Optimization of the Use of Critical Resources in the Development of Offshore Oil Fields

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Abstract. This paper focuses on a real case of connection of subsea oil wells to offshore platforms using pipe laying support vessels. The objective of this study is to maximize the oil production curve through an optimized use of the outsourced fleet. Specific features of this scenario are considered, such as technical constraints of each vessel, the availability of the vessels, materials for connection, and the end of the previous phase, called completion. A mixed integer linear programming model is developed considering several constraints that structure this complex situation, among which a relevant characteristic of the problem: the increase of the production curve using injection wells to fight the natural decline of producing wells over time. This mathematical model was tested in small computational instances, showing adequate behavior, which demonstrates that it faithfully represents the situation portrayed and can be used, combined with more advanced computational resources, to achieve better results.

Keywords: Oil wells · Routing problem · Mixed integer linear programming

1 Introduction

In the past decade, Brazil experienced a significant growth in oil production, especially with important discoveries of oil reserves in the pre-salt layer in ultra-deep waters (over 2,000 meters deep). There are 12 billion barrels of proven oil reserves in the country; in 2018, the offshore oil production amounted to approximately 903 million barrels [1]. Figure 1 shows the evolution of oil production in Brazil between 2004 and 2018. The Brazilian industry has faced an increasing challenge to develop technologies involved in the construction of wells to exploit these new oil reserves [2].

In offshore production, highly complex items that operate in severe conditions are considered critical resources. They also have elevated costs and high manufacturing lead times. Therefore, an advanced planning is required to make sure these items are available in a timely fashion. Offshore activities require the use of specific vessels to build offshore wells (the so-called oil drilling rigs that are used for drilling and completion of offshore wells) and to connect these wells to the production platforms (the pipe laying support vessels, PLSVs). The daily costs of a vessel can amount to US$ 500,000 [3]. There is a limited number of vessels and they are outsourced; thus, it is

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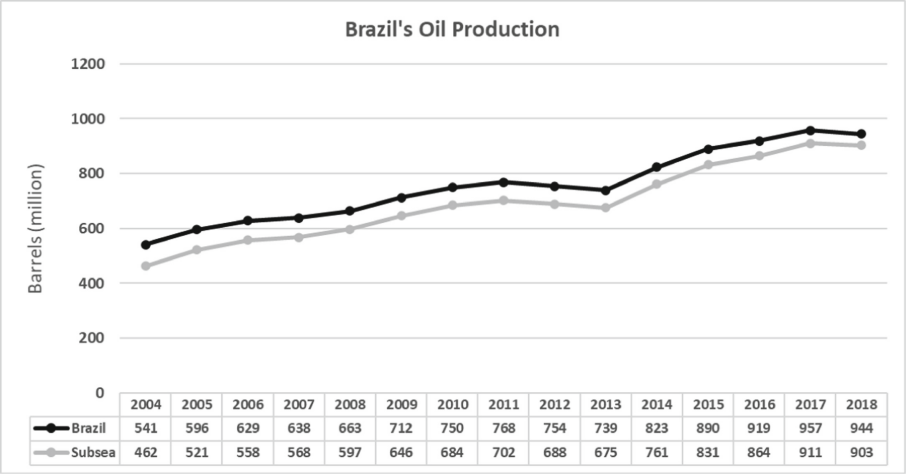

	2004	2005	2006	2007	2008	2009	2010	2011	2012	2013	2014	2015	2016	2017	2018
Brazil	541	596	629	638	663	712	750	768	754	739	823	890	919	957	944
Subsea	462	521	558	568	597	646	684	702	688	675	761	831	864	911	903

Fig. 1. Volumes of oil production between 2004 and 2018: total country production and subsea oil production (according to ref. [1]).

essential to properly schedule them and to find the best routes to the locations where activities will be carried out to optimize physical and monetary resources.

There are several studies in the literature that deal specifically with situations in the maritime environment that require the use of vessels. Nishi and Izuno [4] tackled a problem of routing and scheduling of crude oil ships with split deliveries, aiming to minimize distances while respecting the capacity of the ships that collect oil in various parts of the world and distribute it to some customers. A mathematical model whose constraints are similar to those used in terrestrial/road routing situations (for example, vessel capacity, subtour elimination restrictions, and route start and end restrictions) was presented; and a column-generation-based heuristic was introduced. Another work that describes the transportation of crude oil, with collection and delivery of products, was written by Henning et al. [5]. The addressed problem is characterized by the heterogeneous fleet and the transportation of multiple products. Other considered constraints were the port loading capacities, split loads, and time windows for both collection and delivery activities. A model considering these and other real-life features in which the objective is to minimize fuel costs and port charges was introduced. Lee and Kim [6] addressed a routing problem of a heterogeneous fleet with split deliveries in the context of a steel manufacturing company. They presented a mixed integer linear programming (MILP) model as well as an Adaptive Large Neighbourhood Search (ALNS) heuristic. Assis and Camponogara [7] dealt with the problem of relief ships that transport products between offshore oil platforms and a land terminal. The operation is depicted as a graph in which vertices represent the terminal and the platforms; while arcs represent the travel times. Possible operations for the ships are moving, offloading, uploading, and waiting. From this characterization, an MILP model was proposed. Another MILP approach in the oil industry was proposed by Lin et al. [8] in a problem of scheduling a fleet of marine vessels for crude oil tanker lightering. This

