

Locating Emergency Vehicles: Robust Optimization Approaches

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Abstract. The location of emergency vehicles is crucial for guaranteeing that populations have access to emergency services and that the provided care is adequate. These location decisions can have an important impact on the mortality and morbidity resulting from emergency episodes occurrence. In this work two robust optimization models are described, that explicitly consider the uncertainty that is inherent in these problems, since it is not possible to know in advance how many will be and where will the emergency occurrences take place. These models consider the minimization of the maximum regret and the maximization of the minimum coverage. They are based on a previous work from the same authors, where they develop a model with innovative features like the possibility of vehicle substitution and the explicit consideration of vehicle unavailability by also representing the dispatching of the vehicles. The developed robust stochastic models have been applied to a dataset composed of Monte Carlo simulation scenarios that were generated from the analysis of real data patterns. Computational results are presented and discussed.

Keywords: Emergency vehicles · Location · Optimization · Regret

1 Introduction

The optimal location of emergency vehicles has been largely studied in the past years [1]. Initially, most of the optimization models developed were deterministic, and the uncertainty that is inherent to emergency episodes occurrence was not explicitly considered. Historically, it is usually assigned to Toregas et al. [2] the first publication where a set covering model is used for determining the location of emergency services [3, 4]. The main objective was to guarantee total coverage, minimizing the total number of located services. More recently, other models have been developed, considering in an explicit way uncertainty [5] or survival related features [6, 7].

When considering the location of emergency vehicles, it is important to take into consideration that a vehicle will not be always available to be assigned to an emergency episode. If, for instance, a vehicle is already assigned to an emergency episode that is still taking place, then this vehicle cannot be used for any other episodes simultaneously occurring. One way of dealing with this situation is to consider double coverage.

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This means that a demand point is considered covered only if there are at least two vehicles located within the maximum defined time limit (see, for instance, [8–10]). Another way of dealing with the possibility of several simultaneous emergency calls is to use queue models. Yoon and Albert [11] present a coverage model with different priorities assigned to emergency occurrences, and considers explicitly the existence of queues. It is also possible to consider a given probability of a vehicle being unavailable [12]. McLay [13] considers two different types of vehicles. The demand is classified into three different priority levels, and congestion is explicitly considered. The objective is to maximize the coverage of the more severe occurrences, to maximize survival possibilities.

The occurrence of emergency episodes is inherently uncertain, and it is important to take this uncertainty explicitly into play when deciding where to locate emergency resources. Sung and Lee [14], for instance, develop a model based on a set of scenarios that represent the uncertainty of emergency calls in different time periods. Bertsimas and Ng [15] present a stochastic formulation, minimizing the non-assisted emergency calls. Beraldi, Bruni and Conforti [16] present a stochastic model using probabilistic restrictions that guarantee a determined level of service. In a following work they explicitly consider the equipment's congestion problem [17].

Ingolfsson et al. [18] maximize coverage considering uncertainty in the means availability and travel times. The uncertainty regarding the response time is also considered in Zhang et al. [19]. Boujemaa et al. [20] consider different scenarios and different levels of assistance.

In a previous work [21], we have developed a coverage stochastic optimization model that brings several new advances compared with the existing models. One of the contributes of this model is the explicit consideration of different vehicle types, with different assistance levels, and the substitutability possibilities between these different types. In a real setting, if one given vehicle type is not available, then a vehicle or set of vehicles of different types are sent. The model also allows for the explicit consideration of emergency episodes that need more than one vehicle. The unavailability of a vehicle, if it is already assigned to an occurring episode, is also taken into account by using an incompatibility matrix that defines whether a given vehicle can or cannot be simultaneously assigned to two different emergency episodes, taking into account the time periods of occurrence of these episodes. This model is based on coverage location models, and it maximizes the expected total number of episodes covered. An episode is considered covered if it receives all the needed vehicles (or adequate substitutes) and if they all arrive within a defined time window. The consideration of an objective function of this type does not accommodate the point of view of robustness, that is of great importance in the field of emergency assistance. Actually, when looking at an expected coverage value, it is possible that we are not taking into account what is happening in some of the most complicated scenarios, that can end up having low levels of coverage even if the average value is acceptable. A robust solution will be found looking at other objectives that can better represent the idea that the system must respond as well as possible even in more challenging scenarios.

