Modeling and Identification of Electromechanical Systems Using Orthonormal Jacobi Functions

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Abstract. Solving parameter identification and structure identification in electromechanical systems is relevant in applications for diagnostics of the technical condition of technical systems, as well as in the development of algorithms for controlling machines and mechanisms. Impulse response functions, correlation and autocorrelation coordinate functions are used as dynamic characteristics. The correspondent models unambiguously give an idea of the mathematical description of a linear or linearized electromechanical system. The methods of spectral description of the dynamic characteristics, based on Fourier transforms, are quite effective in relation to physically realizable systems under normal operating conditions. The paper aims to present new approaches to the development of algorithms for identifying electromechanical systems based on the study of the properties of synthesized transformed orthonormal Jacobi functions and the design of spectral models of the impulse response functions of electromechanical systems. The methods of the theory of spectral and operator transformations, as well as functional analysis, are used in the study. The results demonstrate the success of algorithms for nonparametric identification of linear or linearized electromechanical systems based on spectral models of their impulse response functions in the basis of synthesized generalized orthonormal Jacobi functions. The results of the study can be used in practice for the diagnostics and development of control systems for electric drives of machines and installations with changing parameters or structures.

Keywords: Electromechanical systems · Modeling · Identification · Orthonormal functions · Jacobi functions · Impulse response functions · Fourier transforms · Spectral models · Electric drive

1 Introduction

Solving parameter identification and structure identification in electromechanical systems (EMSs) is relevant in applications for diagnostics of the technical condition of technical systems, as well as the development of algorithms for controlling machines and mechanisms [1–5]. This is particularly important in conditions where the parameters and characteristics of individual elements of the system have variability. As an example of such EMSs, one can consider the electric drive systems of machines and

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equipment, the quality of management of which determines not only technological processes but also the success of the whole enterprise [1, 6]. Such machines can be found in the mining industry (excavators, conveyors, mine lifting machines, etc.), as well as in other industries (mechanical engineering, transport, etc.) [7–10]. The elements of the EMSs of these machines change their properties under the influence of external factors. Thus, during the operation of long-distance long-line conveyor systems, through which mineral raw materials are transported, the conveyor belt (the elastic element of the EMS that connects its different masses) under different seasonal climatic conditions has different elastic properties, which objectively affects the procedures for the synthesis and implementation of algorithms for controlling the electric drive systems of the entire conveyor.

In most cases, EMSs belong to the category of physically realizable systems, so their dynamic characteristics can be described by classical methods based on systems of differential equations or one of the forms of their representation – operator schemes or operator linear equations [11, 12]. At the same time, in the formulation of the problem of identifying the parameters or structure of a particular EMS in the conditions of its normal functioning, it becomes important to build equivalent models based on the description of its dynamic characteristics. Impulse response functions (IRFs), correlation and autocorrelation coordinate functions are most commonly used as dynamic characteristics that are mathematically unambiguously related to differential equations, Laplace operator equations of the system (response functions), frequency characteristics [1, 13–15]. Their models unambiguously give the idea of the mathematical description of a linear or linearized EMS. The methods of spectral description of the dynamic characteristics, based on Fourier transforms, are quite effective in physically realizable systems under normal operating conditions. Spectral methods using orthonormal or orthogonal functions as a functional basis are of particular interest [1, 15].

If the IRFs of a dynamical system are used as the dynamic characteristic, then its mathematical model can be expressed as [13, 15]:

$$h_\delta(\tau) = \sum_{j=0}^{\infty} \mu_j \Phi_j(\tau)$$

where $h_\delta(\tau)$ are the IRFs; $\Phi_j(\tau)$ is a system of functions, for example, orthogonal or orthonormal ones; μ_j are the coefficients of the Fourier expansion of $h_\delta(\tau)$ in the basis of $\Phi_j(\tau)$ functions. Here, $\Phi_j(\tau)$ has the properties of the orthonormal or orthogonal function on the interval of the argument τ $[0, \infty)$.

The model scheme is shown in Fig. 1.

The use of orthonormalized Chebyshev functions as such a functional basis makes it possible to ensure the uniqueness of the models, their connection with other operator models (for example, Laplace), stability in determining the model parameters, the implementation of computational procedures, etc. [13]. Chebyshev-Laguerre and Chebyshev-Legendre functions, as well as Chebyshev-Hermite ones, are quite reasonably used as orthonormal Chebyshev functions [13, 16–18]. This class of continuous functions is defined on the time interval $(0, \infty)$ that coincides with the interval of determining the IRF, and also allows designing models of stable dynamical systems. However, most of these functions have a limited set of parameters, suitable to implement the optimization

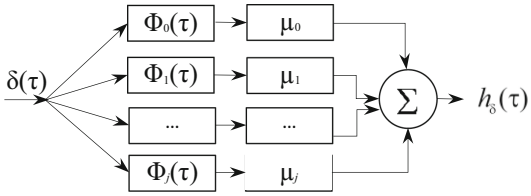


Fig. 1. Model scheme of impulse response functions, where $\delta(\tau)$ is the Dirac function

algorithms for synthesized models. This article is devoted to the study of models of EMSs in the basis of Chebyshev-Legendre functions as a special case of Jacobi functions. The problem of demonstrating new approaches to the design of EMS identification algorithms based on the study of the properties of synthesized transformed orthonormal Jacobi functions and on the construction of spectral models of EMS IRFs is considered in the paper. The methods of the theory of spectral and operator transformations, as well as functional analysis, are used in the paper.

