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Construction of the Regular Dodecahedron with the MATHEMATICA

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Abstract. Man is always curious about the patterns and beauty of platonic solids. Throughout history it has shaped them into artistic works, even today it uses them in technological models; likewise nature has adopted its geometric patterns. Through the development of digital technologies, geometric designs are made using the architecture of Platonic solids. The object of study and construction in this work is the Dodecahedron using Wolfram Mathematica 11.2 Software. For this, an efficient logical language will be used, framed within the discipline of geometry, where relationships and hierarchies are established in reasoning through the programming language, allowing its architecture to be associated with creations of man, objects and organisms of nature.

Keywords: Regular dodecahedron · Programming · Wolfram mathematica.

1 Introduction

The exuberant geometry of regular polyhedra attributed to the nature of their geometry, aesthetic, symbolic, mystical and cosmic, has fascinated man [1]. Although the origin of regular polyhedra is not known exactly, there is a conviction that they were known in the time of the Neolithic peoples. However, it was Pythagoras and his school that systematized and rigorously taught solid sayings. They are attributed the construction of the dodecahedron, the hexahedron and associated them with cosmic figures.

A prominent Greek for his interest in geometry was Platón [2], who had a cosmological view of regular polyhedra, saying, "It is necessary to explain what properties the most beautiful bodies should have [...], they must have the property of dividing equally and similar the surface of the sphere in which it is inscribed." It exposes us the existence of the fifth essence interpreted as the dodecahedron, to which no element of matter is associated.

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But the one who formalized and consecrated the Platonic solids as mathematical elements inscribing them in the sphere was Euclid [3], who made contributions on his constructions accompanied by demonstrations in book XIII from proposition 13 to 17 in his book Elements.

Interest in regular polyhedra grew, especially for the regular dodecahedron also drawn by Leonardo da Vinci, reflected in a book by Luca Pacioli [4], where he describes the regular dodecahedron: "The hollow flat dodecahedron (see Fig. 1(a)) or solid (see Fig. 1(b)) has thirty lines or equal sides that form at sixty surface angles, has twenty solid angles and twelve bases or surfaces that contain it. These are all pentagonal, with sides and angles equal to each other, as can be deduced from their shape".

The German scholar Durero [5] devised an original proto-topological method, developing it on a flat surface so that their faces form a coherent network, which when cut into paper and properly folded their faces will form a three-dimensional model of the Platonic solids and in 1525 published the plane development of the dodecahedron (see Fig. 1(c)) in his book "The four books of measurement". Later his work is highlighted by Panofsky [6], for his strictly scientific treatment when using construction rules of a perspective drawing based on Euclidean concepts and that perspective is an important branch of mathematics.

Another who was very interested was Kepler, he considered these regular solids capable of being inscribed in a sphere, he believed he saw in them the secret architecture of the universe and affirmed the earth is "the measure for the rest of the orbits, it is circumscribed to a dodecahedron [...]" (see Fig. 1(d)).

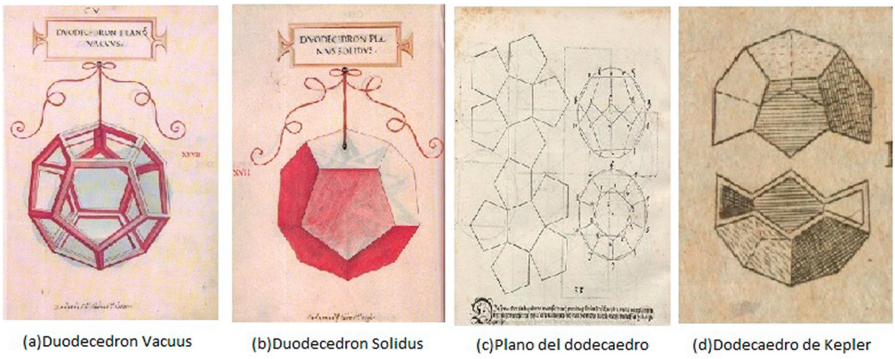

Fig. 1. Representations of the dodecahedron throughout history.