kind of activity is often related to sailing to refineries; it describes the transfer of crude oil from a discharging tanker to smaller vessels to make the tanker lighter. Stanzani et al. [9] addressed a real life multiship routing and scheduling problem with inventory constraints that arises in pickup and delivery operations of crude oil from various offshore platforms to coastal terminals. MILP models were presented to deal with small-to-moderate instances; while a matheuristic was proposed to deal with larger instances.

Some works address problems related to platform supply vessels (PSVs) that support the offshore oil and gas exploration and production activities. The problem consists in delivering goods from an onshore supply base to one or more offshore units and in returning items form these units to the onshore base. Gribkovskaia et al. [10] approached this problem with a single vessel considering the limited loading and unloading capacities at the platforms. A mathematical model, constructive heuristics, and a tabu search algorithm were presented. Kisialiou et al. [11] addressed the determination of the fleet composition and of the vessels' schedules involving flexible departures (multiple voyage departure options every day) from the onshore base. An ALNS algorithm was developed and a comparison with results provided by the resolution of a voyage-based model on small- and medium-sized instances was presented. Cruz et al. [12] proposed a mathematical model and a heuristic solution strategy for the fleet composition and periodic routing problem integrated with the berth allocation problem. In addition to these works, applications of optimization strategies based on MILP models can be found in Mardaneh et al. [13] and Halvorsen-Weare and Fagerholt [14].

Other studies tackle task-scheduling problems in petroleum projects. Bassi et al. [15] considered the problem of planning and scheduling a fleet of rigs to be used for drilling wells or for maintenance activities with uncertain service times. The authors developed a procedure based on simulation and optimization strategies to generate expected solutions. Pereira et al. [16] optimized the usage of PLSVs and drilling rigs in a problem of scheduling oil well development activities. The same problem, taking into account resources' displacement times, was addressed in Moura et al. [17]. In both papers, a Greedy Randomized Adaptive Search Procedure (GRASP) was applied. Also related to the PLSVs programming area, Cunha et al. [18] carried out a study making an analogy between routing activities and scheduling activities in parallel machines. The objective is to reschedule the PLSVs aiming to minimize the impacts caused by operational disruptions. These authors propose an MILP formulation for the PLSVs routing problem that starts from previously created activity blocks (voyages). Additionally, a method based on the Iterated Local Search (ILS) metaheuristic was proposed.

There are three stages involved in the construction of a well, namely, drilling, completion, and connection. This work focuses on the stage of wells' connections by PLSVs, a stage that immediately precedes the beginning of the operation of a well. In general, the connection process consists of three steps: I) loading of the PLSVs with the lines to be launched (this stage occurs in a loading base), II) navigation until the location of a well (where lines will be effectively launched), and III) launching the lines between the well and its associated stationary platform unit (SPU). Figure 2 summarizes this scenario. The problem considered in the present work deals with the usage of

PLSVs for the connection of several wells simultaneously. PLSVs are responsible for connecting flexible ducts and electro-hydraulic umbilical pipelines (EHU) between the SPUs and the producing and injection wells at sea. Thus, the problem consists in allocating activities over time to a heterogeneous fleet of vessels, considering aspects such as the non-concomitant use of different vessels to perform tasks in the same well, the technical viability of a vessel to perform a connection activity, the availability of raw material for a given itinerary, and the relation between vessels' loading constraints and the amount of material that must be loaded into them in order to perform the assigned activities. The goal is to maximize the oil production curve of a given set of wells or project. An innovative feature of the proposed approach is to tackle the decline in the production of producing wells as well as the influence of injection wells in this decline.

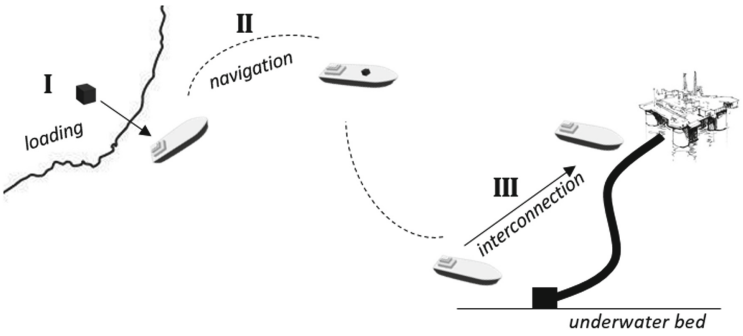

Fig. 2. Schematic representation of the problem: loading, navigation and interconnection of ducts.

