Dynamic Facility Location Problem with Stochastic Setup Times

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Abstract. Dynamic facility location problems consider situations where it is important to decide not only where but also when to open (or close) facilities, given a planning time horizon. Considering explicitly multiple time periods is especially important if assignment costs are expected to change significantly in the future and relocating facilities is expensive. The majority of models presented in the literature considers binary decision variables that define whether facilities are open in a given time period. In addition, if a facility is first opened at some time period then clients can usually be assigned to it from that period forward, without any guarantee that the facility will be operational at the planned time period. In this paper, a new mathematical model is presented where uncertainty in the setup time of facilities is explicitly considered. This means that the decision maker is only capable of deciding where to locate facilities and when to begin all the necessary works to get the facilities operational, but the time elapsed until opening the facility is not known for sure. A primal-dual heuristic is also developed for this problem and its performance is compared with the performance of a commercial exact solver.

Keywords: Dynamic facility location · Primal-dual heuristics · Uncertainty

1 Introduction

Discrete location problems consider situations where there is a finite set of potential locations to install facilities, and there is a finite set of clients that have to be assigned to opened facilities. The objective is to decide what are the best locations to install facilities, and how to assign clients to these facilities, usually

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minimizing total cost (both location costs associated with opening facilities and also assignment costs), although many other objective functions might also be considered. Dynamic location problems consider that facilities can be opened at different time periods, during a given planning horizon. In a dynamic location problem the decisions have to consider not only where to open but also when to open the chosen facilities. These problems are most useful when there are significant expected changes in assignment costs, and relocating facilities to accommodate those changes is too expensive or even impossible. The explicit consideration of the time dimension increases the model relevance, comprehensiveness and connection to reality [11]. The data that are usually used to feed a dynamic location model are fixed location costs and assignments costs, both in the present and future time periods. It is also possible to consider maintenance costs and closing costs in cases where closing an existing facility is allowed. It is not realistic to assume that all these costs are known with certainty at the present time. Actually, even when considering location models where the time dimension is not explicitly included, an estimate of future costs has to be made in order to calculate meaningful present values for all the costs involved [9]. The uncertainty that is most of the times associated with location problems has been acknowledged by several authors, and there is a significant amount of published work in the area of location problems under uncertainty. An introduction to location problems under uncertainty can be found in [15], where the author addresses uncertainty in demands, variable production, transportation prices and selling prices. Snyder [19] provides a good literature review on stochastic location problems until 2006. The author distinguishes risk and uncertainty concepts, and considers approaches where the goal is the optimization of an expected value. Other objectives are also considered, representing robustness measures, like the minimization of the worst-case performance or the minimization of the maximum regret. There are not many works tackling dynamic location problems under uncertainty. A dynamic facility location problem under demand uncertainty is described by Baron et al. [5]. Uncertainty is represented by resorting to box and ellipsoidal uncertainty sets. Hernández et al. [14] consider a multiperiod stochastic model for the location of prison facilities in Chile, with demand uncertainty represented by scenarios. The decisions to consider are when and where to open facilities, with which capacity, and when and where to increase that capacity. The authors apply a branch-and-cluster coordination algorithm to calculate solutions to the model. Dynamic simple plant location problems under fixed and assignment costs uncertainty are tackled by Marques and Dias [17]. The authors consider uncertainty associated with the existence of customers in each time period, represented by scenarios. The model minimizes expected total cost, and a primal-dual heuristic is developed. In a posterior work, the authors introduce a measure of robustness by guaranteeing that the regret associated with each scenario is always upper bounded [18]. A multiobjective approach is also considered, where the authors develop relationships between non-dominated solutions and robust solutions [10]. Albareda-Sambola et al. [1] consider a multiperiod discrete facility location problem with demand and cost uncertainty represented

by scenarios, describing two alternative formulations for the discrete equivalent model (two-stage and multi-stage models). A matheuristic approach is developed. De Rosa et al. [7] develop a model for capacitated facility location with uncertainty in demand, costs and revenues, represented by scenarios. Capacity adjustments during the planning horizon are allowed. The objective function is the minimization of the expected value of relative regret.

In this paper a dynamic facility location problem under uncertainty is studied, considering not only cost and demand uncertainty but also uncertainty related to the moment in which a facility is operational and clients can be assigned to it. Actually, the location decision variables are usually binary variables that define when each facility is opened. It is also assumed that clients can be assigned to a facility from that moment forward. In reality, there are many activities that have to be carried out from the moment a decision to open a facility is made until the moment that the facility is operational: property acquisition, infrastructure construction, preparation of human and material resources, and so on. These activities will consume resources (including time) and delays may occur. The existence of delays in projects is a matter of research by itself, especially in construction projects (see, for instance, [2–4]). Not having a facility ready when it was planned can have important impacts in the assignment of clients to existing facilities. As far as the authors know, this is the first time this issue is introduced in a dynamic location model. A latency period, defined as the number of periods elapsed from the moment it is decided that a facility will be opened until the moment it is really available, has been considered before [1]. However, this latency period was assumed to be known a priori, it was the same for all potential facilities and independent of the opening time period. If this preparation time is deterministic then deciding when to open a facility will immediately determine when the setup work needs to begin.

This paper is organized as follows: in Sect. 2 the developed mathematical model is presented. Then, the primal-dual heuristic developed to tackle the problem is described. Section 4 presents the computational experiments made. Section 5 presents the conclusions and paths for future research.