Where are the dodecahedron present? This Platonic solid has been inspired by artistic works such as Salvador Dalí: The Last Supper painted in 1955 (see Fig. 2(a)); in architecture such as: the Carbonera in Madrid (see Fig. 2(b)) and the Premium planetarium in Córdoba-Argentina (see Fig. 2(c)); Sculptural works such as: the Cosmographic Model (see Fig. 2(d)) that is seen in the Museum of the History of Science in Oxford. In nature, we can find minerals such as: Pyrite

(see Fig. 2(e)): FeS_2, Calcite (see Fig. 2(f)); likewise, in the form of pollen (see Fig. 2(g)). Also, at the molecular level in the crystalline dodecahedral structure that water molecules form when it traps methane forming methane hydrate (see Fig. 2(h)). It is remarkable that the study of geometry develops the capacities of formal abstract thinking: argumentation, demonstration and generalization. In this work the dodecahedron will be studied and for this we will resort to three cognitive processes: visualization, construction and reasoning. These allow us to present concepts, shapes, relationships and properties through the computer using Mathematica 11.2 Software using it as a powerful mathematical and scientific tool.

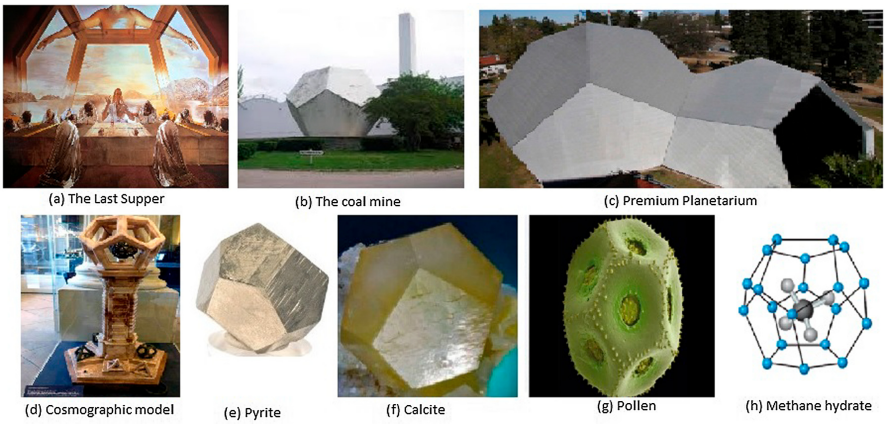

Fig. 2. The dodecahedron in nature and man's creations.

2 Analyzing and Programming in MATHEMATICA 11.2

The study will start from one of the regular geometric solids: the dodecahedron. We will study this one Geometric entity in a rational way and model it through the computer, after reflecting on its building. The analysis carried out makes it possible to establish links between the knowledge acquired and the new ones, using written communication through symbols and vocabulary typical of Geometry. To begin the study of the dodecahedron, let's visualize Fig. 3. For the construction of dodecahedron in Mathematica software, we start from a question: It is possible to know the coordinates of all the vertices of the dodecahedron starting from a canonical circumference of radius 1 u. which rests on the XY plane? It is observed that the vertices of the dodecahedron are vertices of 4 parallel regular pentagons. We will start from the regular pentagon at the base of the dodecahedron, for this we will build a circumference in its canonical form

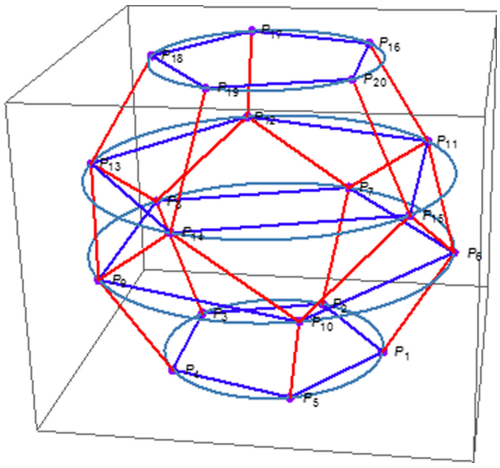

Fig. 3. The dodecahedron.

of radius 1. We can observe the equation of the circumference in its parametric form in (1):

$$cir[t_] := \{Cos[t], Sin[t], 0\}. \tag{1}$$

To graph (1) we will use the following command that we visualize in (2):

$$cirbase = ParametricPlot3D[\{Cos[t], Sin[t], 0\}, \{t, 0, 2Pi\}]. \tag{2}$$

We will build the pentagon inscribed on the circumference described in (1). Since the pentagon is regular (which we will call base pentagon), its central angle measures 72, to find its vertices we will use (3):

$$PuntosPentagonobase = Delete[Table[cir[t], \{t, 0, 2Pi\}, \frac{2Pi}{5}], -1]. \tag{3}$$

Generating the graphical representations of the points of the base pentagon we will use (4):

$$Ptosbase3d = Graphics3D[\{Hue[0.75], PointSize[0.02], Tooltip/Point/ @PuntosPentágonobase\}]. \tag{4}$$