In this work we present two models where robust location decisions are considered. One of the models maximizes the total number of covered episodes in the worst case.

The other model minimizes the maximum regret. These models were applied to a set of scenarios, that were built based on real data, and using Monte Carlo simulation.

This manuscript is organized as follows: in the next section the characteristics of the emergency vehicles location problem that we have considered are briefly described. Section 3 describes the main features of the mathematical model developed. Section 4 presents the results obtained by applying the two robust approaches, and these results are then discussed in Sect. 5. Section 6 presents the conclusions and some paths for future work.

2 Description of the Problem

In this problem we consider a given geographical area, with already existing bases where emergency vehicles can be located. These are the potential locations that can be chosen. There is a fixed fleet of emergency vehicles. We have to decide on which base to locate each one of the existing vehicles. Emergency occurrences can occur anywhere within the geographical area. An emergency episode is considered covered if and only if it receives all the needed vehicles within an acceptable time window.

The geographical area of interest was the Coimbra district, located at the central part of Portugal. At the present moment there are 35 vehicles available that are distributed among 34 existing bases. We also consider this number of vehicles and their corresponding types, and the already existing bases.

The uncertainty that is inherent to the occurrence of emergency episodes is represented by a set of scenarios. The generated scenarios are meant to represent the characteristics of real data. The real data that was used includes information about all the emergency occurrences that required the assignment of advanced life support vehicles during one year, and data published by the National Medical Emergency Institute INEM for the last 3 years, that allowed the estimation of the total number of occurrences for the same period.

The model considers the dispatching decisions also: when an emergency episode occurs, the model will decide which are the vehicles that are sent to that call. Including these dispatching decisions is very important in this kind of model since it will allow a more realistic representation of the unavailability of vehicles when they are already assigned to other emergency calls.

The objective will be to find a robust solution: the location of the vehicles should guarantee the best possible coverage under different scenarios.

2.1 Emergency Vehicles

We consider the existence of five different types of emergency vehicles. Medical Emergency Motorcycles (MEM) are vehicles that have an emergency technician. They are able to provide Basic Life Support (BLS) and are equipped with external automatic defibrillation. They cannot transport victims when they need to receive further assistance in health institutions. Assistance Ambulances (AA) are usually drove by volunteers or professionals with specific training in prehospital emergency techniques. They provide BLS only but have transportation capacity. Medical Emergency

Ambulances (MEA) have, usually, two emergency technicians and have transportation capacity. Immediate Life Support (ILS) Ambulances (ILSA) guarantee more differentiated health care than the previous means, such as resuscitation maneuvers. The crew consists of a nurse and a pre-hospital emergency technician, providing Advanced Life Support (ALS). Medical Emergency and Resuscitation Vehicles (MERV) have a crew composed by a nurse and a medical doctor with competence and equipment for ALS. They do not have the ability to transport victims, and are many times used in conjunction with other vehicles with this capacity.

Considering the type of care that each one of these vehicle types can provide (namely BLS and ALS, and also taking into account the composition of the team), and whether they are or are not able to transport victims, it is possible to define substitutability possibilities between these vehicles. Table 1 presents this information. Each vehicle type presented in a given row can be directly substituted by one single vehicle of another type represented in the columns whenever a 1 is in the corresponding cell.

Table 1. Substitutability between different vehicle types

Vehicle type	AA	MEM	MEA	ILSA	MERV
AA	1	0	1	1	0
MEM	1	1	1	1	0
MEA	1	0	1	1	0
ILSA	0	0	0	1	0
MERV	0	0	0	0	1

There are also other possible substitutions, where a single vehicle can be substitute by more than one vehicle. A vehicle of type ILSA, for instance, can also be substituted by sending simultaneously an AA vehicle and a MERV or a MEM vehicle. These more intricated substitutions are also considered in the optimization model.

2.2 Emergency Episodes

Each emergency episode is characterized by the place where it occurs, the number of vehicles of each type that it will need, the time period in which it takes place (from the moment it begins until the moment when the vehicles that were assigned to it are ready to be assigned to other occurrences).

Each scenario will consider all the episodes that occur in a 24-h period. The generation of these scenarios took into consideration the information gathered from real data.