2 The Mathematical Model

The notation that will be used in this paper is as follows. The time horizon is represented by a finite set of time periods $\mathcal{T} = \{1, ..., t, ..., T\}$. The set of possible future scenarios is denoted by $\mathcal{S} = \{1, ..., s, ..., S\}$, where each scenario $s \in \mathcal{S}$ will occur with probability p^s such that $p^s > 0$ and $\sum_{s \in \mathcal{S}} p^s = 1$. Let the set of potential facility sites be denoted by $J = \{1, ..., j, ..., M\}$ and the set of possible customer locations (or demand points) by $I = \{1, ..., i, ..., N\}$. For $(j, s) \in J \times \mathcal{S}$, let d_{js} be the number of time periods it takes to prepare facility j under scenario s, i.e., the number of time periods required to set up facility j, for it to be ready and to be able to accept customers, under scenario s. Let us define δ_{it}^s as equal to 1 if customer i has a demand that has to be fulfilled during period t for scenario s, and 0 otherwise. Then we have to guarantee that all customers such that

$\delta_{it}^s = 1$ are assigned to an operational facility, for all $(t, s) \in \mathcal{T} \times \mathcal{S}$. In terms of costs: for $(j, t, s) \in J \times \mathcal{T} \times \mathcal{S}$, let f_{jt}^s be the fixed cost associated with facility j, if this facility is opened in $t + d_{js}$, under scenario s, being the setup works initiated in t. This fixed cost considers both opening costs and operating costs in all the time periods during which the facility is operational, under scenario s; for $(i, j, t, s) \in I \times J \times \mathcal{T} \times \mathcal{S}$, c_{ijt}^s represents the assignment cost of customer i to facility j in period t and under scenario s. We assume that once it has been decided that a given facility will be opened, all the works necessary to have the facility operational will effectively take place, and once the facility is operational it will stay operational until the end of the planning horizon.

The decisions to be made are where and when to locate new facilities, and how to assign the existing customers to operational facilities over the whole planning horizon and under each scenario. Thus, the following binary decision variables are defined: x_{jt} equals 1 if facility j begins to be prepared at the beginning of period t, and 0 otherwise; y_{ijt}^s equals 1 if customer i is assigned to facility j in period t and under scenario s, and 0 otherwise. The objective is to minimize expected total cost including fixed and assignment costs over all scenarios. The dynamic facility location problem under uncertainty considering setup times can then be formulated as follows:

$$\min \quad \sum_{t\in\mathcal{T}}\sum_{j\in J}\sum_{s\in\mathcal{S}} p^s f_{jt}^s\, x_{jt} \;+\; \sum_{s\in\mathcal{S}}\sum_{t\in\mathcal{T}}\sum_{i\in I}\sum_{j\in J} p^s c_{ijt}^s\, y_{ijt}^s \tag{1}$$

subject to

$$\sum_{j\in J} y_{ijt}^s = \delta_{it}^s \qquad \forall i \in I, t \in \mathcal{T}, s \in \mathcal{S}, \tag{2}$$

$$\sum_{\tau\in\mathcal{T}:\, \tau+d_{js}\le t} x_{j\tau} - y_{ijt}^s \ge 0 \qquad \forall i \in I, j \in J, t \in \mathcal{T}, s \in \mathcal{S}, \tag{3}$$

$$\sum_{t\in\mathcal{T}} x_{jt} \le 1 \qquad \forall j \in J, \tag{4}$$

$$x_{jt} \in \{0, 1\} \qquad \forall j \in J, t \in \mathcal{T}, \tag{5}$$

$$y_{ijt}^s \in \{0, 1\} \qquad \forall i \in I, j \in J, t \in \mathcal{T}, s \in \mathcal{S}. \tag{6}$$

The objective function (1) minimizes expected total costs (fixed plus variable costs). Constraints (2) require that in every time period and under each scenario a customer with demand is assigned to exactly one operational facility. These constraints should be further analyzed. Actually, it is not possible to guarantee that these constraints are indeed all satisfied. The explicit consideration of uncertain setup times will possibly delay the opening of facilities in some scenarios. For instance, if all the potential facilities to be opened require setup times greater than zero, then there will be no operational facilities in the first time period of the planning horizon, nor possibly in other periods of time. Consequently, constraints (2) are not satisfied at least for $t = 1$. In order to ensure feasibility it is sufficient to consider that there is at least one facility $j \in J$ such

that $d_{js} = 0, \forall s \in \mathcal{S}$. This means that at least one facility will surely be opened and assignments will be guaranteed from period 1 forward. Another possibility is to consider the existence of at least one facility already operational at the beginning of the planning horizon.

Constraints (3) impose that a customer with demand can only be assigned to operational facilities. A customer can be assigned to different facilities at different time periods and different scenarios. However, a customer can only be assigned to a facility j under scenario s if d_{js} time periods have already elapsed since the time the facility began to be prepared. Constraints (4) ensure that each facility begins to be built at most once during the time horizon (located at the same site in all scenarios, but with possible different building times). Finally, (5)–(6) restrict the decision variables to be binary.