To generate labels in the display of points in (4) we will use (5):

$$listPuntosPentágonobase = Graphics3D[Table[Text[P_i, Puntos Pentágonobase[[i]] + \{0.2, 0, 0\}], \{i, Length[PuntosPentágonobase]\}]]. \tag{5}$$

To visualize the graphic representation of (4) and (5) we will use (6):

$$Fig.\,1 = Show[listPuntosPentágonobase, Ptosbase3d]. \tag{6}$$

Now we will find the vertices of the base pentagon, but in the generated list we repeat the first point at the end in order to close the regular base pentagon. See (7):

$$Puntosbase = Table[cir[t], \{t, 0, 2Pi, \frac{2Pi}{5}\}]. \tag{7}$$

With the list (7) we generate the graphic representation of the base pentagon (8):

$$fig2 = Graphics3D[\{Thick, Hue[0.7], Line[Puntosbase]\}]. \tag{8}$$

Now we graph the regular pentagon called base, inscribed in the circuit generated by (1), using (9) and we will visualize its graphic representation in Fig. 4:

$$a1 = Show[fig1, cirbase, fig2]. \tag{9}$$

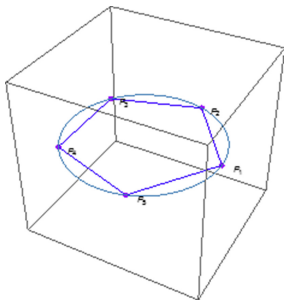

Fig. 4. Regular pentagons base and near base

To find the next regular pentagon parallel to the base of the dodecahedron, we need the measure of its side that coincides with the distance between points P_1 and P_3 with (10):

$$distP1P3 = Norm[PuntosPentágonobase[[1]] - PuntosPentágono\\ base[[3]]]//Simplify, \tag{10}$$

getting: $\sqrt{\frac{1}{2}(5+\sqrt{5})}$.

Let's look at the regular polygon inscribed in a circle, having as data the central angle and the measure of its side given in (10). Note Fig. 5: For the calculation of the radius R of the circumference that inscribes the regular pentagon called pentagon near the base shown in Fig. 4, we will use (11):

$$Solve[\sin[\frac{Pi}{5}] == \frac{\frac{L}{2}}{r}], \tag{11}$$

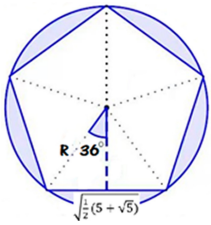

Fig. 5. Second regular pentagon that we will call a pentagon near the base.

getting: $\{\{r \to \sqrt{\{\frac{5+\sqrt{5}}{5-\sqrt{5}}\}}\}$.
Simply when the previous expression by (12) we get:

$$R = \sqrt{\frac{5+\sqrt{5}}{5-\sqrt{5}}}//FullSimplify, \tag{12}$$

we obtain: $R = \frac{1+\sqrt{5}}{2}$. We will find the parametric equation of the circumference that inscribes the second regular pentagon, previously we will calculate the radius. See Fig. 6:

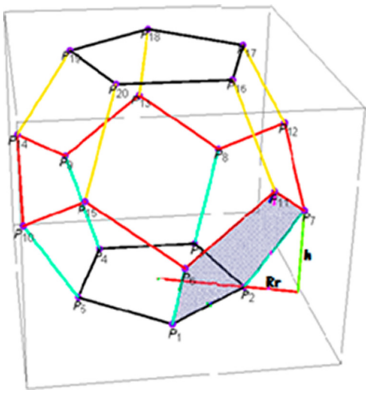

Fig. 6. Visualizing the dodecahedron the distance h and Rr to be calculated

We calculate the measure of the side of the pentagon, for this we will use (13):

$$distP1P2 = Norm[PuntosPentágonobase[[1]] - PuntosPentágono\ base[[2]]]//Simplify, \tag{13}$$

from where we get: $\sqrt{\frac{5-\sqrt{5}}{2}}$. Now let's calculate the difference of the radii of circles that inscribes the pentagon of the base and the pentagon near the base,

for them we will use the command (14):

$$Rr = R - 1//FullSimplify, \tag{14}$$

obtaining result $\frac{-1+\sqrt{5}}{2}$.
Using the Pythagorean Theorem we calculate the height of the circle that contains the points of the pentagon near the base, see (15):

$$h = \sqrt{distP1P2^2 - Rr^2}//FullSimplify, \tag{15}$$

getting result 1. Now we will find the parametric equation of the circumference that inscribes the regular pentagon called near the base, whose vertices belong to the dodocahedron in (16):

$$cir1[t] := \{\frac{1}{2}(1+\sqrt{5})Cos[t], \frac{1}{2}(1+\sqrt{5})Sin[t], 1\}. \tag{16}$$