The region of interest considered in this work presents different emergency occurrence patterns in different subregions, either because these subregions correspond to urban centres or because they are traversed by heavy traffic roads, for instance. These differences were also taken into account, by splitting the region into different polygons with different characteristics considering the number and severity of the emergency occurrences (Fig. 1).

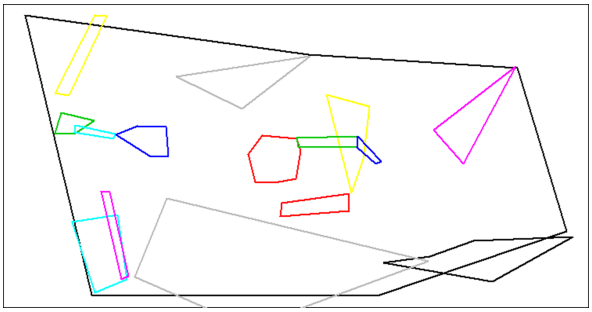

Fig. 1. Definition of different areas with different patterns regarding the occurrence of emergency episodes. Each polygon defines a subregion with specific patterns regarding the occurrence of emergency episodes.

To generate the emergency episodes that constitute one given scenario, the number of episodes that will occur is randomly generated, considering a Poisson distribution. Each episode is then allocated to one of the defined subregions, considering the emergency episodes occurrence probabilities for each one. Within this subregion, a particular location is randomly generated (with latitude and longitude coordinates). Then, the number and types of vehicles that will be needed for that episode are also randomly generated. The road driving times between each one of the existing bases and the location of the emergency episodes are calculated using Google Maps API. This allows the creation of a binary coverage matrix, considering a defined time limit. The time limits were defined as 30 min for rural areas and 15 min for urban centres. This matrix will dictate which vehicles are within the coverage radius of each emergency episode. The occurrence time period associated with each episode is also randomly generated (from the moment the episode begins to the moment where all the assigned vehicles are already operational and can assist other episodes). More detailed information regarding the generation of scenarios can be found in [21].

3 Modelling the Problem

The mathematical model is a two-stage stochastic model, with uncertainty included in the model using a set of different scenarios There will be decisions that are scenario independent, and others that are scenario dependent. The location variables will not be scenario dependent (these decisions are first level decisions, since they must be made without knowing what the future will bring). Variables related to the dispatching of vehicles to emergency episodes will depend on each particular scenario.

The mathematical model has to consider a very large number of constraints, so that it represents, as faithfully as possible, the real context of emergency vehicles dispatching. The constraints considered guarantee that:

- An episode belonging to a given scenario is considered “covered” if and only if it receives all the necessary vehicles of all the needed types in that scenario.

- An emergency vehicle can only be assigned to an episode if this episode is within the coverage radius of the vehicle, and it is available when the episode occurs.
- An emergency vehicle can only be assigned to two different episodes if their occurrence time periods do not intersect.
- It is not possible to anticipate the future in each scenario, considering the assignment decisions. This means that the dispatching decisions will be made based on the emergency occurrences that have already taken place, not considering the ones that will happen in the future. The inclusion of non-anticipative constraints is very important to guarantee the least biased results in terms of coverage.
- There is a maximum number of vehicles that can be located at each base, and vehicles can only be assigned to a base that has been prepared.
- Each emergency vehicle is located in one, and only one base.

The implementation of these conceptual restrictions gives rise to a high-dimensional mathematical programming problem. Details can be found in [21]. The developed model considered an objective function of maximization of the expected total number of covered episodes.

In the emergency location setting it can be better to look for robust solutions other than looking for the best expected coverage values. There are many interpretations for the concept of robustness. It is usually related with finding solutions that perform well under different scenarios. Performing well can be related with admissibility (finding solutions that are admissible under most of the scenarios) or with optimality (finding solutions that are near-optimal under most scenarios). In the current setting, the defined restrictions have to be always fulfilled, because they guarantee that the calculated solution makes sense. The robustness, in the current context, has to do with optimality.