This model is an extension of the model presented in [17]. The main difference is the consideration of uncertainty in setup times, represented in the present model by parameters $d_{js}, (j, s) \in J \times \mathcal{S}$. As far as the authors know, it is the first time this feature is taken into account. Furthermore, in the above model the setup time of a given facility is only scenario dependent, but it could also depend on the initial building time of that facility. For this latter option, setup times would be defined as $d^s_{jt}, (j, t, s) \in J \times \mathcal{T} \times \mathcal{S}$, and in terms of problem formulation, constraints (3) have to be slightly changed to

$$\sum_{\tau \in \mathcal{T}:\, \tau + d^s_{j\tau} \leq t} x_{j\tau} - y^s_{ijt} \geq 0 \qquad \forall i \in I, j \in J, t \in \mathcal{T}, s \in \mathcal{S}.$$

For ease in the exposition, we will assume the situation described earlier throughout this text.

3 Primal–Dual Heuristic

Primal–dual heuristics have proven their value when dealing with facility location problems [6,8,12,16,17,21]. One of the advantages of primal-dual heuristics relies on their capacity of providing both upper and lower bounds for the optimal solution. Even in cases where the heuristic is unable to find the optimal solution (or it is not possible to prove it is optimal), it still allows the obtained solution to be evaluated, and a maximum gap to be calculated. The dual problem and complementary slackness conditions used by the heuristic procedures are presented next.

3.1 Dual Problem and Complementary Slackness Conditions

Consider restrictions (4) rewritten as $\sum_{t \in \mathcal{T}}(-x_{jt}) \geq -1, \forall j \in J$ and the linear programming relaxation of the primal problem defined by (1)–(4), where restrictions (5) and (6) are replaced by nonnegativity constraints. Let us define in (1) $\mathcal{F}^s_{jt} = p^s f^s_{jt}$ and $\mathcal{C}^s_{ijt} = p^s c^s_{ijt}$. Considering dual variables v^s_{it}, w^s_{ijt} and u_j associated with the restrictions (2), (3) and (4), respectively, the condensed dual

problem can thus be formulated as follows (since it is possible to demonstrate that $w^s_{ijt} = \max\{0, v^s_{it} - \mathcal{C}^s_{ijt}\}, \forall i, j, t, s$):

$$\max \quad \sum_{i\in I}\sum_{t\in\mathcal{T}}\sum_{s\in\mathcal{S}} \delta^s_{it}\, v^s_{it} - \sum_{j\in J} u_j \tag{7}$$

subject to

$$\sum_{i\in I}\sum_{s\in\mathcal{S}}\sum_{\tau=t+d_{js}}^{T} \max\{0, v^s_{i\tau} - \mathcal{C}^s_{ij\tau}\} - u_j \le \sum_{s\in\mathcal{S}} \mathcal{F}^s_{jt} \qquad \forall j, t, \tag{8}$$

$$u_j \ge 0 \qquad \forall j. \tag{9}$$

The corresponding slack variables π_{jt} for constraints (8) are given by:

$$\pi_{jt} = \sum_{s\in\mathcal{S}} \mathcal{F}^s_{jt} - \sum_{i\in I}\sum_{s\in\mathcal{S}}\sum_{\tau=t+d_{js}}^{T} \max\{0, v^s_{i\tau} - \mathcal{C}^s_{ij\tau}\} + u_j \qquad \forall j, t. \tag{10}$$

Then, the complementary slackness conditions are:

$$\pi_{jt}\; x_{jt} \;=\; 0 \qquad \forall j, t, \tag{11}$$

$$v^s_{it} \left(\sum_j y^s_{ijt} - \delta^s_{it}\right) \;=\; 0 \qquad \forall i, t, s, \tag{12}$$

$$w^s_{ijt} \left(\sum_{\tau\in\mathcal{T}:\, \tau+d_{js}\le t} x_{j\tau} - y^s_{ijt}\right) \;=\; 0 \qquad \forall i, j, s, t, \tag{13}$$

$$u_j \left(1 - \sum_t x_{jt}\right) \;=\; 0 \qquad \forall j, \tag{14}$$

$$y^s_{ijt} \left(v^s_{it} - \mathcal{C}^s_{ijt} - w^s_{ijt}\right) \;=\; 0 \qquad \forall i, j, t, s. \tag{15}$$

As it is well known from duality theory, if the dual and primal solutions satisfy all complementary slackness conditions, then the solutions are optimal. If not, the corresponding primal solution is said to have gap.

3.2 Algorithm

We reindex, for each scenario s, $\mathcal{C}^s_{ijt}$ for each (i, t) in nondecreasing order as $\mathcal{C}^{s(k)}_{it}$, for $k = 1, 2, ..., k^s_{it}$, where k^s_{it} denotes the number of facility-to-customer links for (i, t) under scenario s. For instance, $\mathcal{C}^{s(1)}_{it} = \min_{j\in J}\{\mathcal{C}^s_{ijt}\}$. For convenience, we also include $\mathcal{C}^{s(k^s_{it}+1)}_{it} = +\infty, \forall\ (i, t, s)$. Let I^+ be the set of *pseudo customers*

(i,t,s) corresponding to the dual variables v_{it}^s that the dual ascent procedure will try to increase. Initially, I^+ will be equal to all possible combinations $(i,t,s) \in I \times \mathcal{T} \times \mathcal{S}$, except those such that $\delta_{it}^s = 0$. Later, I^+ will be set within the respective procedures. The main steps of the heuristic are as follows:

1. Set $v_{it}^s = \min_{j \in J}\{\mathcal{C}_{ijt}^s\}$, $\forall\ (i,t,s)$, and $u_j = 0, \forall\ j$.
   Set $I^+ = \{(i,t,s) \in I \times \mathcal{T} \times \mathcal{S} : \delta_{it}^s = 1\}$.
2. Execute the dual ascent procedure.
3. Execute the primal procedure. If an optimal solution is found, then stop.
4. Execute the primal–dual adjustment procedure. The heuristic stops when the optimal solution is found or when there are no primal or dual improvements after a given number of trials within the adjustment procedure.