From the above equation, we find the vertices of the regular pentagon near the base and generate its graphical representation in the same way as the base regular pentagon. Now we join the two parallel regular pentagons, where we can visualize 10 of the vertices of the dodecahedron, see (17) and its graph in Fig. 7:

$$b1 = Show[a1, a2]. \tag{17}$$

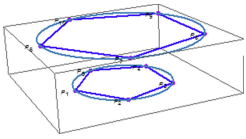

Fig. 7. Regular pentagons base and near base

Let's build the third regular pentagon that is close to the ceiling, see Fig. 8:

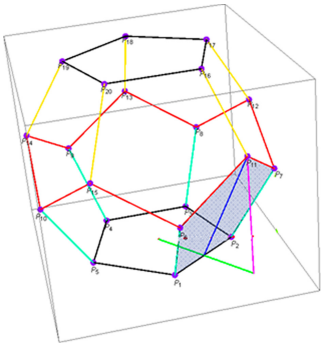

Fig. 8. Regular pentagons base and near base

Let's calculate the midpoint between points P_1 and P_2, using (18):

$$\begin{aligned} p1 &= PuntosPentágonobase[[1]] \\ p2 &= PuntosPentágonobase[[2]] \\ Pm &= \frac{p1+p2}{2}, \end{aligned} \tag{18}$$

where it was obtained: $\{\frac{1+\frac{-1+\sqrt{5}}{4}}{2}, \frac{1}{2}\sqrt{\frac{5}{8}+\frac{\sqrt{5}}{8}}, 0\}$.
Since the distance from P_{11} to P_m is the same from P_4 to P_m, we calculate it by (19):

$$\begin{aligned} P4 &= PuntosPentágonobase[[4]] \\ distPmP4 &= Norm[Pm - P4]//Simplify, \end{aligned} \tag{19}$$

is obtained: $\frac{1}{2}\sqrt{\frac{5(3+\sqrt{5})}{2}}$.
Let us calculate the height of the third regular pentagon with respect to the XY plane, using (20):

$$H = \sqrt{(distPmP4^2 - Rr^2)}//Simplify, \tag{20}$$

we obtain: $\frac{1}{2}\sqrt{\frac{3(1+\sqrt{5})}{2}}$.
With the value found in (20), the parametric equation of the circle that inscribes the third regular pentagon that we will call the pentagon near the ceiling is defined in (21):

$$cir2[t_] := \left\{ \frac{(1+\sqrt{5})Cos[t]}{2}, \frac{(1+\sqrt{5})Sin[t]}{2}, \frac{3+9\sqrt{5}}{2} \right\}. \tag{21}$$

We proceed to make the programming as the first regular pentagon. Now we join the three parallel regular pentagons, whose vertices represents 15 of the 20 vertices of the dodecahedron, using (21) the Fig. 9 is generated:

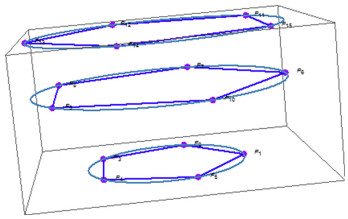

Fig. 9. Regular pentagons: base, base wax, and close to ceiling.

Let's build the last inscribed regular pentagon, representing the roof. It is known that heights of the pentagon near the ceiling is H and the height of the east and the ceiling is 1, see (22)

$$\begin{aligned} h &= 1 \\ H &= \frac{1}{2}\sqrt{\frac{3+9\sqrt{5}}{2}}. \end{aligned} \tag{22}$$

Since the measurements of the heights of the pentagon near the ceiling is H and the height from the east to the ceiling pentagon is 1, we can find the parametric equation of the circumference that inscribes the fourth regular pentagon. See (23):

$$cirtecho = ParametricPlot3D[\{Cos[t], Sin[t], 1 + \frac{1}{2}\sqrt{\frac{3+9\sqrt{5}}{2}}, \{t, 0, 2Pi\}]. \tag{23}$$

It is programmed as the first regular pentagon called the base. Later we make a graphic representation of the 4 regular pentagons, together visualize the 20 vertices of the dodecahedron. See Fig. 10 which is generated by (24)

$$b4 = Show[b2, a4]. \tag{24}$$

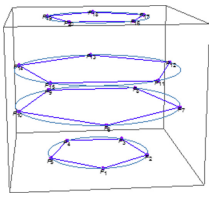

Fig. 10. The 4 parallel regular pentagons whose vertices belong to the dodecahedron.