2 Synthesis and Study of Orthonormal Jacobi Functions

Standardized Jacobi polynomials can be expressed as

$$J_n(x;a,b) = \frac{\Gamma(b+n+1)\cdot\Gamma(a+n+1)}{2^n} \times \sum_{k=0}^{n} \frac{(x-1)^{n-k}(1+x)^k}{k!\Gamma(n-k+1)\Gamma(a+n-k+1)\Gamma(b+k+1)}, \tag{1}$$

where $x \in (-1,\ 1)$, $a > -1,\ b > -1$; $\Gamma(\cdot)$ is Euler's gamma function [19].

The weight function of the polynomial is defined by the expression

$$h(x) = (1-x)^a(1+x)^b. \tag{2}$$

The Rodrigue formula for Eq. (1) is represented as

$$J_n(x;a,b) = \frac{(-1)^n}{2^n n!}(1-x)^{-a}(1+x)^{-b}\frac{d^n}{dx^n}\Big[(1-x)^{a+n}(1+x)^{b+n}\Big]. \tag{3}$$

Using the Rodrigue formula (3), the integration in parts is performed $n-1$ times to obtain the following integral expression of the square of the norm:

$$\begin{aligned}\|J_n\|^2 &= \frac{(-1)^2}{2^n\cdot\Gamma(n+1)}\int_{-1}^{1}\frac{d^n}{dx^n}\Big[(1+x)^a(1-x)^b\Big]J_n(x;a,b)\\ &= \frac{\Gamma(a+n+1)\Gamma(b+n+1)}{\Gamma(n+1)\cdot(a+b+2n+1)\Gamma(a+b+n+1)}2^{a+b+1}.\end{aligned} \tag{4}$$

In accordance with Eq. (4), the expression for the orthonormal Jacobi polynomial can be written

$$\begin{aligned}\hat{J}_n(x;a,b) &= \sqrt{\Gamma(n+1)\cdot(a+b+2n+1)\Gamma(a+b+n+1)\Gamma(b+n+1)\cdot\Gamma(a+n+1)}\\ &\times 2^{-\frac{1}{2}(a+b+1)-n}\sum_{k=0}^{n}\frac{(x-1)^{n-k}(1+x)^k}{\Gamma(k+1)\cdot\Gamma(n-k+1)\Gamma(a+n-k+1)\Gamma(b+k+1)}.\end{aligned} \tag{5}$$

In order to obtain functionals that are orthonormal on the interval $(0; \infty)$ and have satisfactory approximation properties, the modification of the standardized Jacobi polynomials was performed.

At the first stage, the substitution of $x = 1 - 2y$ in $J_n(x)$ was realized.

It allows forming the displaced polynomial that has the properties of orthogonality in the interval (0; 1):

$$\dot{J}_n(y; a, b) = (-1)^n \Gamma(a + n + 1)\Gamma(b + n + 1) \times \sum_{k=0}^{n} \frac{y^{n-k}(y-1)^k}{\Gamma(k+1)\Gamma(n-k+1)\Gamma(a+n-k+1)\Gamma(b+k+1)}. \quad (6)$$

The binomial formula $(1 - y)^k = \sum_{j=0}^{k} \frac{(-1)^j k!\, y^j}{j!(k-j)!}$ allows obtaining a different formula for biased orthogonal Jacobi polynomials:

$$\dot{J}_n(y; a, b) = (-1)^n \Gamma(a + n + 1)\Gamma(b + n + 1) \times \sum_{k=0}^{n} \frac{1}{\Gamma(k+1)\Gamma(n-k+1)\Gamma(a+n-k+1)\Gamma(b+k+1)} \times \sum_{j=0}^{k} \frac{(-1)^j \Gamma(k+1) \cdot y^{j+n-k}}{\Gamma(j+1) \cdot \Gamma(k-j+1)}. \quad (7)$$

The next transformations of the considered polynomials are possible by implementing the substitution of $y = e^{-u \cdot t}$ in Eq. (6) or (7), where u is the scale parameter.

After these transformations, the expression follows:

$$\overline{J}_n(a, b, u, t) = (-1)^n \Gamma(b + n + 1)\Gamma(a + n + 1) e^{-unt} \times \sum_{k=0}^{n} \frac{[1-e^{ut}]^k}{\Gamma(k+1)\Gamma(n-k+1)\Gamma(a+n-k+1)\Gamma(b+k+1)}. \quad (8)$$

Equation (8) defines transformed orthogonal Jacobi functions that have orthogonality properties on the interval $(0; \infty)$.