Dual Ascent Procedure. In what follows, $(i,t,s)_q$, with $q \leq |I \times \mathcal{T} \times \mathcal{S}|$, represents a given, but arbitrary, sequence of pseudo customers.

1. Consider any dual feasible solution $\{v_{it}^s\}$ such that $v_{it}^s \geq \mathcal{C}_{it}^{s(1)}, \forall\ (i,t,s)$, and $\pi_{jt} \geq 0, \forall\ (j,t)$.
   For each (i,t,s) define $k(i,t,s) = \min\{k : v_{it}^s \leq \mathcal{C}_{it}^{s(k)}\}$. If $v_{it}^s = \mathcal{C}_{it}^{s(k(i,t,s))}$, then $k(i,t,s) \leftarrow k(i,t,s) + 1$.
2. $(i,t,s) \leftarrow (i,t,s)_1$ and $q \leftarrow 1$; $r = 0$.
3. If $(i,t,s) \notin I^+ \vee \delta_{it}^s = 0$, then go to step 7.
4. Set $\Delta_{it}^s = \min\left\{+\infty, \min_j\{\pi_{j\tau} : v_{it}^s - \mathcal{C}_{ijt}^s \geq 0, \tau \leq t - d_{js}, t \geq 1 + d_{js}\}\right\}$.
5. If $\Delta_{it}^s > \mathcal{C}_{it}^{s(k(i,t,s))} - v_{it}^s$, then $\Delta_{it}^s = \mathcal{C}_{it}^{s(k(i,t,s))} - v_{it}^s$; $r = 1$; $k(i,t,s) \leftarrow k(i,t,s) + 1$.
6. For all $j \in J$ with $v_{it}^s - \mathcal{C}_{ijt}^s \geq 0$, set $\pi_{j\tau} = \pi_{j\tau} - \Delta_{it}^s, \tau \leq t - d_{js}$; set $v_{it}^s = v_{it}^s + \Delta_{it}^s$.
7. If $q < |I^+|$, then $q \leftarrow q + 1$, $(i,t,s) \leftarrow (i,t,s)_q$, and return to step 3.
8. If $r = 1$, then return to step 2, otherwise stop.

The ascent procedure starts with a dual feasible solution and it tries to increase iteratively the values of dual variables v_{it}^s, increasing the dual objective function value. Some slacks have to be decreased namely those associated with facilities that can be operational (steps 4 and 6). Looking at step 4 it is possible to realize that, for a given (i,t,s), $\Delta_{it}^s = +\infty$ if, and only if, there is no facility j under the conditions stated in that step, meaning that those facilities can not be operational during that t and s due to the magnitude of the respective setup times. In such cases, the value Δ_{it}^s will be updated in the next step. This procedure stops when it is not possible to further improve the dual objective function value because all dual variables are blocked from increasing by at least one slack. The output of this procedure is a dual feasible solution $\{v_{it}^{s+}\}$, with an objective function value v_D^+, and an associated set of candidate facility locations $\{\pi_{jt}^+\}$, defined by the slacks that are equal to zero. This dual solution is the base for building a primal feasible solution using the procedure that is described next.

Primal Procedure. Let us first define the following sets and notation:
$J^* = \{(j,t) \in J \times \mathcal{T} : \pi^+_{jt} = 0\}$;
$J^{s*}_t = \{j \in J : (j,\tau) \in J^*, \tau \leq t - d_{js}\}$, $\forall t, s$;
$J^{s+}_t = \{j \in J :$ facility j is operational at time t and scenario $s\}$, $\forall t, s$.
$t_1(j) = \min\{\gamma - d_{js} : j \in J^{s+}_\gamma, \gamma - d_{js} \geq 1\}$ and, if $t_1(j) \geq 1$,
$t_2(j) = \max\{\gamma \leq t_1(j) : (j,\gamma) \in J^*\}$.
$J^+ = \{(j, t_2(j)) \in J \times \mathcal{T} : j \in J^{s+}_\tau$ for some τ and s$\}$.