This paper is organized as follows. Section 2 describes the problem and presents the problem description and a mixed integer linear programming model. Section 3 presents numerical experiments with a set of small-sized instances, given that optimal solutions to the real instances of the problem are not expected to be obtained within a reasonable execution time. Section 4 presents the final considerations and next steps suggested for this study.

2 Problem Description and a Mathematical Model

In the present section, a detailed description of the considered problem and its MILP formulation are presented. There are two types of wells: producing wells and injection wells. Each injection well is associated with one or more producing well. Producing wells are effectively responsible for oil production; while injection wells are responsible for repressurizing the oil reservoir and, consequently, for increasing the volume generated by the associated producing wells. Before a production well can start producing, there are three ducts or lines to be connected to it: two flexible ducts (one for

production and one for service) and one EHU. On the other hand, injection wells must be connected to two or three lines depending on whether they inject one or two types of fluid (one or two for the fluids plus one EHU). All lines must be installed between the wellhead and the associated platform (not necessarily in this order); this means that there is no intermediate equipment working as a line hub. There is only one loading base whose position, as well as the positions of wells and platforms, is fixed. Thus, distances between the loading base, the wells, and the platforms are all known a priori. There is no precedence relationship between the activities and, once started, an activity cannot be interrupted. Concomitance is not allowed, i.e. it is not possible to perform two or more activities at the same time in the same well. Each well has a date (end of its completion stage) at which the connection stage activities at the well can start to be executed. Each duct and EHU has an arrival date to the loading base and, naturally, they can only be loaded into a PLSV after their arrival to the base. Not every PLSV can perform every task (technical viability) since each PLSV has a storage capacity, an upper bound on the water depth of the activities it can perform, and a duct launch capacity.

After the end of the connection stage of a producing well, a testing stage, whose duration (commissioning time) is known, must be executed; and, after the stage of testing, the well starts its production (independently of the other wells being considered). The productivity of each well, in barrels per day (bpd), as a function of time, is also known a priori. One of the key points and differential aspect of this study is considering the decline of the well production and the effect of the inclusion of injection wells over time. A producing well begins to operate at its maximum volume, which is called the initial production potential. Over time, this volume is reduced due to the natural decline of production of the reservoir, which is explained by the decrease in pressure. The injection wells, whether water- or gas-operated, repressurize the reservoir, adding a fraction of oil production to that field in subsequent periods. Figure 3 illustrates the natural decline of a producing well including the effect of an injection well. In the illustrated example, the producing well has an initial production potential of 50,000 bpd and starts operating at time 10. The production decreases at a known linear rate. The injection well starts operating at time 20 and it has the capacity of increasing the productivity of the producing well by 6%. Therefore, if there were no injection well, the production would continue to decline, as depicted by the solid line. More than one injection well might be associated with a producing well. In this case, the production is increased by the sum of the fractions added by each injection well. As a whole, the productivity of a set of wells as a function of time can be computed and the objective is, satisfying all the problem constraints, to assign activities to the PSLVs and to determine the date these activities will be executed in order to maximize the total productivity of the wells within the considered period of time.

Figure 4 shows, with the help of a Gantt chart, a feasible solution for an instance with two vessels and three wells. In the figure, *Well # (act.i)* represents activity *i* to be performed on well #. The dotted lines represent the instant at which the three lines of a well were launched and, therefore, the well is ready to be commissioned (beginning of the test phase). It is also possible to observe, in this example, that activities are not concomitant, i.e. there is no time intersection between the activities of the same well.

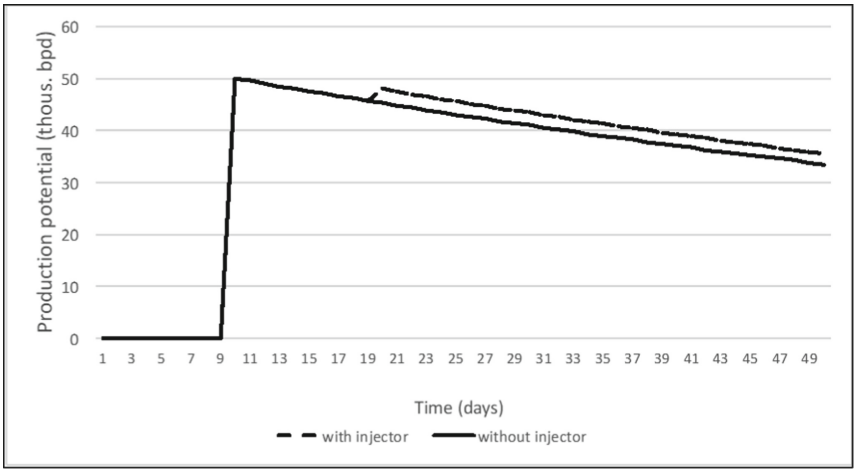

Fig. 3. Natural decline and effect of the use of injection wells on the volume of a producing well.