All the vertices obtained in the constructed regular pentagons are listed, together these are vertices of the dodecahedron. See (25).

$$
\begin{aligned}
&Pto = \{\{1,0,0\},\{\frac{1}{4}(-1+\sqrt{5}),\sqrt{\frac{5}{8}+\frac{\sqrt{5}}{8}},0\},\\
&\{\frac{1}{4}(-1+\sqrt{5}),\sqrt{\frac{5}{8}+\frac{\sqrt{5}}{8}},0\},\{\frac{1}{4}(-1-\sqrt{5}),-\sqrt{\frac{5}{8}-\frac{\sqrt{5}}{8}},0\},\\
&\{\frac{1}{4}(-1+\sqrt{5}),-\sqrt{\frac{5}{8}+\frac{\sqrt{5}}{8}},0\},\{\frac{1}{4}(-1+\sqrt{5}),-\sqrt{\frac{5}{8}+\frac{\sqrt{5}}{8}},0\},\\
&\{\frac{1}{8}(-1+\sqrt{5})(1+\sqrt{5}),\frac{1}{2}\sqrt{\frac{5}{8}+\frac{\sqrt{5}}{8}}(1+\sqrt{5}),1\},\\
&\{\frac{1}{8}(-1-\sqrt{5})(1+\sqrt{5}),\frac{1}{2}\sqrt{\frac{5}{8}-\frac{\sqrt{5}}{8}}(1+\sqrt{5}),1\},\\
&\{\frac{1}{8}(-1-\sqrt{5})(1+\sqrt{5}),-\frac{1}{2}\sqrt{\frac{5}{8}-\frac{\sqrt{5}}{8}}(1+\sqrt{5}),1\},\\
&\{\frac{1}{8}(-1+\sqrt{5})(1+\sqrt{5}),-\frac{1}{2}\sqrt{\frac{5}{8}+\frac{\sqrt{5}}{8}}(1+\sqrt{5}),1\},\\
&\{\frac{1}{4}(3+\sqrt{5}),\frac{1}{4}\sqrt{\frac{1}{2}(5-\sqrt{5})}(1+\sqrt{5}),\frac{1}{2}\sqrt{\frac{3}{2}(1+3\sqrt{5})}\},\\
&\{-\frac{1}{2},\frac{1}{4}(1+\sqrt{5})\sqrt{\frac{1}{2}(5+\sqrt{5})},\frac{1}{2}\sqrt{\frac{3}{2}(1+3\sqrt{5})}\},\\
&\{-\frac{1}{2},\frac{1}{4}(1+\sqrt{5})\sqrt{\frac{1}{2}(5+\sqrt{5})},\frac{1}{2}\sqrt{\frac{3}{2}(1+3\sqrt{5})}\},\\
&\{-\frac{1}{2},\frac{1}{4}(1+\sqrt{5})\sqrt{\frac{1}{2}(5+\sqrt{5})},\frac{1}{2}\sqrt{\frac{3}{2}(1+3\sqrt{5})}\},\\
&\{\frac{1}{4}(3+\sqrt{5}),-\frac{1}{4}\sqrt{\frac{1}{2}(5-\sqrt{5})}(1+\sqrt{5}),\frac{1}{2}\sqrt{\frac{3}{2}(1+3\sqrt{5})}\},\\
&\{\frac{1}{4}(3+\sqrt{5}),-\frac{1}{4}\sqrt{\frac{1}{2}(5-\sqrt{5})}(1+\sqrt{5}),\frac{1}{2}\sqrt{\frac{3}{2}(1+3\sqrt{5})}\},\\
&\{\frac{1}{4}(1-\sqrt{5}),\sqrt{\frac{5}{8}+\frac{\sqrt{5}}{8}},\frac{1}{4}(4+\sqrt{6+18\sqrt{5}})\},\{-1,0,\frac{1}{4}(4+\sqrt{6+18\sqrt{5}})\},\\
&\{\frac{1}{4}(1-\sqrt{5}),-\frac{1}{2}\sqrt{\frac{1}{2}(5+\sqrt{5})},\frac{1}{4}(4+\sqrt{6+18\sqrt{5}})\},\\
&\{\frac{1}{4}(1+\sqrt{5}),-\sqrt{\frac{5}{8}-\frac{\sqrt{5}}{8}},\frac{1}{4}(4+\sqrt{6+18\sqrt{5}})\}\}.
\end{aligned}
\tag{25}
$$

Let's find the distance between the base regular pentagon and the roof regular pentagon. See (26):

$$hz = \frac{1 + \frac{1}{2}\sqrt{\frac{3}{2}(1 + 3\sqrt{5})}}{2}, \tag{26}$$

moving the points of the dodecahedron to the coordinate center. See (27) y (28).