Set J^* represents all (j,t) such that j can begin to be prepared (built) at the beginning of t without violating (11); set J^{s*}_t includes all j that can be operational in t under scenario s; set J^{s+}_t corresponds to all j that are actually operational during t and s; set $J^+ \subseteq J^*$ corresponds to all j that start being prepared at the beginning of some t, i.e., J^+ dictates which facilities are actually built and when should the preparing works begin (location decisions). Notice that if $j \in J^+$, then j starts to be prepared at the beginning of $t_2(j)$, a time period selected taking into account all the time period(s) and scenario(s) during which j is operational and also the setup time(s) required in that scenario(s). The definition of $t_2(j)$ ensures that facility j starts to be built such that it will be operational when needed, considering the primal assignment variables, since it is limited by time period $t_1(j)$. Furthermore, the slack (10) associated with time period $t_2(j)$ should also be zero. The steps of the primal procedure are as follows:

1. Set $J^+ = J^{s+}_t = \emptyset$, $\forall t, s$. Build J^* and J^{s*}_t, $\forall t, s$.
2. For each $t \in \mathcal{T}$, if $j \in J^{s*}_t$ such that $\exists (i,s) : v^{s+}_{it} \geq \mathcal{C}^s_{ijt}$ and $v^{s+}_{it} < \mathcal{C}^s_{ij't}, \forall\, j' \in J^{s*}_t \backslash \{j\}$, then: $J^{s+}_\tau = J^{s+}_\tau \cup \{j\}$, $\forall \tau \geq t$; if $j \in J^{s'*}_t$, then $J^{s'+}_\tau = J^{s'+}_\tau \cup \{j\}$, $\forall \tau \geq t$.
3. For each (i,t,s), if $\nexists j \in J^{s+}_t$ with $v^{s+}_{it} \geq \mathcal{C}^s_{ijt}$, select $j \in J^{s*}_t : \mathcal{C}^s_{ijt} = \min\{\mathcal{C}^s_{ij't} : v^s_{it} \geq \mathcal{C}^s_{ij't}\}$. Then: $J^{s+}_\tau = J^{s+}_\tau \cup \{j\}$, $\forall \tau \geq t$; if $j \in J^{s'*}_t$, then $J^{s'+}_\tau = J^{s'+}_\tau \cup \{j\}$, $\forall \tau \geq t$.
4. Build J^+.
5. Update J^{s+}_t, $\forall t, s$. Assign each (i,t,s) to facility $j \in J^{s+}_t$ with lowest $\mathcal{C}^s_{ijt}$.

The facilities that are considered first (step 2) are the ones that at a given time t should be assigned to a given customer (i,s), according to conditions (13), called *essential* facilities. Other facilities are only opened if strictly necessary, that is, if there is a customer that cannot be assigned to an *essential* facility (step 3). At the end, for each scenario s and period t, each customer i will be assigned to the facility operating in t and s with the lowest assignment cost.

Notice that the decision of where and when to locate facilities will not immediately determine in which time periods the facilities can be assigned to customers, since these time periods will not be the same under all scenarios. It is necessary to take into account the uncertainty in setup times, meaning that the same facility can be operational in a given time period under a given scenario but not prepared to be assigned to clients in the same time period under other scenario (setup times are scenario dependent). The assignment of customers to

facilities has to take into consideration that a facility j that has began to be prepared at period t will only be operational under scenario s from $t + d_{js}$ forward.

Primal–Dual Adjustment Procedure. This procedure tries to change the current dual solution, by decreasing the value of at least one variable v^s_{it} (and thus possibly decreasing the value of some variables w^s_{ijt}), such that at least two slacks will be increased. The changes in the slacks' values may lead to the increase of other dual variables increasing the dual objective function value. Let us first consider the additional sets:

$J^{s*}_{it} = \{j : \exists \tau \leq t - d_{js} \mid (j,\tau) \in J^* \text{ and } v^s_{it} \geq \mathcal{C}^s_{ijt}\}$, $\forall (i,t,s)$;
$J^{s+}_{it} = \{j : \exists \tau \leq t - d_{js} \mid (j,\tau) \in J^+ \text{ and } v^s_{it} > \mathcal{C}^s_{ijt}\}$, $\forall (i,t,s)$;
$I^+_{jt} = \{(i,\tau,s) : J^{s*}_{i\tau} = \{j\} \text{ for } \tau \geq t\}$, $\forall (j,t)$.

The best source and the second-best source for (i,t,s) in J^{s+}_t will be represented by $j(i,t,s)$ and $j'(i,t,s)$, respectively, and such that $\mathcal{C}^s_{ij(i,t,s)t} = \min_{j \in J^{s+}_t}\{\mathcal{C}^s_{ijt}\}$, $\forall (i,t,s)$, and $\mathcal{C}^s_{ij'(i,t,s)t} = \min_{j \in J^{s+}_t, j \neq j(i,t,s)}\{\mathcal{C}^s_{ijt}\}, \forall (i,t,s)$ for $|J^{s+}_{it}| > 1$. We also define $\mathcal{C}^{s\,-}_{it} = \max_j\{\mathcal{C}^s_{ijt} : v^s_{it} > \mathcal{C}^s_{ijt}\}$. For a given (i,t,s), set J^{s*}_{it} represents all facilities j that can be open at period t (because a slack $\pi_{j\tau}$ is equal to zero for some $\tau \leq t - d_{js}$) and such that if j is open then customer i can be assigned to j at period t under scenario s. Similarly, for a given (i,t,s), the set J^{s+}_{it} considers all facilities that are in operation during period t in the current primal solution, and such that customer i would have to be assigned to j in period t under scenario s to guarantee the satisfaction of (13). If $|J^{s+}_{it}| > 1$, for some (i,t,s), then a complementary slackness condition (13) is violated. In such case, the decrease of the variable v^s_{it} causes the increase of at least two slacks $\pi_{j\tau}$, associated with distinct facilities (step 4). Set I^+_{jt} corresponds to all variables $v^s_{i\tau}$ whose value can be increased with the increase of slacks $\pi_{j\tau}, \tau \leq t - d_{js}$. This set is then used in the dual ascent procedure (step 5). As stated above, the heuristic stops when the optimal solution is found or when there are no primal or dual improvements after a given number of trials within the adjustment procedure. The steps of the primal-dual adjustment are:

1. $(i,t,s) \leftarrow (i,t,s)_1, q \leftarrow 1$; set $v_D = v^+_D$ and $v_P = v^+_P$; set $r = 0$.
2. If $|J^{s+}_{it}| \leq 1$, then go to step 9.
3. If $I^+_{j(i,t,s)t} = \emptyset$ and $I^+_{j'(i,t,s)t} = \emptyset$, then go to step 9.
4. For each (j,τ), with $\tau \leq t - d_{js}$ and $v^s_{it} > \mathcal{C}^s_{ijt}$, set $\pi_{j\tau} = \pi_{j\tau} + v^s_{it} - \mathcal{C}^{s\,-}_{it}$; set $v^s_{it} = \mathcal{C}^{s\,-}_{it}$.
5. (a) Set $I^+ = I^+_{j(i,t,s)t} \cup I^+_{j'(i,t,s)t}$ and execute the dual ascent procedure.
   (b) Set $I^+ = I^+ \cup \{(i,t,s)\}$ and execute the dual ascent procedure.
   (c) Set $I^+ = I \times \mathcal{T} \times \mathcal{S}$ and execute the dual ascent procedure.
6. If v^s_{it} is changed, then return to step 2.
7. Execute the primal procedure.
8. If neither $v^+_D > v_D$ nor $v^+_P < v_P$, then $r \leftarrow r + 1$; otherwise $r \leftarrow 0$ and update v_D and v_P.
9. If $v_D \geq v_P$, or $r = r_{max}$ or $q = |I \times \mathcal{T} \times \mathcal{S}|$, then stop; otherwise $q \leftarrow q + 1, (i,t,s) \leftarrow (i,t,s)_q$, and return to step 2.

4 Computational Experiments

Let us consider that there are only 4 scenarios, equally likely, 5 time periods, 4 possible facility locations and 10 possible customers. Facility 1 has no setup time under all scenarios ($d_{1s} = 0, \forall s \in \mathcal{S}$). Facility 2 has a setup time that is equal to 1 ($d_{2s} = 1, \forall s \in \mathcal{S}$). Facility 3 has a setup time equal to 1 under scenarios 1 and 2, and equal to 3 under scenarios 3 and 4. Facility 4 has a setup time that is equal to 1 in all scenarios but scenario 4 where it is equal to 2. Setup times for facilities 2, 3 and 4 were generated by using a PERT distribution with a most probable time (d_{mode}) equal to 1, 3 and 1 respectively. All the needed data were randomly generated. The optimal solution for this instance is given in the first line of Table 1, called stochastic solution (SS). We report the location decisions, total fixed costs, total assignment costs for each scenario and expected total assignment costs, and also the optimal objective function value f. To illustrate the impact of ignoring the uncertainty associated with setup times, we have also searched for the optimal solution of the problem where a deterministic setup time is used. First, the dynamic location problem under uncertainty with setup times fixed to d_{mod} has been solved. This problem is equivalent to problem (1)–(6) with d_{js} replaced by d_{mode}. The optimal location decisions obtained from this problem were recorded. Then, with the location decisions fixed to the values previously found, the assignment costs for each scenario were calculated (it is equivalent to solve problem (1)–(6) with fixed location decisions). The optimal solution is reported in the last line of Table 1, called deterministic solution (DS).

Table 1. Illustrative example: stochastic and deterministic solutions.

	$(j,t) : x_{jt} = 1$	Fixed costs	Assignment costs					o.f
			s = 1	s = 2	s = 3	s = 4	Total	f
SS	(1,1); (3,1)	4017	3056,5	2671,3	2858,8	2373,0	10959,5	14976,5
DS	(1,1); (2,2)	4144	3477,0	2958,3	2806,3	2294,8	11536,3	15680,3

The optimal solutions provided by the two approaches differ on the location decisions: the deterministic approach chooses to build facility 2, whilst the stochastic approach chooses facility 3. Facility 3 would hardly be a choice for the deterministic approach, given the *more pessimistic* d_{mod}. The option of ignoring uncertainty leads to a cost that is 4.49% higher than the stochastic solution ($(f_{DS} - f_{SS})/f_{DS}$). If, for instance, both d_{33} and d_{34} increase one time period to 4, the optimal SS found also dictates $x_{22} = 1$, in addition to $x_{11} = 1$ and $x_{13} = 1$. Consequently, the fixed costs increase to 5298, but the assignment costs decrease to 10074, leading to $(f_{DS} - f_{SS})/f_{DS} = 1.97\%$. The value of using the stochastic approach instead of considering deterministic setup times will be highly dependent of the problem data, namely costs, variability and magnitude of setup times.