The established functional dependencies allow synthesizing spectral models of dynamic systems, but these models will not be unique. This situation will require additional procedures to check the control algorithms created on the basis of these models and identify EMSs with different properties. For example, for systems with different oscillations. Therefore, it is rational to use orthonormal functions while constructing spectral models.

The norm of the resulting functional can also be determined from Eq. (4).

The necessary orthonormality condition for classical orthogonal complexes is defined as follows [19]:

$$\int_a^b J_n(x) J_m(x) h(x) dx = 0 \text{ for } n \neq m \text{ and } \int_a^b J_n(x) J_m(x) h(x) dx = 1 \text{ for } n = m \quad (9)$$

where a and b are the orthogonality interval, and $h(x)$ is the weight function of the polynomial.

The transformations of the weight function (2) were performed in the same sequence as for the transformation of the classical Jacobi polynomials:

$$H(a,b,t,u) = h(x)_{x=1-2e^{-u\cdot t}}\Big| = 2^{a+b}e^{-uta}\left(1-e^{-ut}\right)^{b}. \tag{10}$$

The general formula defining the transformed generalized synthesized orthonormal Jacobi functions (SOJFs) was obtained considering the above conditions of the existence of the orthonormal functional, Eq. (8) for the transformed Jacobi functions, and Eq. (9) for its norm,

$$\hat{J}_n(u,t;a,b) = (-1)^{\frac{2n+b}{2}}\sqrt{u\,\Gamma(a+n+1)\Gamma(n+1)\Gamma(a+b+n+1)(a+b+2n+1)}$$
$$\times\sqrt{\Gamma(a+n+1)}\times e^{-u\,t\,\frac{a+b+2n+1}{2}}\cdot\sum_{k=0}^{n}\frac{\left[1-e^{(ut)}\right]^{\frac{2k+b}{2}}}{\Gamma(k+1)\Gamma(n-k+1)\Gamma(a+n-k+1)\Gamma(b+k+1)}. \tag{11}$$

Using the binomial Newton formula allows obtaining a different expression for SOJF:

$$\hat{J}_n(u,t;a,b) = (-1)^{\frac{2n+b}{2}}\sqrt{u\,\Gamma(a+n+1)\Gamma(n+1)\Gamma(a+b+n+1)(a+b+2n+1)}$$
$$\times\sqrt{\Gamma(a+n+1)}e^{-u\,t\,\frac{a+b+2n+1}{2}}\cdot\sum_{k=0}^{n}\frac{1}{\Gamma(k+1)\Gamma(n-k+1)\Gamma(a+n-k+1)\Gamma(b+k+1)}$$
$$\times\sum_{j=0}^{\left(\frac{2k+b}{2}\right)}\frac{(-1)^j\Gamma\left(\frac{2k+b}{2}+1\right)e^{utj}}{\Gamma(j+1)\left[\Gamma\left(\frac{2k+b}{2}\right)-j+1\right]} \tag{12}$$

or

$$\hat{J}_n(u,t;a,b) = (-1)^{\frac{2n+b}{2}}\sqrt{u\,\Gamma(a+n+1)\Gamma(n+1)\Gamma(a+b+n+1)(a+b+2n+1)}$$
$$\times\sqrt{\Gamma(a+n+1)}\sum_{k=0}^{n}\frac{1}{\Gamma(k+1)\Gamma(n-k+1)\Gamma(a+n-k+1)\Gamma(b+k+1)}$$
$$\times\sum_{j=0}^{\left(\frac{2k+b}{2}\right)}\frac{(-1)^j\Gamma\left(\frac{2k+b}{2}+1\right)e^{ut\left(j-\frac{a+b+2n+1}{2}\right)}}{\Gamma(j+1)\,\Gamma\left[\left(\frac{2k+b}{2}\right)-j+1\right]} \tag{13}$$

The graphical interpretation of two orthonormal transformed Jacobi functions for $n = 1$ and $n = 5$ is shown in Fig. 2. The type of the plots indicates that the order of the transformed functions corresponds to the number of transitions of the corresponding function through zero.

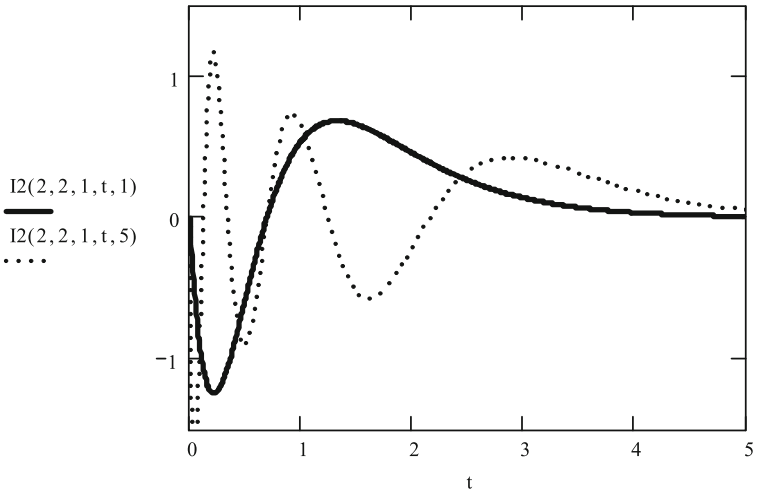


Fig. 2. Dependence of transformed Jacobi functions on the parameter t ($J(2, 2, 1, t, 1)$ at $a = 2$, $b = 2$, $u = 1$, and n = 1; $J(2, 2, 1, t, 5)$ at $a = 2$, $b = 2$, $u = 1$, and $n = 5$)

3 Some Properties of Orthonormal Jacobi Functions

Theorem 1. On the recurrent formula of transformed generalized orthonormal Jacobi functions.