Many stochastic robust models consider an objective function of the type max-min: they are concerned with finding the solution that is the optimal one for the worst case possible. In this case we would like to locate the emergency vehicles in order to maximize the coverage in the scenario where coverage is the worst among all the scenarios. The drawback of this approach is that, sometimes, this point of view is too conservative. Actually, the worst scenario is usually not the one with the highest probability of occurring and looking only at this scenario can ignore solutions that could behave better for the majority of the future situations.

This drawback can be overcome if another measure of robustness is considered, namely the minimization of the maximum regret. Regret has already been used in the location optimization field [22, 23]. It measures how much we are losing for not being capable of predicting the future and choosing the optimal solution for what will occur. We must make a decision without being able to anticipate what the future will be. This means that the chosen solution will not, with a very high probability, be the best one for that particular situation.

If we have a set of different scenarios, we can calculate the best solution for each one. This will give us the maximum possible coverage for that scenario. Then we want to find the solution that minimizes the maximum regret: our greatest loss for not doing what would be the best thing if we could anticipate the future.

These two approaches were considered and the model presented in [21] was changed accordingly. As the max-min coverage and min-max regret objective functions

are non-linear, it is necessary to linearize them using auxiliary variables and constraints, increasing the dimension of the problem.

In the next section we present some computational results and discuss the obtained results.

4 Computational Experiments

We have solved different model instances. First, we have fixed the vehicles' locations to their current locations to see how the current solution behaves under the two different objective functions. Then we have solved both the min-max regret and the max-min coverage problems and we have compared the obtained solutions. Each instance considered 30 scenarios, corresponding to 1978 emergency occurrences in total. The instances were solved by Cplex, version 12.7, using Intel Xeon Silver 4116, 2.1 GHz, 12-core processor, 128 GB RAM. These instances took around 3 h of computational time. When the location decisions are fixed, the computational time is of few minutes only.

Table 2 presents the current vehicles' locations and the ones calculated by the two different modeling approaches. Table 3 presents the coverage results obtained.

Using the same scenario set to calculate the optimal vehicles location and to assess their true adequacy to the real setting can be misleading. Actually, we are comparing solutions that are definitely the best, for that particular objective function and for that particular set of scenarios. But what we really want to assess is their ability to perform well under different and unseen situations. This is why we have generated an extra set of 15 scenarios, with a total of 951 emergency episodes, and tested all the solutions under this unseen set. Table 4 presents the results for this out-of-sample set. Table 3 and Table 4 present the expected coverage, the worst coverage and the worst regret for four different solutions: the current one (where we have fixed all the vehicles' location variables in their current positions), the max-min coverage, the min-max regret and the maximization of the expected coverage (these last results consider the original model applied in [21]).

Looking at the differences on the vehicles' locations between the current and the calculated solutions, and knowing the characteristics of each of the considered regions, it is possible to observe that some vehicles were relocated to areas with a higher population density, with higher rates of emergency occurrences. These changes try to correct some situations were high level of occurrences were not being properly covered. There are also some changes considering the location of vehicles with transportation capacity, positioning them in bases where they can be more useful.

These changes increase the response capacity in both the 30 scenario and 15 scenario sets. The current locations present worse results than the coverage results presented by the other three alternative solutions.

Table 2. Current vehicles' locations and the ones calculated by the two different modeling approaches