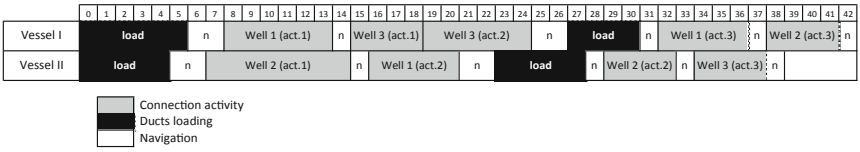

Fig. 4. Gantt chart of a feasible solution for an instance with two vessels and three wells.

An MILP model for the problem is now presented. In order to avoid a large number of indices in the variables, activities to be performed on the wells are numbered consecutively. This means that, for example, if there are three activities to be performed on wells 1 and 3 and two activities to be performed on well 2 then the activities of the first well will be numbered 1, 2, and 3; the activities of the second well will be numbered 4 and 5; and the activities of the third well will be numbered 6, 7, and 8. We denote by n the total number of activities, $N = \{1, 2, \ldots, n\}$, and $N^+ = \{0, 1, 2, \ldots, n, n+1\}$, where 0 and $n+1$ represent the activities of leaving from and returning back to the loading base, respectively. Since solutions include the possibility of multiple itineraries for each PLSV, activities 0 and $n+1$ are "special" activities that might be performed more than once. It is worth noting that an itinerary consists in a closed route leaving from and returning back to the loading base. It is the definition of routes that reflects in the model the fact that the raw material to be used in a set of activities must be loaded in the PLSV before starting the route that contains them. P denotes the number of wells and the sets P^* and P' are a partition of $\{1, \ldots, P\}$ that represent the indices of the producing wells and the injection wells, respectively. $A = \{(p^*, p') \mid p^* \in P^* \text{ and } p' \in P'\}$ represents the existent associations between the producing and injection wells. A description of the additional instance data follows. T is the number of days in the analyzed period and $\prod = (1, 2, \ldots, T)$ is the set of possible operation

periods of producing wells; B is the number of PLSVs and $V = (1, 2, \ldots, B)$ is the set of PLSVs; R is an upper bound on the allowed number of routes per PLSV; d_{ij} is the duration (in days) of the transfer between the sites of activities i and j; s_i is the duration (in days) of activity i; m_i^b is a binary constant whose value is one if the PLSV b is able to perform activity i and zero otherwise; if $p*$ is a producing well, $W_{p^*}^{\pi}$ is the number of barrels per day produced by well $p*$ in its π-th producing day; if p' is an injection well and $(p*, p') \in A$, $Inj_{p'}^{\pi}$ is the fraction of increment (between zero and one) in the production of well $p*$ in its π-th producing day; DC_i is the time from which activity i can be executed (this constant is identical for all activities to be performed in the same well and it corresponds to the end of the completion stage of the well); DM_i is the time at which the material required to execute activity i becomes available; q_i is the length (in km) of the duct required for activity i; Q_b is the storage capacity of vessel b (in km of ducts); β is the loading time (in days) per kilometer of duct; and c is the duration in days of the testing phase of a duct.

The description of indexes and variables used in the proposed MILP model and the complete model are presented next.

Indexes

i, j, h: activities;
b: vessel;
r: route;
p: well;
p:* producing well;
p': injection well;
π : period of operation (ordinal) of a producing well

Variables

X_{ij}^{rb}: 1, if vessel b, in route r, performs activity j right after activity i; 0, otherwise;
Y_i^{rb}: 1, if activity i is in route r of vessel b; 0, otherwise;
t_i^{rb}: auxiliary variable to determine the start time of activity i that is performed in route r of the vessel b; especially for activity 0, it is the time when the vessel leaves the loading base in each route r of each vessel b;
g_i^{rb}: actual start time of activity i if this activity is performed in route r of the vessel b; 0, otherwise;
$a_{i1,i2}$: 1, if activity $i1$ occurs before activity $i2$, being $i1$ and $i2$ activities that will take place at the same well; 0, otherwise;
σ^{rb} : duration of loading of the ducts used in the activities that make up route r of a vessel b;
k_p: finalization time of the connection stage of well p;
$\mu_{p^*}^{\pi}$: 1, if producing well $p*$ has its π-th operation period in the analysis interval; 0, otherwise;
$\gamma_{p'}^{\pi}$: 1, if injection well p' operates in the π-th operation period of its respective producing well in the analysis interval; 0, otherwise

$$\text{maximize} \sum_{\pi=1}^{T} \sum_{\forall p^*} \left(W_{p^*}^{\pi} \left(\mu_{p^*}^{\pi} + \sum_{p'/(p^*,p')\in A} Inj_{p'}^{\pi} \gamma_{p'}^{\pi} \right) \right) \tag{1}$$

subject to:

$$\sum_{j \in N^+ \setminus \{0\}} X_{ij}^{rb} = Y_i^{rb} \qquad i \in N, \forall r, \forall b \tag{2}$$