For the transformed generalized orthonormal Jacobi functions, the recurrent formula is true

$$\begin{aligned} &2 \cdot \sqrt{\frac{(a+n+1)(a+b+n+1)(n+1)(b+n+1)}{(a+b+2n+3)(a+b+2n+1)(a+b+2n+1)^2}}\hat{J}_{n+1}(u, t; a, b) \\ &= \left(1 - 2 \cdot e^{-u\,t} - \frac{b^2-a^2}{(a+b+2n)(a+b+2n+2)}\right)\hat{J}_n(u, t; a, b) \\ &- 2 \cdot \sqrt{\frac{n(a+n)(a+b+n)(b+n)}{((a+b+2n-1)(a+b+2n+1)(a+b+2n)^2}}\hat{J}_{n-1}(u, t; a, b). \end{aligned} \tag{14}$$

To prove this theorem, the recurrent formula for orthonormal polynomials was used, according to which the condition is fulfilled [19]:

$$\lambda_n \hat{P}_{n+1}(x) = (x - \eta_n)\hat{P}_n(x) - \lambda_{n-1}\hat{P}_{n-1}(x), \tag{15}$$

where $\lambda_n = \frac{\mu_n}{\mu_{n+1}}$; μ_n is the highest coefficient of the orthonormal polynomial $\hat{P}_n(x)$;

η_n is the coefficient determined by the type of the polynomial.

For orthonormal Jacobi polynomials (5), the expansion coefficients $\hat{P}_n(x) = \mu_n x^n + \nu_n x^{n-1} \ldots$ can be determined using the following expressions:

$$\mu_n = \frac{\Gamma(a+b+2n+1)}{\sqrt{2^{a+b+2n+1}\Gamma(a+b+n+1)(a+b+2n+1)\Gamma(n+1)\Gamma(a+n+1)\Gamma(b+n+1)}},$$
$$\nu_n = \frac{(a-b)\Gamma(a+b+2n)\sqrt{n(a+b+2n+1)}}{\sqrt{2^{a+b+2n+1}\Gamma(a+b+n+1)\Gamma(n)\Gamma(a+n+1)\Gamma(b+n+1)}}.$$

In the next step, the parameter λ_n in Eq. (15) was determined

$$\lambda_n = \frac{\mu_n}{\mu_{n+1}} = 2\sqrt{\frac{(n+1)(a+n+1)(a+b+n+1)(b+n+1)}{(a+b+2n+2)^2(a+b+2n+3)(a+b+2n+1)}}. \tag{16}$$

Solving the equations composed under the condition that the coefficients are equal for the same powers of the argument, the expression for η_n can be determined as:

$$\lambda_n \nu_{n+1} = \nu_n - \eta_n \mu_n.$$

Solving this equation, the following expression can be obtained:

$$\eta_n = \frac{b^2 - a^2}{(a+b+2n+2)(a+b+2n)}. \tag{17}$$

The expression equivalent to Eq. (14) can be obtained from the three-term recurrent formula taking into account the substitutions carried out earlier when performing transformations on Jacobi polynomials. Thus, the theorem on the recurrent formula of transformed orthonormal Jacobi polynomials can be considered proven.

The recurrent formula (14) can be represented so that it will be possible to calculate the n-th function based on the values of the previous two functions:

$$\begin{aligned} &2 \cdot \sqrt{\tfrac{(a+n+1)(a+b+n+1)(n+1)(b+n+1)}{(a+b+2n+3)(a+b+2n+1)(a+b+2n+1)^2}} \cdot \hat{J}_n(u,t;a,b) \\ &= \left(1 - 2 \cdot e^{-ut} - \tfrac{b^2-a^2}{(a+b+2n)(a+b+2n+2)}\right) \times \hat{J}_{n-1}(u,t;a,b) \\ &- 2 \cdot \sqrt{\tfrac{n(a+n)(a+b+n)(b+n)}{(a+b+2n-1)(a+b+2n+1)(a+b+2n)}} \cdot \hat{J}_{n-2}(u,t,a,b). \end{aligned}$$

The identified recurrent formula is essential in determining the parameters of spectral models in the conditions of limited information, as well as in the formalization of mathematical operators in the structure of mathematical models.