In order to analyze the proposed model and to assess the efficiency of the proposed heuristic, several test problems were randomly generated, with $S \in \{10, 20\}$ (number of scenarios), $T \in \{10, 15\}$ (number of time periods), $M \in \{10, 20, 50\}$ (number of possible facility locations) and $N \in \{100, 200\}$ (number of possible customers). For each combination of (S, T, M, N), five instances were randomly generated. Two classes of test problems were generated considering different magnitudes in terms of setup times: shorter and longer setup times. The approach used in the generation of all test problems can be summarized as follows. First the networks (nodes and costs) were randomly generated using the algorithm described in [17]. Afterwards, setup times were also randomly generated. For each instance, we have selected one facility j such that $d_{js} = 0, \forall s \in \mathcal{S}$, ensuring that there is at least one operating facility in $t = 1$ (feasible problems). For each potential facility site j setup times were randomly generated from a PERT distribution (see, for instance, [13]). Although the use of this distribution is not exempted from criticism [20], we considered it as a good alternative for the stochastic generation of setup times. The PERT distribution requires three parameters: optimistic, d_{min}, most likely, d_{mode}, and pessimistic, d_{max}, setup times for the present case. These have been randomly generated from Uniform distributions $\mathcal{U}[l, u]$, with parameters l and u given in Table 2 dependent on the given time horizon T.

Table 2. Parameters used in the random generation of setup times.

	d_{mode}	d_{min}	d_{max}
Shorter	$[1, \frac{T}{4}]$	$[1, d_{mode}]$	$[d_{mode}, 3\frac{T}{4}]$
Longer	$[\frac{T}{4}, 3\frac{T}{4}]$	$[\frac{T}{4}, d_{mode}]$	$[d_{mode}, T]$

A total of 240 instances were generated (120 in each class) that were solved by the heuristic and by an exact algorithm, CPLEX MIP optimizer, v12.4. After some preliminary tests, we have established as stopping criterium the quality of the best solution achieved by the algorithms (2% for all test problems). This is a fairer criteria to compare a heuristic with a general solver, since many times the general solver will spend much time slightly improving an already very good solution or trying to prove the optimality of the optimal solution already found. The algorithm was coded in C–language and the computational experiments were carried out on a Intel(R) Core(TM)2 Duo CPU T9400 2.53 GHz with 4.00 GB of RAM. Codes and all instances are available from the authors upon request.

Tables 3, 4, 5 and 6 summarize the computational results obtained in terms of solution quality and computational time. Each table corresponds to a given class of setup times and a number S of scenarios. We report the minimum, average and maximum gap on the five instances solved for each combination of (S, T, M, N), as well as the minimum, average and maximum time (in seconds) spent by the algorithms to solve those instances (not including the time required to read the problems' data). Gap is given, in percentage, by the difference between the

Table 3. Computational results (shorter setup times and 10 scenarios).

T	M	N	Gap (%)						Time (sec)					
			Heur			Cplex			Heur			Cplex		
			Min	Aver	Max	Min	Aver	Max	Min	Aver	Max	Min	Aver	Max
10	10	100	0,00	0,03	0,15	0,00	0,00	0,00	0,02	0,03	0,05	3,57	4,13	5,04
	10	200	0,00	0,03	0,12	0,00	0,31	1,07	0,06	0,07	0,09	8,71	9,75	10,87
	20	100	0,00	0,24	0,51	0,00	0,04	0,20	0,04	0,05	0,05	6,77	8,89	14,01
	20	200	0,00	0,04	0,07	0,00	0,00	0,00	0,10	0,11	0,12	18,95	20,23	21,72
	50	100	0,13	0,51	0,92	0,19	0,88	1,59	0,07	29,4	86,7	31,87	42,17	51,18
	50	200	0,38	0,71	1,01	0,22	1,41	2,55	0,17	0,19	0,21	84,1	145,2	246,1
Aver			**0,08**	**0,26**	**0,46**	**0,07**	**0,44**	**0,90**	**0,08**	**4,9**	**14,5**	**25,6**	**38,4**	**58,2**
15	10	100	0,00	0,04	0,15	0,00	0,00	0,00	0,04	0,06	0,08	5,49	6,22	6,80
	10	200	0,00	0,00	0,00	0,00	0,00	0,00	0,11	0,12	0,14	15,94	18,13	19,06
	20	100	0,00	0,05	0,18	0,00	0,07	0,34	0,09	0,09	0,09	11,83	14,11	18,83
	20	200	0,00	0,01	0,04	0,00	0,00	0,00	0,19	0,20	0,22	31,28	33,88	39,48
	50	100	0,44	0,95	1,59	0,00	1,62	3,32	0,14	50,8	113,1	40,9	85,8	127,4
	50	200	0,07	0,34	0,59	0,00	11,00	43,53	0,30	0,33	0,37	80,97	212,4	501,6
Aver			**0,09**	**0,23**	**0,42**	**0,00**	**2,11**	**7,87**	**0,14**	**8,6**	**19,0**	**31,1**	**61,8**	**118,9**
			0,09	**0,25**	**0,44**	**0,03**	**1,28**	**4,38**	**0,11**	**6,79**	**16,8**	**28,4**	**50,08**	**88,5**

Solver was unable to solve one of the instances with $(T, M, N) = (15, 50, 200)$.

Table 4. Computational results (shorter setup times and 20 scenarios).