$$\sum_{b=1}^{B} \sum_{r=1}^{R} Y_i^{rb} = 1 \qquad i \in N \tag{3}$$

$$\sum_{i \in N^+} X_{ih}^{rb} - \sum_{j \in N^+} X_{hj}^{rb} = 0 \qquad h \in N, \forall r, \forall b \tag{4}$$

$$\sum_{i \in N^+} X_{0i}^{rb} = 1 \qquad \forall r, \forall b \tag{5}$$

$$\sum_{i \in N^+} X_{i(I+1)}^{rb} = 1 \qquad \forall r, \forall b \tag{6}$$

$$\sum_{i \in N} q_i Y_i^{rb} \leq Q_b \qquad \forall r, \forall b \tag{7}$$

$$t_i^{rb} + s_i + d_{ij} - M\left(1 - X_{ij}^{rb}\right) \leq t_j^{rb} \qquad i \in N^+, j \in N^+, \forall r, \forall b \tag{8}$$

$$t_0^{rb} \geq DM_i Y_i^{rb} + \sigma^{rb} \qquad i \in N, \forall r, \forall b \tag{9}$$

$$\sigma^{rb} = \beta \sum_{i \in N} q_i Y_i^{rb} \qquad \forall r, \forall b \tag{10}$$

$$t_{n+1}^{rb} + \sigma^{(r+1)b} \leq t_0^{(r+1)b} \qquad r = 1, \ldots, R-1; \forall b \tag{11}$$

$$Y_i^{rb} \leq m_i^b \qquad i \in N, \forall r, \forall b \tag{12}$$

$$t_i^{rb} \geq DC_i \qquad i \in N, \forall r, \forall b \tag{13}$$

$$g_i^{rb} \geq t_i^{rb} - M\left(1 - Y_i^{rb}\right) \qquad i \in N, \forall r, \forall b \tag{14}$$

$$g_i^{rb} \leq t_i^{rb} + M\left(1 - Y_i^{rb}\right) \qquad i \in N, \forall r, \forall b \tag{15}$$

$$g_i^{rb} \leq M Y_i^{rb} \qquad i \in N, \forall r, \forall b \tag{16}$$

$$\begin{aligned} k_p &\geq \sum_{r=1}^{R} \sum_{b=1}^{B} g_{(3p-2)}^{rb} + s_{3p-2} + c \\ k_p &\geq \sum_{r=1}^{R} \sum_{b=1}^{B} g_{3p-1}^{rb} + s_{3p-1} + c \\ k_p &\geq \sum_{r=1}^{R} \sum_{b=1}^{B} g_{3p}^{rb} + s_{3p} + c \qquad \forall p \end{aligned} \tag{17}$$

$$\begin{aligned} &\sum_{b=1}^{B} \sum_{r=1}^{R} g_{3p-2}^{rb} - \sum_{b=1}^{B} \sum_{r=1}^{R} g_{3p-1}^{rb} + s_{3p-2} \\ &\qquad \leq M\left(1 - a_{3p-2,3p-1}\right) \\ &\sum_{b=1}^{B} \sum_{r=1}^{R} g_{3p-1}^{rb} - \sum_{b=1}^{B} \sum_{r=1}^{R} g_{3p-2}^{rb} + s_{3p-1} \leq M a_{3p-2,3p-1} \\ &\sum_{b=1}^{B} \sum_{r=1}^{R} g_{3p-1}^{rb} - \sum_{b=1}^{B} \sum_{r=1}^{R} g_{3p}^{rb} + s_{3p-1} \leq M\left(1 - a_{3p-1,3p}\right) \qquad \forall p \end{aligned} \tag{18}$$

$$\begin{aligned} &\sum_{b=1}^{B} \sum_{r=1}^{R} g_{3p}^{rb} - \sum_{b=1}^{B} \sum_{r=1}^{R} g_{3p-1}^{rb} + s_{3p} \leq M a_{3p-1,3p} \\ &\sum_{b=1}^{B} \sum_{r=1}^{R} g_{3p-2}^{rb} - \sum_{b=1}^{B} \sum_{r=1}^{R} g_{3p}^{rb} + s_{3p-2} \leq M\left(1 - a_{3p-2,3p}\right) \\ &\sum_{b=1}^{B} \sum_{r=1}^{R} g_{3p}^{rb} - \sum_{b=1}^{B} \sum_{r=1}^{R} g_{3p-2}^{rb} + s_{3p} \leq M a_{3p-2,3p} \end{aligned}$$

$$\mu_{p^*}^{\pi} \leq \frac{T - k_p}{\pi} \qquad \forall \pi, \forall p^* \in P^* \tag{19}$$

$$\pi \gamma_{p'}^{\pi} + M\left(1 - \gamma_{p'}^{\pi}\right) \geq k_{p'} - k_{p^*} + 1 \qquad \forall \pi, \forall (p^*, p') \in A \tag{20}$$