4 Spectral Models Based on Orthonormal Jacobi Functions

IRF decomposition coefficients are determined using the method of the spectral decomposition of the IRF in the basis of SOJF, Eqs. (11–13), in accordance with the following expression

$$\gamma_i = \int_0^\infty h_\delta(\tau)\hat{J}_i(u,t;a,b)d\tau. \tag{18}$$

The spectral model is defined as

$$h_\delta(\tau) = \sum_{j=0}^{\infty} \gamma_j \hat{J}_j(u,t;a,b). \tag{19}$$

Equation (11) allows obtaining the general formula for the orthogonal spectral model of the IRF in the basis of orthonormal Jacobi functions:

$$h_\delta(\tau)=\sum_{j=0}^{\infty}\left\{\begin{array}{l}\gamma_j(-1)^{\frac{2j+b}{2}}\sqrt{u\,\Gamma(a+j+1)\Gamma(j+1)\Gamma(a+b+j+1)(a+b+2j+1)}\\ \times\sqrt{\Gamma(a+j+1)}e^{-u\,t\,\frac{a+b+2j+1}{2}}\\ \times\sum\limits_{k=0}^{j}\frac{\left[1-e^{(ut)}\right]^{\frac{2k+b}{2}}}{\Gamma(k+1)\Gamma(j-k+1)\Gamma(a+j-k+1)\Gamma(b+k+1)}\end{array}\right\}. \tag{20}$$

The application of the binomial Newton formula allows obtaining the different expression for the IRF model:

$$h_\delta(\tau)=\sum_{j=0}^{\infty}\left\{\begin{array}{l}\gamma_j(-1)^{\frac{2j+b}{2}}\sqrt{u\,\Gamma(a+j+1)\Gamma(j+1)\Gamma(a+b+j+1)(a+b+2j+1)}\\ \times\Gamma(a+j+1)\sum\limits_{k=0}^{j}\frac{1}{\Gamma(k+1)\Gamma(j-k+1)\Gamma(a+j-k+1)\Gamma(b+k+1)}\\ \times\sum\limits_{i=0}^{\binom{2k+b}{2}}\frac{(-1)^i\Gamma\left(k+\frac{1}{2}b+1\right)e^{-\frac{1}{2}ut(a+b+2n-2i+1)}}{\Gamma(i+1)\Gamma\left(k+\frac{1}{2}b-i+1\right)}\end{array}\right\} \tag{21}$$

It can be shown that n-th Jacobi function in the space of Laplace transformations can be defined by the following formula:

$$\begin{array}{l}L\left[\hat{J}_n(u,a,b;p)\right]==(-1)^{\frac{2n+b}{2}}\sqrt{u\,\Gamma(a+n+1)\Gamma(n+1)\Gamma(a+b+n+1)}\\ \times\sqrt{(a+b+2n+1)\Gamma(b+n+1)}\\ \times\sum\limits_{k=0}^{n}\left\{\frac{-\Gamma\left(-\frac{p}{u}-\frac{a+b+1}{2}-n\right)\Gamma\left(k+\frac{b}{2}+1\right)}{\Gamma(k+1)\Gamma(n-k+1)\Gamma(a+n-k+1)\Gamma(b+k+1)\Gamma\left(-\frac{p}{u}+k-\frac{a-1}{2}-n\right)}\right\},\end{array} \tag{22}$$

where L $(\cdot)$ is the notation of Laplace transformations.

Some simplifications in order to develop final algorithms were introduced. Let $a\neq 0,\quad b=0$, then on the basis of Eqs. (22), the following operator expression for SOJF was obtained

$$\begin{array}{l}L\left[\hat{J}_n(u,a,b=0;p)\right]=(-1)^{n+1}\sqrt{u\,(a+2n+1)}\\ \times\frac{\Gamma\left(-\frac{p}{u}-\frac{a-1}{2}-n\right)\Gamma\left(-\frac{p}{u}+\frac{a+1}{2}-n\right)}{u\left(-\frac{p}{u}-\frac{a-1}{2}-n\right)\Gamma\left(-\frac{p}{u}-\frac{a-1}{2}\right)\Gamma\left(-\frac{p}{u}+\frac{a+1}{2}\right)}.\end{array} \tag{23}$$

The ratio of operator transforms of the orthonormal functions with the ordinal numbers $j+1$ and j for SOFJ at $a\neq 0,\quad b=0$ was determined:

$$\frac{L\left\{\hat{J}_{n+1}(u,a,b=0;p)\right\}}{L\left\{\hat{J}_n(u,a,b=0;p)\right\}}=\sqrt{\frac{a+2n+3}{a+2n+1}}\left(\frac{u\frac{a+2n+1}{2}-p}{u\frac{a+2n+3}{2}+p}\right). \tag{24}$$