Locations	Current locations					Minimizing the maximum regret					Max the minimum coverage				
	AA	MEM	MEA	ILSA	MERV	AA	MEM	MEA	ILSA	MERV	AA	MEM	MEA	ILSA	MERV
Base 1	1	0	0	0	0	0	0	1	0	0	2	0	1	0	0
Base 2	1	0	0	0	0	0	1	0	1	0	1	0	0	0	0
Base 3	1	0	0	0	0	1	0	0	0	0	0	0	1	0	0
Base 4	0	0	0	0	1	1	0	0	0	0	1	0	0	0	1
Base 5	0	0	0	0	1	1	0	0	0	1	1	0	0	0	0
Base 6	0	0	0	0	0	1	0	0	0	0	1	0	0	0	0
Base 7	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Base 8	0	1	3	0	0	0	0	1	0	1	1	0	0	0	1
Base 9	1	0	0	0	0	0	0	0	0	0	0	1	0	0	0
Base 10	1	0	0	0	0	1	0	0	0	0	1	0	0	0	0
Base 11	1	0	0	0	0	0	0	0	1	0	0	0	0	0	0
Base 12	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0
Base 13	1	0	0	0	0	1	0	0	0	0	0	0	0	0	0
Base 14	0	0	0	1	1	1	0	0	0	1	0	0	1	0	1
Base 15	1	0	0	0	0	0	0	1	0	0	0	0	0	0	0
Base 16	0	0	0	0	0	1	0	0	0	0	1	0	0	0	0
Base 17	0	0	0	0	0	1	0	0	0	0	1	0	0	0	0
Base 18	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Base 19	1	0	0	0	0	1	0	0	0	0	1	0	0	0	0
Base 20	1	0	0	0	0	1	0	0	0	0	1	0	0	1	0
Base 21	1	0	0	0	0	1	0	0	0	0	1	0	0	0	0
Base 22	1	0	0	0	0	1	0	0	0	0	1	0	0	0	0
Base 23	1	0	0	0	0	1	0	0	0	0	1	0	0	0	0
Base 24	1	0	0	0	0	1	0	0	0	0	0	0	0	0	0
Base 25	1	0	0	0	0	0	0	1	0	0	1	0	0	0	0
Base 26	1	0	0	0	0	1	0	0	0	0	1	0	0	0	0
Base 27	1	0	0	0	0	1	0	0	0	0	1	0	0	0	0
Base 28	1	0	0	0	0	0	0	0	0	0	1	0	0	0	0
Base 29	1	0	0	0	0	0	0	0	0	0	1	0	0	0	0
Base 30	1	0	0	0	0	1	0	0	0	0	1	0	0	0	0
Base 31	1	0	0	0	0	1	0	0	0	0	0	0	0	1	0
Base 32	1	0	0	0	0	1	0	0	0	0	1	0	0	0	0
Base 33	1	0	0	0	0	1	0	0	0	0	1	0	0	0	0
Base 34	1	0	1	1	1	2	0	0	0	1	1	0	1	0	1

Table 3. Summary of the results considering the 30 scenarios set

	Average coverage	Worst case coverage	Maximum regret
Current location	84.20%	69.60%	10
Max min coverage	84.70%	83.10%	10
Min max regret	87.70%	78.80%	3
Expected coverage	89.50%	78.79%	4

Table 4. Summary of the results considering the 15 scenarios set

	Average coverage	Worst case coverage	Maximum regret
Current location	84.40%	78.33%	6
Max min coverage	87.42%	80.85%	6
Min max regret	88.52%	81.67%	6
Expected coverage	87.95%	81.67%	6

As expected, in the 30-scenario set, each solution is the best one among the four considering the specific objective function that was used in the calculation of the respective solution. The best value for the maximum regret is obtained with the min-max regret solution, the best expected coverage is obtained with the optimal solution of this model, and so on. Looking at the values in Table 3, it is difficult to decide on what solution to choose. Should we consider as most important the expected coverage, or is it more important to guarantee that we are better off in the worst situation? This question has not got a simple answer. But since each of these solutions is being tested with the same data that was used for the calculation of the optimal solution of each of the objective functions, this analysis can be biased.

Looking at Table 4, it is possible to conclude that one solution stands out as the best one when compared with the others. The solution that minimizes the maximum regret obtains the best average coverage and also the best coverage in the worst case. The maximum regret is the same for all the solutions. Assessing the solutions in these independent scenario set allows for an unbiased analysis that gives a better idea of how the solutions would behave in the real setting.

5 Discussion

When looking at the implementation of changes in the vehicle's locations, there are several issues that have to be taken into consideration and that can make these decisions hard to put into practice.

One of these barriers is the fact that different vehicles belong to different institutions (volunteer firemen, red cross, national emergency institute, and so on). Some institutions will not feel comfortable in letting their vehicles being positioned somewhere else. This is particularly true for vehicles of the AA type, that are usually located at firemen departments. This observation motivated us to analyse the impact of restricting the locations of these vehicles to their current positions. We have run all of the models, fixing the locations of all the AA vehicles. Table 5 presents the obtained results for the 30-scenarios set and Table 6 for the out-of-sample scenario set.