$$\mu_{p^*}^{\pi} \geq \gamma_{p'}^{\pi} \qquad \forall \pi, \forall (p^*, p') \in A \tag{21}$$

$$t_i^{rb} \geq 0 \qquad i \in N^+, \forall r, \forall b \tag{22}$$

$$g_i^{rb} \geq 0 \qquad i \in N^+, \forall r, \forall b \tag{23}$$

$$k_p \geq 0 \qquad \forall p \tag{24}$$

$$\sigma^{rb} \geq 0 \qquad \forall r, \forall b \tag{25}$$

$$X_{ij}^{rb} \quad binary \qquad i \in N^+, j \in N^+, \forall r, \forall b \tag{26}$$

$$Y_i^{rb} \quad binary \qquad i \in N^+, \forall r, \forall b \tag{27}$$

$$a_{i1,i2} \quad binary \qquad i1, i2 \in N \tag{28}$$

$$\mu^{\pi}_{p^*} \quad binary \qquad \forall \pi, \forall p^* \tag{29}$$

$$\gamma^{\pi}_{p'} \quad binary \qquad \forall \pi, \forall p' \tag{30}$$

The objective function (1) aims at increasing the production volume in the considered period. At this point, it is crucial to clearly define time π: π is the period of operation of the producing well; and it necessarily starts at period 1. Thus, in each operation period, the objective function multiplies the production potential of the producing well ($W^{\pi}_{p^*}$) by the variables that indicate in which periods the well will be producing ($\mu^{\pi}_{p^*}$) and, moreover, in which of these times the corresponding injection well will be operating, which results in a production increase factor given by $Inj^{\pi}_{p'}\gamma^{\pi}_{P'}$. It is important to point out that π is the operating time of the producing well even when the wells at hand are injectors. In this case, π starts counting when the corresponding producing well begins operating.

Constraints (2) express that, if any activity is performed by a given vessel route ($Y^{rb}_i = 1$), there must be an exit arc for an activity j (which may be an activity in the base, $n + 1$, or for the execution of an activity in any other location), that is, one, and only one $X^{rb}_{ij} = 1$. Constraints (3) imply that all activities must be performed. It is important to note that activities of departure and arrival to the loading base are not considered in this restriction since they must occur once in each itinerary of each vessel, as it will be seen later in Constraints (5) and (6). Constraints (4) are responsible for the existence of flow in the routes of each vessel. Given that an activity h is being performed, equalities (4) ensure that, prior to it, an activity i was performed and, after it, an activity j will be performed. Activities i and j can be activities performed on departure and arrival at the base (0 and $n + 1$, respectively). Constraints (5) and (6) establish the requirement of departure and arrival at the base in each route of each PLSV, respectively. Constraints (7) guarantee that the number of pipelines to be loaded in each route of each vessel does not exceed the capacity Q_b of the PLSV. Constraints (8) ensure the time continuity within the same itinerary of the same vessel. If activity j follows activity i in the route r of the vessel b, then X^{rb}_{ij} is equal to 1. Consequently, $t^{rb}_i + s_i + d_{ij} \leq t^{rb}_j$, i.e., the start time of the next activity, j, is greater than or equal to the start time of the previous activity, i, added to the execution time of activity i (s_i) and the transfer time between the place where activity i is performed and where the activity j will be performed (d_{ij}). Otherwise, the value t^{rb}_j is not limited by the constraint. Constraints (8) also avoid the existence of subtours, given that the following activity must always take place after the previous activity.

Constraints (9) state that a PLSV can only leave the base to perform a route once all necessary lines of the route are available (DM_i) and loaded (σ^{rb}). Constraints (10) calculate the time to be spent in loading the lines of a route (σ^{rb}); the components of this computation are: the length of the line of each activity i (q_i), which is activated in the multiplication if Y^{rb}_i is 1, which represents that activity i is part of route r of vessel b; and a parameter β representing the time, in days, spent to load each km of line.

Constraints (11) refer to the transition between routes in one vessel. The constraints state that the next route can begin at least at the end of the previous route (PLSV returns to the loading base), plus the loading time of the next load. In a scenario where the required resources are already available, the inequality constraint holds as equality. Constraints (12) evaluate if a vessel can perform a given activity. A matrix of parameters m_i^b is provided to check which PLSVs can be used for each one of the tasks to be performed. The set of constraints (13) prevents connection activities from being started before the completion activity of that well has been completed. Constraints (14), (15), and (16) are used to compute the times g_i^{rb} that are, in fact, used in each of the routes. Variables t_i^{rb} are also used for intermediate calculations, so variables g_i^{rb} are established since these variables can only take positive values when the activity is performed. If $Y_i^{rb} = 1$, g_i^{rb} can only take a value equal to t_i^{rb}, on the other hand if $Y_i^{rb} = 0$, Constraints (16) force g_i^{rb} to be zero. These variables are important to establish Constraints (18) that avoid the concomitance of two vessels in the same well. The set of constraints (17) narrows the finalization time of the connection stage of each well (k_p). Knowing that each well is composed of 3 connection activities, these constraints calculate the earliest completion date of each activity as if it was the critical activity of the well. These 3 equations refer to the first, second, and third activity of the well p, respectively. For example, the term $\sum_{r=1}^{R}\sum_{b=1}^{B} g_{(3p-2)}^{rb}$, considering that there is only one positive $g_{(3p-2)}^{rb}$, indicates the time when activity *3p-2* begins. The duration of the activity s_{3p-2} is added to this sum to determine the end time of the activity. In addition, the commissioning time (c) is computed in each case. The second and third equations of set (17) perform similarly. The earliest date k_p is, therefore, defined as the highest value of these three equations, for each p. The set of constraints (18) prevents more than one vessel from operating at the same time in one well. Since each well has three activities, these inequalities allow one connection activity to either be terminated before the start of another activity or to start after the end of another activity at the same well. Variables $a_{i1,i2}$ indicate the order in which the activities take place; and variables g_i^{rb} are used because they are not influenced by intermediate calculations, as variables t_i^{rb}.