The parameters of the chain former of Jacobi functions at the value of the parameter $b = 0$ at each stage were set as:

$$W_{J0}(p) = \frac{L\left\{\hat{J}_1(u,t,a,b=0;p)\right\}}{L\left\{\hat{J}_0(u,t,a,b=0;p)\right\}} = \sqrt{\frac{a+3}{a+1}}\left[\frac{\frac{u}{2}(a+1)-p}{\frac{3u}{2}(a+3)+p}\right];$$

$$W_{P1}(p) = \frac{L\left\{\hat{J}_2(u,t,a,b=0;p)\right\}}{L\left\{\hat{J}_1(u,t,a,b=0;p)\right\}} = \sqrt{\frac{a+5}{a+3}}\left[\frac{\frac{u}{2}(a+3)-p}{\frac{u}{2}(a+5)+p}\right];$$

$$W_{P2}(p) = \frac{L\left\{\hat{J}_3(u,t,a,b=0;p)\right\}}{L\left\{\hat{J}_2(u,t,a,b=0;p)\right\}} = \sqrt{\frac{a+7}{a+5}}\left[\frac{\frac{u}{2}(a+5)-p}{\frac{u}{2}(a+7)+p}\right];$$

$$W_{P3}(p) = \frac{L\left\{\hat{P}_4(u,t)\right\}}{L\left\{\hat{P}_3(u,t)\right\}} = \sqrt{\frac{a+9}{a+7}}\left[\frac{\frac{u}{2}(a+7)-p}{\frac{u}{2}(a+9)+p}\right] \quad \ldots$$

Figure 3 shows the block diagram of the former of orthonormal Jacobi functions for the corresponding parameter values.

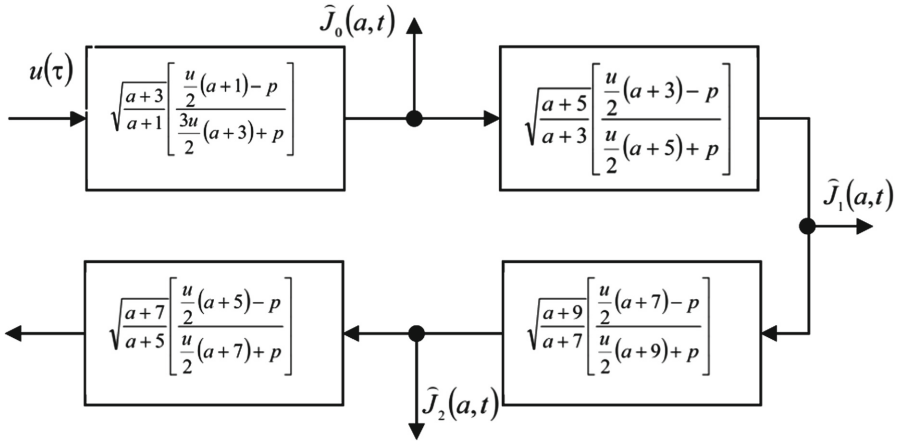


Fig. 3. The block diagram of the former of orthonormal Jacobi functions

If $u(\tau) = \delta(\tau)$, where $\delta(\tau)$ is the Dirac function, then the corresponding Jacobi functions can be observed at the output of each filter cascade.

The formula for the spectral orthogonal model for the IRF, Eq. (20), also allows synthesizing the block diagram of the orthogonal EMS model in the SOJF basis (Fig. 4).

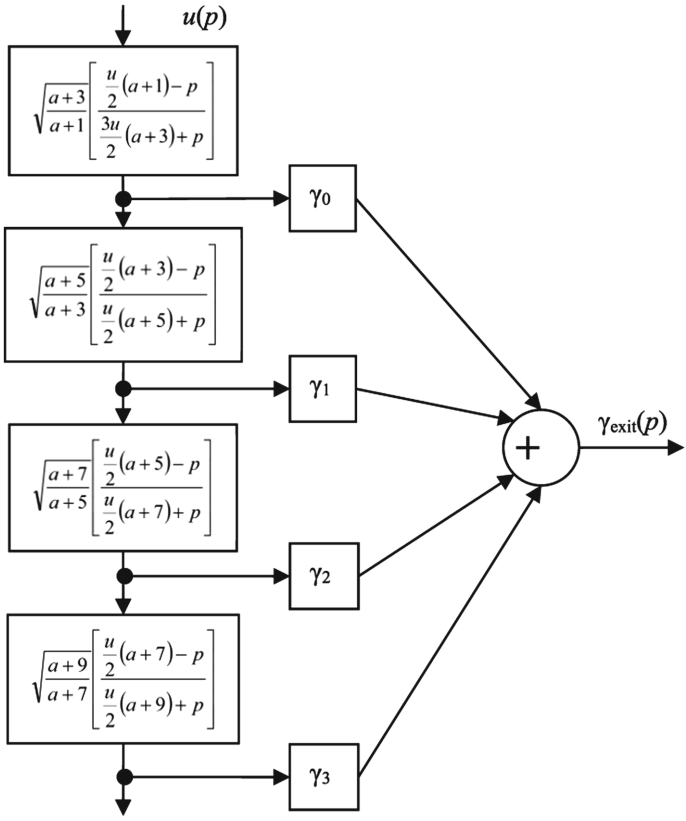


Fig. 4. The block diagram of the orthogonal EMS model in the SOJF basis

5 Nonparametric Identification Based on Spectral Models in the Basis of Orthonormal Jacobi Functions

The relationship between the components of the IRF spectral model in the SOJF basis and the parameters of the transfer function was determined and can be represented as the ratio of two polynomials:

$$W(p) = \frac{A(p)}{G(p)} = \frac{a_0 + a_1 p + a_2 p^2 + \ldots a_n p^n}{g_0 + g_1 p + g_2 p^2 + \ldots g_n p^n}. \tag{25}$$

The identification problem is reduced in this case to determining the coefficients of the polynomials $A(p) = (a_0,\ a_1,\ a_2\ \ldots\ a_n)$ and $B(p) = (g_0,\ g_1,\ g_2\ \ldots\ g_n)$.

