prospects and problems of the Arctic shelf oil and gas field development. Drilling Oil (Burenie i neft') **11**, 4–10 (2012)
4. Blanch, J.O.; Robertsson. J.O.A., Symes, W.W.: Modeling of a constant Q: Methodology and algorithm for an efficient and optimally inexpensive viscoelastic technique. Geophysics **60**(1), 176–184 (1995)
5. Carcione, J.M.: Seismic modeling in viscoelastic media. Geophysics **5**, 110–120 (1993)
6. Cheverda, V.A., Kostin, V.I.: R-pseudoinverses for compact operators in hilbert spaces:existence and stability. J. Inverse Ill-Posed Prob. **3**(2), 131–148 (1995)
7. Christensen R.M.: Theory of viscoelasticity - An introduction. 2nd edn, Academic Press, Cambridge, 364 p (1982)
8. Coleman, B.D., Noll, W.: Foundations of linear viscoelasticity. Rev. Mod. Phys. **33**(2), 239–249 (1961)
9. Curtin, M.E., Sternberg, E.: On the linear theory of visscoelasticity. Arch. Ration. Mech. Anal. **11**, 291–356 (1962)
10. Day, S.M., Minster, J.B.: Numerical simulation of attenuated wavefields using a Pade approximant method. Geophys. Res. **67**, 5279–5291 (1962)
11. Dmitrenko, I., et al.: Recent changes in shelf hydrography in the Siberian arctic: potential for subsea permafrost instability. J. Geophys. Res. **116**, C10027 (2011)
12. Dugarov G.A., Duchkov A.A., Duchkov A.D., Drobchik A.N.: Laboratory study of the acoustic properties of hydrate-bearing sediments. Sci. Rep. Facuilty Phys. Moscow Univ. (5), 1750812 (2017)
13. Efimova E.S.: Reliability of attenuation properties recovery for viscoelastic media. Open J. Appl. Sci. **3**(1B1), 84–88 (2013)
14. Fichtner, A.: Full Seismic Waveform Modeling and Inversion. Springer, Berlin (2011) https://doi.org/10.1007/978-3-642-15807-0
15. Hao, Q., Greenhalgh, S.: The generalized standard-linear-solid model and the corresponding viscoacoustic wave equations revisited. Geophys. J. Int. **219**, 1939–1947 (2019)
16. Hölemann, J.A., Kirillov, S., Klagge, T., Novikhin, A., Kassens, H., Timokhov, L.: Near-bottom water warming in the Laptev sea in response to atmospheric and sea-ice conditions in 2007. Polar Res. **30**, 6425 (2011)
17. Janout, M.A., Hölemann, J., Krumpen, T.: Cross-shelf transport of warm and saline water in response to sea ice drift on the Laptev Sea shelf. J. Geophys. Res. Oceans **118**, 563–576 (2013)

18. Janout, M., et al.: Episodic warming of near-bottom waters under the Arctic sea ice on the central Laptev Sea shelf. Geophys. Res. Lett. **43**(1), 264–272 (2016)
19. Kostov, C., Neklyudov, D., Tcheverda, V.: waveform inversion for macro velocity model recon-struction in look-ahead off-set VSP: numerical SVD-based analysis. Geophys. Prospect. **61**(6), 1099–1113 (2016)
20. Hak, B., Mulder, W.A.: Seismic attenuation imaging with causality. Geophys. J. Int. **184**, 439–451 (2011)
21. Hicks, G.J., Pratt, R.G.: Reflection waveform inversion using local descent methods: Estimating attenuation and velocity over a gas-sand deposit. Geophysics **66**(2), 598–612 (2001)
22. Liu, H.-P., Anderson, D.L., Kanamori, H.: Velocity dispersion due to anelasticity; implications for seismology and mantle composition. Geophysics **47**, 41–58 (1976)
23. McDonal, F.J., Angona, F.A., Mills, R.L., Sengbush, R.L., van Nostrand, R.G., White, J.E.: Attenuation of shear and compressional waves in Pierre shale. Geophysics **23**, 421–439 (1958)
24. McGuire, A.D., et al.: Sensitivity of the carbon cycle in the arctic to climate change. Ecol. Monogr. **79**(4), 523–555 (2009)
25. Mulder, W.A.: Velocity and attenuation perturbations can hardly be determined simultaneously in acoustic attenuation scattering. In: SEG Houston International Exposition and Annual Meeting, pp. 3078–3082 (2009)
26. Mulder, W.A., Hak, B.: An ambiguity in attenuation scattering imaging. Geophys. J. Int. **178**(3), 1614–1624 (2009)
27. Nemirovskaya, I.A.: Petroleum hydrocarbons in the ocean (in russina: Neftjanye uglevodorody v okeane). Priroda **3**, 17–27 (2008)
28. Parra, J.O., Hackert, C.: Wave attenuation attributes as flow unit indicators. Lead. Edge **21**(6), 564–572 (2002)
29. Rachold, V., et al.: Near-shore arctic subsea permafrost. Trans. EOS: Trans. Am. Geophys. Union **88**(13), 149–156 (2007)
30. Romanov, V.G.: The two-dimensional inverse problem for the equation of viscoelasticity. Siberian Math. Mag. **53**(6), 1401–1412 (2012)
31. Ruppel, C.D., Kessler, J.D.: The interaction of climate change and methane hydrates. Rev. Geophys. **55**(1), 126–168 (2017)
32. Sun, Y.F., Goldberg, D. Hydrocarbon signatures from high-resolution attenuation profiles. In: SEG Technical Program Expanded Abstracts, pp. 996–999 (1998)
33. Zener, C.: Elasticity and Anelasticity of Metals, p. 170. University of Chicago Press, Chicago (1948)
34. White, R.E.: The acuracy of estimating Q from seismic data. Geophysics **57**, 1508–1511 (1992)
35. Zhang, D., Lamoureux, M., Margrave, G., Cherkaev, E.: Rational approximation for estimation of quality q factor and phase velocity in linear, viscoelastic, isotropic media. Comput. Geosci. **15**, 117–133 (2011)