The set of constraints (19) determines the production periods of the wells, $\mu_{p^*}^{\pi}$, according to the finalization time of the connection stage, k_p. Since the potential of each producing well (considering the decline of production) is function of the period π of production, the establishment of these periods is crucial for the evaluation of the objective function. If a well p produces for 30 periods, variables $\mu_{p^*}^{1}, \ldots, \mu_{p^*}^{30}$ should be 1, while the others should be 0. Note that the value of the right-hand-side of Constraints (19), $\left(T - k_p\right)/\pi$, decreases as π increases. Therefore, the value of the denominator will gradually narrow the value of $\mu_{p^*}^{\pi}$. Since it is a maximization problem, $\mu_{p^*}^{\pi}$ tends to always be 1 until the moment of inversion, when $\left(T - k_p\right)/\pi$ becomes smaller than 1, which forces $\mu_{p^*}^{\pi}$ to become 0. This inversion occurs when π converges to the values greater than $T - k_p$ meaning that the end of the analysis period was reached. Constraints (20) determine the periods in which the influence of the injection wells should be computed. The calculation of $\gamma_{p'}^{\pi}$ is based on the finalization time of the connection

stage of their corresponding producing well (k_{p^*}). Observe that π is the operating period of the producing well. If $k_{p'} - k_{p^*} + 1$ (right-hand-side of the constraint) has a value greater than the coefficient π that multiplies $\gamma^{\pi}_{p'}$, the variable $\gamma^{\pi}_{p'}$ will assume value 0 and will not add any value to the objective function. Otherwise, when expression $k_{p'} - k_{p^*} + 1$ is smaller than or equal to the coefficient π, $\gamma^{\pi}_{p'}$ will be 0 or 1 to comply with the considered constraint. In this case, the model forces $\gamma^{\pi}_{p'}$ to be 1 because this decision increases the objective function. For example, if the producing well has its connection finalization at time 5 (i.e. $k_{p^*} = 5$) and its corresponding injection well at time 9 (i.e. $k_{p'} = 9$), the following scenario would occur: the first four production periods of the producing well would not have an injection well in operation ($\gamma^{1}_{p'} = = \gamma^{3}_{p'} = \gamma^{4}_{p'} = 0$). The terms from $\gamma^{5}_{p'}$ on must be positive (up to the limit T of analysis) due to the fact that the injection well is already operating and that it can add value to the objective function.

The set of constraints (21) prevents an injection well from adding value to the objective function before the beginning of the operation of the corresponding producing well or after the end of the analysis period. Constraints (22–30) define the domain of the variables.

3 Numerical Experiments

In this section, we aim to evaluate the performance of the proposed method in a set of small-sized instances in order to obtain its optimal solution. With this purpose, a set of thirty instances, all of them with two wells (one injection and one producing well) and six activities, was generated. Instances are divided into six sets of five instances each. In all the thirty instances, the capacity of the PLSVs is 12 km of ducts; the loading time per kilometer of duct is 0.5 day; all PLSVs can perform all activities; the production potential of the producing well is 180 bpd at its first producing day and it decreases at the rate of one barrel per day; the fraction of increment given by the injection well is 0.03; the duration of the testing phase of a duct (commissioning time) is 3 days; and the considered period is equal to 180 days. Tables 1 and 2 complete the description of the instances in Set I. Table 1 displays the transfer times between the activities; while Table 2 displays, for each activity, the duration of the activity (s_i), the length of the duct (q_i), the completion time (DC_i) of the associated well (i.e. the time at which the activity can start to be executed), and the time at which the duct is made available (DM_i). Sets II to VI correspond to variations of Set I as described in Table 3.

Numerical experiments were run on a Dell Studio XPS 8100 computer with an Intel Core i7 2.93 GHz and 16 Gb of RAM memory. The commercial solver CPLEX (version 12.1), with all its default parameters, was able to find optimal solutions for all the instances. Table 3 shows the results. It is worth noting that, in some instances, CPLEX used more than seven hours of CPU time. The analysis of the optimal solutions shows a stabilization of the objective function values in instances with three to five PLSVs within the same set, especially in Sets V and VI.