$$Tras[P : \{_, _, _\}] := P - \{0, 0, \sqrt{\frac{3}{2}(1 + 3\sqrt{5})}\}, \tag{27}$$

$$ptotD = Tras/@Pto. \tag{28}$$

Building the dodecahedron with the regular pentagons inscribed in the circles called: base, near the base, near the ceiling and ceiling. See (29) y (30).

$$\begin{array}{l} line1 = Graphics3D[Hue[0], Thick, Line[\{ptotD[[1]], ptotD[[6]]\}], \\ Line[\{ptotD[[2]], ptotD[[7]]\}], Line[\{ptotD[[3]], ptotD[[8]]\}], \\ Line[\{ptotD[[4]], ptotD[[9]]\}], Line[\{ptotD[[5]], ptotD[[10]]\}]\} \\ line2 = Graphics3D[Hue[0], Thick, Line[\{ptotD[[6]], ptotD[[11], \\ ptotD[[7]], ptotD[[12], ptotD[[8]], ptotD[[13], ptotD[[9]], ptotD[[14], \\ ptotD[[10]], ptotD[[15], ptotD[[6]]\}]\} \\ line3 = Graphics3D[Hue[0], Thick, Line[\{ptotD[[11]], ptotD[[16]\}], \\ Line[\{ptotD[[12]], ptotD[[7]]\}], Line[\{ptotD[[13]], ptotD[[18]]\}], \\ Line[\{ptotD[[14]], ptotD[[19]]\}], Line[\{ptotD[[15]], ptotD[[20]\}]\}], \end{array} \tag{29}$$

$$\begin{array}{l} pent1 = Graphics3D[\{Hue[0.], Opacity[0.8], Polygon[\{ptotD[[1]], \\ ptotD[[2]], ptotD[[3]], ptotD[[4]], ptotD[[5]]\}]\} \\ pent2 = Graphics3D[\{Hue[0.4], Opacity[0.4], Polygon[\{ptotD[[6]], \\ ptotD[[7]], ptotD[[8]], ptotD[[9]], ptotD[[10]]\}]\} \\ pent3 = Graphics3D[\{Hue[0.5], Opacity[0.4], Polygon[\{ptotD[[11]], \\ ptotD[[12]], ptotD[[13]], ptotD[[14]], ptotD[[15]]\}]\}] \\ pent4 = Graphics3D[\{Hue[0.5], Opacity[0.8], Polygon[\{ptotD[[16]], \\ ptotD[[17]], ptotD[[18]], ptotD[[19]], ptotD[[20]]\}]\}] \\ Show[b4, line1, line2, line3, pent1, pent2, pent3, pent4]. \end{array} \tag{30}$$

Let's visualize its graphical representation Fig. 10 by (31).

$$Show[b4, line1, line2, line3, pent1, pent2, pent3, pent4]. \tag{31}$$

We will build the dodecahedron. See (32) (Fig. 11).

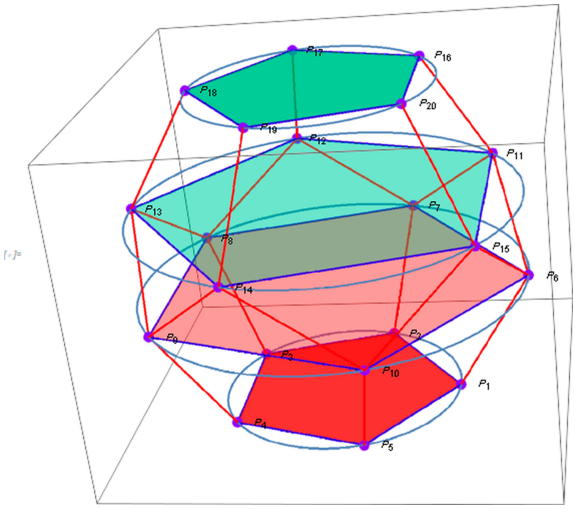

Fig. 11. vertexes of the dodecahedron.