To demonstrate the solution, the nonparametric model based on the first three SOJFs with parameter values $a \neq 0;\quad b = 0$ was considered. The general operator expression for the transfer function will have the form:

$$\begin{aligned} W_1(p) &= \gamma_0 2\frac{\sqrt{u(a+1)}}{2p+u(a+1)} + \gamma_1 2\frac{[u(a+1)-2p]\sqrt{u(a+3)}}{[2p+u(a+1)][2p+u(a+3)]} \\ &+ \gamma_2 2\frac{\sqrt{u(a+5)}[u(a+1)-2p][u(a+3)-2p]}{[2p+u(a+1)][2p+u(a+3)][2p+u(a+5)]}. \end{aligned} \tag{26}$$

Equating the coefficients for the same degree of p, the expressions for the coefficients of the polynomials $A(p)$ and $B(p)$ in Eq. (25) were obtained:

$$a_0 = \gamma_0 u^2 \sqrt{u(a+1)}(a+5)(a+3) + \gamma_2 u^2 \sqrt{u(a+5)}(a+3)(a+1)$$
$$+ \gamma_1 \left[u^2 \sqrt{u(a+3)} a(a+6) + \tfrac{5}{2} u^3 (a+3) \right],$$
$$a_1 = u^{\frac{3}{2}} \left\{ 4\left[\gamma_0 \sqrt{a+1} - \gamma_2 \sqrt{a+5} \right] - 8\left[\gamma_2 \sqrt{a+5} - 2\gamma_0 \sqrt{a+1} + \gamma_1 \sqrt{a+3} \right] \right\},$$
$$a_2 = 4\sqrt{u}\left(\sqrt{a+1}\gamma_0 - \sqrt{a+3}\gamma_1 + \sqrt{a+5}\gamma_2 \right)$$
$$g_0 = \tfrac{1}{2} u^3 (a+1)(a+3)(a+5),\ g_1 = u^2\left(3a^2 + 18a + 23\right), g_2 = 6u(a+3),\ g_3 = 4.$$

Similarly, using the above method, the expressions for determining the relationship between the parameters of the approximating transfer functions at different orders of SOJF can be obtained.

6 Synthesis and Study of Spectral Models of Two-Mass EMSs in the Basis of Orthonormal Jacobi Functions

The main aspects of the design of spectral models of two-mass EMSs based on the spectral decomposition of the IRF in the SOJF basis were studied. The case of the open two-mass EMS was considered, the differential equation of which has the form [12]:

$$\begin{cases} J_1 \cdot \frac{d\omega_1}{dt} + c_1 \cdot (\varphi_1 - \varphi_2) = M_d; \\ J_2 \cdot \frac{d\omega_2}{dt} - c_1 \cdot (\varphi_1 - \varphi_2) = -M_c; \\ (u - \omega_1 \cdot k_c) \cdot k_a = M_d + T_e \cdot \frac{dM_d}{dt}, \end{cases} \tag{27}$$

where J_1 and J_2 are the moments of inertia of the first and second EMS masses, respectively; c_1 is the rigidity coefficient of the connection between the first and second EMS masses; ω_1, φ_1, ω_2, and φ_2 are the coordinates (angular velocity and angle of rotation) of the first and second EMS masses, respectively; k_c, k_a and T_e are the design constants of the electric motor device; M_d is the moment of the electric motor device; M_c is the moment of resistance on the executive body; u is the controlling coordinate.

The roots of the characteristic equation have the form:

$$p_{1,2} = -10.36 \pm 31.308 \cdot j,$$

$$p_{1,2} = -7.7 \pm 22.89 \cdot j, \text{ where } j = \sqrt{-1}$$

The coefficients of IRF expansion of the two-mass EMS in the eighth-order SOJF basis, determined with the help of Eq. (18), are presented in Fig. 5 for the fixed value of the scale parameter $u = 7$.

The data presented in Fig. 5 indicate the satisfactory convergence of the spectral model, as well as the effectiveness of the algorithmic support for calculating the model parameters.