Table 5. Summary of the results considering the 30 scenarios set and fixing the location of all the AA vehicles to their current positions

	Average coverage	Worst case coverage	Maximum regret
Current location	84.20%	69.60%	10
Max min coverage	83.59%	82.76%	10
Min max regret	86.20%	77.27%	4
Expected coverage	88.90%	78.79%	5

Table 6. Summary of the results considering the 15 scenarios set and fixing the location of all the AA vehicles to their current positions

	Average coverage	Worst case coverage	Maximum regret
Current location	84.40%	78.33%	6
Max min coverage	84.34%	83.33%	9
Min max regret	69.31%	57.58%	21
Expected coverage	89.28%	83.33%	5

It is interesting to observe the behaviour of the min-max regret solution in the out-of-sample set. This solution is clearly more dependent on particular changes that are done in the location of some vehicles. Not being able to go through with those needed changes, namely fixing the location of all the AA vehicles, makes the solution present very bad coverage metrics in the out-of-sample set. The model that seems to behave better when all the AA vehicles' locations are fixed is the maximization of the expected coverage model. The results in out-of-sample, for this model where we are considering these AA vehicles fixed, are even better than the solution where these restrictions were not imposed. Actually, the location of AA vehicles is the result of a long time-period adaptation of different institutions to the specificities of their regions. These locations may implicitly consider an empirical knowledge about the emergency occurrences characteristics. However, the results are much better than the ones obtained with the current solution, meaning that the change in the location of some vehicles can have an important impact in terms of coverage.

These results motivate further studies, namely considering enlarged scenario sets.

Another issue that is not being considered in these models has to do with the episodes that are not being covered. These episodes will not receive any emergency vehicles. This would never happen in a real setting. In a real situation all emergency episodes will receive assistance, but probably later than they should or resorting to less adequate vehicles. It is possible that the impact of this limitation is not very significant, since the model is not also accounting for vehicles that are positioned in the boundaries of the region of interest, and that can cover emergency occurrences that take place in the frontier of this region. However, it will be important to take these features explicitly into account in future works.

The placement of vehicles nearer some health institutions can also increase the affluence of victims to these institutions, that may not be prepared to this increased

demand. This can jeopardize the quality of care, or increase the costs in order to cope with this increase.

The generation of scenarios did not take into consideration differences in the emergency occurrence patterns in different days of the week, or different times of the year. The models do not also consider the possibility of dynamically changing the vehicles' location during the day. This is a possibility that could increase coverage, since the number and location of occurrences have different patterns during the day. However, such dynamic changes are usually not well accepted by the emergency technicians and medical doctors, so these solutions are even harder to put into practice.

6 Conclusions

In this work we extend the model presented in [21], by considering two different objective functions that allow the calculation of robust solutions: one model considers the maximization of the worst case coverage; the other considers the minimization of the maximum regret.

Assessing the different solutions considering an unseen set of 15 scenarios, it is possible to conclude that the solution that minimizes the maximum regret seems to be the best one under all the different criteria. From a decision-making point of view, it is also a decision that is supported by the reasoning of being as close as possible to the optimal solution in each situation using the available resources. However, when we fix all the AA type vehicles to their current locations, this is the model that suffers the most in terms of worsening the coverage metrics.

The presented models suffer from some limitations that have been identified. These limitations motivate the pursuing of new developments. It is important, for instance, to consider the possibility of partial coverage: a situation where an emergency occurrence does not receive the most adequate resources and/or they do not arrive within the defined time window. These situations are not being considered, but they should be as they consume resources that cannot be assigned to other occurrences and can thus somehow bias the coverage results.

We are not considering the possibility of acquiring new vehicles, or preparing new potential locations to receive emergency vehicles. This is also an interesting line of research.