Table 1. Transfer times (d_{ij}) between activities (in days).

		j							
		0	1	2	3	4	5	6	7
i	0	0	2	2	2	3	3	3	0
	1	2	0	0	0	1	1	1	2
	2	2	0	0	0	1	1	1	2
	3	2	0	0	0	1	1	1	2
	4	3	1	1	1	0	0	0	3
	5	3	1	1	1	0	0	0	3
	6	3	1	1	1	0	0	0	3
	7	0	2	2	2	3	3	3	0

Table 2. Input parameters.

p	*Type*	***i***	s_i	q_i	DC_i	DM_i
1	Producing well	1	3	5	4	3
		2	4	4	4	8
		3	5	6	4	7
2	Injection well	4	5	6	6	2
		5	6	4	6	10
		6	7	5	6	12

4 Final Considerations

This paper addresses the process of interconnecting oil wells to offshore production units in the high seas. Analyzing the features and technical areas involved in this problem, it is possible to understand how complex it is from the geological study phase until the production from the oil wells after each duct is interconnected with the vessels (PLSV). Given the growth in the Brazilian offshore oil production in the past years and, especially, the contribution of wells from the pre-salt layer to this outcome, studies intended to characterize and improve this process are very relevant. Following this trend, our research study developed and evaluated techniques to optimize the use of the physical resources of an oil company, aiming to maximize the oil production curve.

A mixed integer linear programming model was developed to tackle a series of practical constraints of the problem to be solved. It should be highlighted that additional aspects in relation to the literature were considered, mainly with respect to the increase in the production curve due to the operation of injection wells and to the natural decline of producing wells during their operational lifespan. Numerical experiments with small instances were conducted and the proposed model was successfully validated. These numerical experiments showed consistent results, showing an increase in the value of the objective function with a greater number of PLSVs available. This fact indicates that, with a larger number of vessels, more activities can

Table 3. Comparison of the optimal solutions.

Instances description							Optimal results	
Set	#PLSVs	#routes	#binary variables	#real variables	#constraints	main features	obj. function	processing time (s)
I	1	6	786	111	1459	See Tables 1 and 2	16,059.4	1,406.00
	2	3	786	111	1458		16,357.8	776.00
	3	2	786	111	1457		16,401.8	191.00
	4	2	926	145	1750		16,427.3	190.00
	5	2	1066	179	2043		16,432.5	6,162.00
II	1	6	786	111	1459	$d_{0j} = d_{7j} = 3$, $j = 1, 2, 3$ and $d_{0j} = d_{7j} = 2$ $j = 4, 5, 6$	15,963.0	4,871.00
	2	3	786	111	1458		16,291.9	541.00
	3	2	786	111	1457		16,369.7	155.00
	4	2	926	145	1750		16,396.4	214.00
	5	2	1066	179	2043		16,417.3	1,779.00
III	1	6	786	111	1459	$q_i = 4$, $i = 1, \ldots, 6$	16,155.3	6,470.00
	2	3	786	111	1458		16,387.3	5,312.00
	3	2	786	111	1457		16,426.6	407.00
	4	2	926	145	1750		16,452.1	22,031.00
	5	2	1066	179	2043		16,452.1	28,737.00
IV	1	6	786	111	1459	$q_i = 8$, $i = 1, \ldots, 6$	15,712.7	3.24
	2	3	786	111	1458		16,268.0	1.06
	3	2	786	111	1457		16,356.2	0.82
	4	2	926	145	1750		16,381.1	1.85
	5	2	1066	179	2043		16,401.6	1.98
V	1	6	786	111	1459	$(s_1, \ldots, s_6)$ = (10,12,15,7,9,10)	14,866.9	1,032.00
	2	3	786	111	1458		15,489.6	218.00
	3	2	786	111	1457		15,567.4	91.17
	4	2	926	145	1750		15,567.4	108.00
	5	2	1066	179	2043		15,567.4	3,925.00
VI	1	6	786	111	1459	$DC_i = 20$. $i = 1, \ldots, 3$	15,712.1	829.00
	2	3	786	111	1458		16,061.9	798.00
	3	2	786	111	1457		16,129.8	60.02
	4	2	926	145	1750		16,129.8	6.57
	5	2	1066	179	2043		16,129.8	24.37

occur in parallel, allowing the anticipation of the start-up of producing wells and increasing the volume of production in the analyzed period.

As future topic for research, we suggest the elaboration of a constructive heuristic in order to validate the behavior of this model for bigger instances. Also, it is possible the application of neighborhood search strategies to investigate activity movements after the initial allocation indicated by the heuristic procedure. Since our target problem has many constraints, preparing an advanced procedure represents a great challenge because the solution can easily lose its viability characteristics.

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