To assess the reliability of synthesis of the IRF model, the square of the normalized variance of the IRF spectral model is used, which is determined by the expression [15]:

$$\sigma_n^2(u) = \frac{\int\limits_0^\infty \left[\hat{h}_\delta(\tau) - \sum\limits_{i=0}^{n} \vartheta_i \hat{H} e_i(u,\tau)\right]^2 d\tau}{\int\limits_0^\infty \hat{h}_\delta^2(\tau) d\tau}. \tag{28}$$

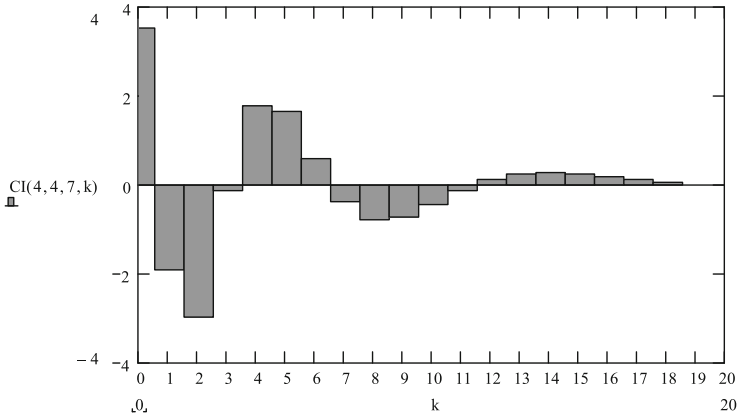


Fig. 5. The values of IRF expansion coefficients of the two-mass EMS control channel (IRF EMS spectral model), established using SOJF ($a = 4$, $b = 4$) of 8th order at the value of the scale parameter $u = 7$

The best approximation of the spectral model Eq. (19) is achieved by choosing the optimal values of the parameter u. As a criterion, the minimum condition of Eq. (28) is used:

$$\min\left\{\sigma_n^2(u)\right\}.$$

The dependencies of the square of the normalized variance of the spectral IRF model over the control channel of the two-mass EMS on the values of the scale parameters of the transformed orthonormal Jacobi functions for different orders of the spectral model are shown in Fig. 6.

The dependencies of the square of the normalized variance on the scale parameter u indicate the stability of the method of the design of IRF spectral models of EMSs in the basis of orthonormal Jacobi functions presented in this paper.

The proposed methodological approach to the spectral models of electromechanical systems based on orthonormal Jacobi functions has a certain practical significance. The following applications may be noted:

- the possibility of simplifying the linear complex and multi-coupled models of electromechanical multi-mass systems, for example, described by Eq. (27) to the level of the spectral sequential model shown in Fig. 4;

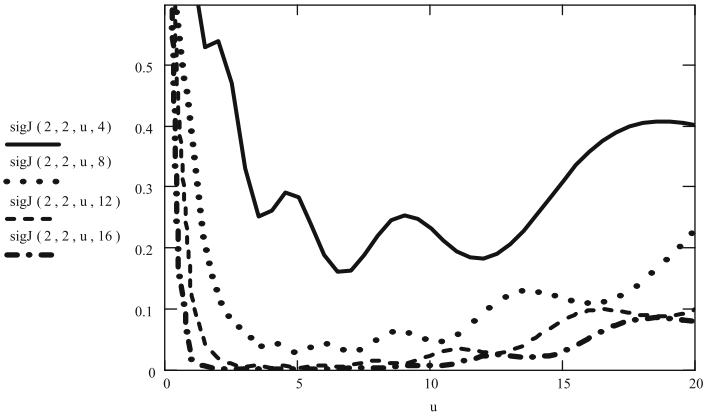


Fig. 6. The dependence of the square of the normalized dispersion of the IRF spectral model of the two-mass EMS control channel on the value of the scale parameter SOJF: ($a = 2$, $b = 0$) sigJ (0.0, u, 4) – $n = 4$; sigJ (0.0, u, 8) – $n = 8$; sigJ (0.0, u, 12) – $n = 12$; sigJ (0.0, u,16) – $n = 16$, where a, b are the parameters of the spectral model; u is the scale parameter of the optimization procedure; n is the order of the spectral model)

- diagnostics of electric drive systems of machinery (for example, excavators, belt conveyors, etc.) based on the algorithms of parametric identification presented in the paper;
- new opportunities in the interpretation of experimental studies of the dynamic characteristics of electromechanical systems.

The above-mentioned trends will be the subject of further study by the author.

7 Conclusion

The studies presented in this paper demonstrate the success of using the synthesized transformed orthonormal Jacobi functions to solve the problems of constructing IRF spectral models of EMSs.

The parameters of IRF spectral models of EMSs with a high degree of confidence can be used to synthesize algorithms for nonparametric identification of EMSs.

The applicability of the presented Jacobi functions in nonparametric identification algorithms using examples of a two-mass EMS was examined in the paper. This creates the prerequisites for solving the class of problems, for example, the control problems in electric drive systems and the systems of automation of technological processes.

8 Key Focus of Further Study

Having determined the class of EMS mathematical models based on IRF spectral representation in the SOJF basis, it is considered appropriate to solve a number of problems in further study:

- to define parametric identification algorithms based on the data on the IRF EMS spectral model in the SOJF basis;
- to identify the impact of SOJF parameters on the quality of the EMS parametric identification procedure;
- to evaluate the influence of the parameters a and b of SOJF on the optimization procedure of the IRF EMS spectral model;
- to determine the influence of reliability indicators of parametric and nonparametric identification procedures based on IRF EMS spectral models on the quality of control algorithms applied to closed control systems for electric drives of machines and installations;
- to develop algorithmic and methodological support for diagnostics systems of the parameters and structure of electric drive systems of machines and equipment in normal operation.

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