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References

1. Brotcorne, L., Laporte, G., Semet, F.: Ambulance location and relocation models. Eur. J. Oper. Res. **147**(3), 451–463 (2003)
2. Toregas, C., Swain, R., ReVelle, C., Bergman, L.: The location of emergency service facilities. Oper. Res. **19**(6), 1363–1373 (1971)

3. Daskin, M.S., Dean, L.K.: Location of health care facilities. In: Brandeau, M.L., Sainfort, F., Pierskalla, W.P. (eds.) Operations Research and Health Care. International Series in Operations Research & Management Science, vol. 70, pp. 43–76. Springer, Boston (2005)
4. Li, X., Zhao, Z., Zhu, X., Wyatt, T.: Covering models and optimization techniques for emergency response facility location and planning: a review. Math. Meth. Oper. Res. **74**(3), 281–310 (2011)
5. Zhou, Y., Liu, J., Zhang, Y., Gan, X.: A multi-objective evolutionary algorithm for multi-period dynamic emergency resource scheduling problems. Transp. Res. Part E Logist. Transp. Rev. **99**, 77–95 (2017)
6. Knight, V.A., Harper, P.R., Smith, L.: Ambulance allocation for maximal survival with heterogeneous outcome measures. Omega **40**(6), 918–926 (2012)
7. Leknes, H., Aartun, E.S., Andersson, H., Christiansen, M., Granberg, T.A.: Strategic ambulance location for heterogeneous regions. Eur. J. Oper. Res. **260**(1), 122–133 (2017)
8. Moeini, M., Jemai, Z., Sahin, E.: Location and relocation problems in the context of the emergency medical service systems: a case study. CEJOR **23**(3), 641–658 (2014). https://doi.org/10.1007/s10100-014-0374-3
9. Moeini, M., Jemai, Z., Sahin, E.: An integer programming model for the dynamic location and relocation of emergency vehicles: a case study. In: International Symposium on Operational Research in Slovenia, SOR, pp. 343–350 (2013)
10. Pulver, A., Wei, R.: Optimizing the spatial location of medical drones. Appl. Geogr. **90**, 9–16 (2018)
11. Yoon, S., Albert, L.A.: An expected coverage model with a cutoff priority queue. Health Care Manage. Sci. **21**(4), 517–533 (2017). https://doi.org/10.1007/s10729-017-9409-3
12. Daskin, M.S.: A maximum expected covering location model: formulation, properties and heuristic solution. Transp. Sci. **17**(1), 48–70 (1983)
13. McLay, L.A.: A maximum expected covering location model with two types of servers. IIE Trans. **41**(8), 730–741 (2009)
14. Sung, I., Lee, T.: Scenario-based approach for the ambulance location problem with stochastic call arrivals under a dispatching policy. Flex. Serv. Manuf. J. **30**, 1–18 (2016). https://doi.org/10.1007/s10696-016-9271-5
15. Bertsimas, D., Ng, Y.: Robust and stochastic formulations for ambulance deployment and dispatch. Eur. J. Oper. Res. **279**(2), 557–571 (2019)
16. Beraldi, P., Bruni, M.E., Conforti, D.: Designing robust emergency medical service via stochastic programming. Eur. J. Oper. Res. **158**(1), 183–193 (2004)
17. Beraldi, P., Bruni, M.E.: A probabilistic model applied to emergency service vehicle location. Eur. J. Oper. Res. **196**(1), 323–331 (2009)
18. Ingolfsson, A., Budge, S., Erkut, E.: Optimal ambulance location with random delays and travel times. Health Care Manage. Sci. **11**(3), 262–274 (2008)
19. Zhang, B., Peng, J., Li, S.: Covering location problem of emergency service facilities in an uncertain environment. Appl. Math. Model. **51**, 429–447 (2017)
20. Boujemaa, R., Jebali, A., Hammami, S., Ruiz, A., Bouchriha, H.: A stochastic approach for designing two-tiered emergency medical service systems. Flex. Serv. Manuf. J. **30**(1), 123–152 (2017). https://doi.org/10.1007/s10696-017-9286-6
21. Nelas, J., Dias, J.: Optimal emergency vehicles location: an approach considering the hierarchy and substitutability of resources. Eur. J. Oper. Res. **287**, 583–599 (2020)
22. Marques, M.C., Dias, J.: Dynamic location problem under uncertainty with a regret-based measure of robustness. Int. Trans. Oper. Res. **25**, 1361–1381 (2015)
23. Dias, J.M., do Céu Marques, M.: A multiobjective approach for a dynamic simple plant location problem under uncertainty. In: Murgante, B., et al. (eds.) ICCSA 2014. LNCS, vol. 8580, pp. 60–75. Springer, Cham (2014). https://doi.org/10.1007/978-3-319-09129-7_5