$$
\begin{aligned}
&Dodecaedro = \\
&Graphics3D[\{Hue[0], Thick, Line[\{PtotD[[6]], PtotD[[11]], \\
&PtotD[[7]], PtotD[[12]], PtotD[[8]], PtotD[[13]], PtotD[[9]], PtotD[[14]], \\
&PtotD[[10]], PtotD[[15]], PtotD[[6]]\}], Line[\{PtotDPto[[1]], \\
&PtotD[[6]]\}], Line[\{PtotD[[1]], PtotDPto[[2]], PtotD[[3]], PtotD[[4]], \\
&PtotD[[5]], PtotD[[1]]\}], Line\{PtotD[[16]], PtotD[[17]], PtotD[[18]], \\
&PtotD[[19]], PtotD[[20]], PtotD[[16]]\}], Line[\{PtotD[[2]], PtotD[[7]]\}], \\
&Line[\{PtotD[[15]], PtotD[[20]]\}], Line[\{PtotD[[14]], PtotD[[19]]\}], \\
&Line[\{PtotD[[13]], PtotD[[18]]\}], Line[\{PtotD[[12]], PtotD[[17]]\}], \\
&Line[\{PtotD[[11]], PtotD[[16]]\}], Line[\{PtotD[[3]], PtotD[[8]]\}], \\
&Line[\{PtotD[[4]], PtotD[[9]]\}], Line[\{PtotD[[5]], PtotD[[10]]\}]\}].
\end{aligned}
\tag{32}
$$

Let's generate the points of the vertices (33) and their (34) labels of the dodecahedron.

$$
\begin{aligned}
&PtosDodecaedro = Graphics3D[\{Hue[0.75], PointSize[0.02], \\
&Tooltip/@Point/@PtotD\}],
\end{aligned}
\tag{33}
$$

$$
\begin{aligned}
&listPuntosDodecaedro = Graphics3D[Table[Text[P_i, \\
&Pto[[i]] + \{0.2, 0, 0\}], \{i, Length[PtotD]\}]].
\end{aligned}
\tag{34}
$$

Let's visualize its graphical representation Fig. 12 by (35).

$$figd = Show[Dodecaedro, PtosDodecaedro, listPuntosDodecaedro]. \quad (35)$$

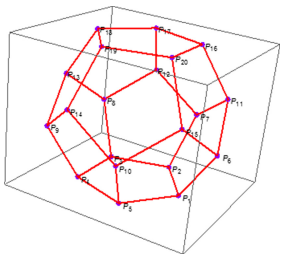

Fig. 12. The dodecahedron

Generating the vertices of the dodecahedron and its labels. See (36).

$$\begin{aligned} &PtosDodecaedro12 = Graphics3D[\{Hue[0.75], PointSize[0.02], \\ &Tooltip/@Point/ptotD\}] \\ &listPtosDodecaedro12 = Graphics3D[Table[Text[Subscript[P, i], \\ &PtotD[[i]] + 0.2, 0, 0], i, Length[PtotD]]]; . \end{aligned} \quad (36)$$

Building the Platonic solid called the dodecahedron. See (37).

$$\begin{aligned} &Solidoplatonico12 = ConvexHullMesh[ptostrasldode, MeshCellHighlight \\ &-> 2, All-> Opacity[0.65, Hue[0.]]]. \end{aligned} \quad (37)$$

Calculating the radius of the sphere that inscribes the dodecahedron. See (38):

$$RadioEsferaID = Norm[\{1, 0, -\frac{1}{2}(1 + \frac{1}{2}\sqrt{\frac{3}{2}(1 + 3\sqrt{5})})\}], \quad (38)$$

we obtain: $RadEID = \sqrt{1 + \frac{1}{4}(1 + \sqrt{\frac{3}{2}(1 + 3\sqrt{5})}^{2}} \sim 1.6800978811170325$.
Using (39) we find the graphical representation of the sphere.

$$\begin{aligned} &EsfID = ParametricPlot3D[RadEID\{Cos[u]Cos[v], Sin[u]Cos[v], Sin[v]\}, \\ &\{u, 0, 2 \quad Pi\}, \{v, -\frac{Pi}{2}, \frac{Pi}{2}\}, \\ &PlotStyle-> \{Opacity[0.4], Yellow\}. \end{aligned} \quad (39)$$

Graphing the inscribed dodecahedron inside a sphere Fig. 13 using (40).

$$Show[Solidoplatonico12, EsfID, PtosDodecaedro12, listPuntosDodecaedro12]. \tag{40}$$

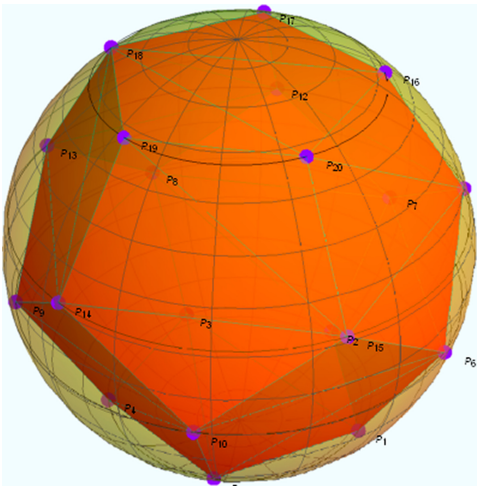

Fig. 13. The dodecahedron

A new question arises: Can we generate the dodecahedron, starting from a sphere of center (0, 0, 0) and of radius R? Using the (41) we can find the list of the vertices of the Dodecahedron with its graphic representation and its circumscribed sphere, given its radius of said sphere

$$\begin{aligned} &PtosDodeER[R_] := Modele[pttos, Pttos = \frac{R}{\sqrt{1+\frac{1}{64}(4+\sqrt{6+18\sqrt{5}})^2}}\\ &ptotD//FullSimplify//PowerExpand;\\ &Print[Pttos]; Show[\{ConvexHullMesh[Pttos, MeshCellHighlight\\ &-> \{\{2, All\} -> Opacity[0.65, Hue[0.]]\}], Graphics3D[\{PointSize\\ &[0.03], Hue[0.75], Point/@Pttos\}], ParametricPlot3D[R\{Cos[u]Cos[v],\\ &Sin[u]Cos[v], Sin[v]\}, \{0, 0, 2Pi\}, \{v, -\frac{Pi}{2},\\ &\frac{Pi}{2}\}, PlotStyle -> Opacity[0.5]]\}]]. \end{aligned} \tag{41}$$

In the analysis another question arises, can we find the sphere inscribed in the dodecahedron? Doing the respective analysis, we have to calculate the centroids of the dodecahedron faces using the following (42):

$$C(c_x, c_y, c_z) = \left(\frac{x_1 + x_2 + x_3 + x_4 + x_5}{5}, \frac{y_1 + y_2 + y_3 + y_4 + y_5}{5}, \frac{z_1 + z_2 + z_3 + z_4 + z_5}{5}\right). \tag{42}$$

Using the (43) we can find the list of the vertices of the Dodecahedron with its graphic representation and its inscribed sphere, given its radius of said sphere.

$$PtosDodeEsfI[r_] := Modele[pttosIE, PttosIE = \frac{r}{\sqrt{\frac{1}{4} + \frac{1}{2\sqrt{5}} + \left(\frac{1}{2} + \frac{1}{\sqrt{5}}\right)^2 + \frac{\left(4+3\sqrt{6+18\sqrt{5}}\right)^2}{1600}}}$$
$$ptotD//FullSimplify//PowerExpand;$$
$$Print[PttosIE]; Show[\{ConvexHullMesh[Pttos, MeshCellHighlight -> \{\{2, All\} -> Opacity[0.65, Hue[0.]]\}], Graphics3D[\{PointSize[0.03], Hue[0.75], Point/@PttosIE\}], ParametricPlot3D[r\{Cos[u]Cos[v], Sin[u]Cos[v], Sin[v]\}, \{0, 0, 2Pi\}, \{v, -\frac{Pi}{2}, \frac{Pi}{2}\}, PlotStyle -> Opacity[0.5]]\}]]. \tag{43}$$

3 Conclusions

In this article algorithms have been developed to calculate characteristics of the geometry of the Regular dodecahedron encoded in the language of Mathematica v. 11.2. The following characteristics have been observed: The dodecahedron can be built from parallel regular pentagons as specified below: The regular pentagon (called the base) inscribed in a circumference of radius 1, is parallel to the regular pentagon (near the base). The latter can be generated by moving the regular pentagon (called the base) one unit up and scaling the radius of the circle that inscribes it to the golden number. The regular pentagon (called near ceiling) of radio number Golden, can be generated by moving the regular pentagon (called near the base) upwards by a distance of approximately 0.7001695429 and rotating 36. The regular pentagon (called the ceiling) of Radius 1 can be generated by moving the regular pentagon (called the ceiling fence) up a distance of 1 and shrinking down to the radius 1 measurement. To construct the inscribed dodecahedron in a sphere with a center at the coordinate origin given its radius, it is

necessary to have the information about the center of the original dodecahedron and the distance from the center to any vertex. To construct the circumscribed dodecahedron in a sphere centered at the coordinate origin given its radius, it is necessary to have the information about the centroids of the faces of the original dodecahedron and the distance from the center to any centroid point. The ratio of the radii of the circumscribed and inscribed spheres to the dodecahedron is approximately 1.2725887427162601970982. It is suggested to extend the investigation and create packages of the Platonic solids and others, later applying them to the design of the polyhedral architecture of different artificial objects or existing organisms from nature.

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