T	M	N	Gap (%)						Time (sec)					
			Heur			Cplex			Heur			Cplex		
			Min	Aver	Max	Min	Aver	Max	Min	Aver	Max	Min	Aver	Max
10	10	100	0,00	0,00	0,02	0,00	0,00	0,00	0,10	0,12	0,16	8,25	9,34	10,23
	10	200	0,00	0,00	0,00	0,00	0,00	0,00	0,20	0,23	0,26	23,1	26,25	30,65
	20	100	0,18	0,27	0,46	0,00	0,16	0,79	0,15	0,16	0,17	16,8	20,1	22,4
	20	200	0,00	0,01	0,02	0,00	0,18	0,91	0,31	0,33	0,35	44,8	49,7	65,2
	50	100	0,18	0,80	1,37	0,00	0,53	1,88	0,26	160,9	450,1	90,4	145,4	190,8
	50	200	0,33	1,13	1,97	0,00	19,10	38,20	0,57	32,24	158,8	291,7	467,2	643,2
Aver			**0,12**	**0,37**	**0,64**	**0,00**	**3,33**	**6,96**	**0,27**	**32,3**	**101,7**	**79,1**	**119,7**	**160,4**
15	10	100	0,00	0,02	0,11	0,00	0,00	0,00	0,17	0,24	0,29	16,16	18,81	23,35
	10	200	0,00	0,00	0,00	0,00	0,00	0,00	0,42	0,49	0,56	37,00	41,95	49,22
	20	100	0,00	0,06	0,20	0,00	0,00	0,00	0,30	0,31	0,32	28,42	35,14	43,48
	20	200	0,00	0,05	0,20	0,00	0,00	0,00	0,60	0,67	0,80	74,88	81,74	87,88
	50	100	0,70	1,35	2,69	0,00	10,79	41,53	42,49	371,88	754,31	159,73	282,60	352,50
	50	200	0,52	0,71	1,06	–	–	–	0,98	1,10	1,21	–	–	–
Aver			**0,20**	**0,37**	**0,71**	**0,00**	**2,16**	**8,31**	**7,49**	**62,45**	**126,3**	**63,24**	**92,05**	**111,29**
			0,16	**0,37**	**0,68**	**0,00**	**2,80**	**7,57**	**3,88**	**47,39**	**113,9**	**71,88**	**107,11**	**138,08**

Solver was unable to solve: three of the instances with $(T, M, N) = (10, 50, 200)$; one of the instances with $(T, M, N) = (15, 50, 100)$; any with $(T, M, N) = (15, 50, 200)$.

best objective function value found by the algorithm and the best known lower bound on the optimal value divided by this best known lower bound. The average

Table 5. Computational results (longer setup times and 10 scenarios).

T	M	N	Gap (%)						Time (sec)					
			Heur			Cplex			Heur			Cplex		
			Min	Aver	Max	Min	Aver	Max	Min	Aver	Max	Min	Aver	Max
10	10	100	0,00	0,11	0,26	0,00	0,00	0,00	0,03	0,04	0,04	2,23	2,46	2,76
	10	200	0,00	0,02	0,05	0,00	0,00	0,00	0,08	0,08	0,08	5,10	6,26	7,16
	20	100	0,00	0,09	0,22	0,00	0,05	0,27	0,04	0,06	0,08	4,74	5,45	6,46
	20	200	0,00	0,10	0,36	0,00	0,16	0,78	0,11	0,12	0,13	12,04	14,44	18,13
	50	100	0,80	1,13	1,75	0,00	0,85	1,83	14,54	37,28	94,65	14,06	20,38	25,76
	50	200	0,39	0,63	0,94	0,00	0,37	1,46	0,19	0,20	0,22	42,74	60,95	84,74
Aver			**0,20**	**0,35**	**0,60**	**0,00**	**0,24**	**0,72**	**2,50**	**6,30**	**15,9**	**13,5**	**18,3**	**24,17**
15	10	100	0,00	0,00	0,00	0,00	0,00	0,00	0,06	0,07	0,11	3,70	4,03	4,66
	10	200	0,00	0,00	0,00	0,00	0,00	0,00	0,13	0,15	0,17	9,53	10,75	12,01
	20	100	0,00	0,06	0,11	0,00	0,00	0,00	0,09	0,10	0,12	7,96	9,00	11,58
	20	200	0,00	0,02	0,10	0,00	0,00	0,00	0,20	0,22	0,26	20,67	22,21	23,62
	50	100	0,01	0,56	1,51	0,00	0,70	1,87	0,15	32,5	161,8	20,3	30,2	47,5
	50	200	0,15	0,46	0,99	0,00	0,09	0,26	0,31	0,37	0,42	51,64	85,40	125,05
Aver			**0,03**	**0,18**	**0,45**	**0,00**	**0,13**	**0,36**	**0,16**	**5,6**	**27,1**	**18,9**	**26,9**	**37,4**
			0,11	**0,26**	**0,52**	**0,00**	**0,19**	**0,54**	**1,33**	**5,9**	**21,5**	**16,2**	**22,6**	**30,8**

results for gap and computational time are also reported for each T, as well as for each S, in the last row of the corresponding table. Statistics refer only to those instances that were solved. The heuristic was able to solve all the instances whereas the solver could not solve some problems due to lack of memory. We report these cases at the bottom of each table. Both algorithms present gaps higher than 2% for some instances, which is justified as follows: for the general solver, this happens due to lack of memory to proceed with the calculations; for the heuristic this happens when the maximum number of trials to improve a solution within the adjustment procedure was reached. The computational results show that the heuristic is capable of finding very good quality solutions in reasonable computational times. The worst gap equals 2.69% and was observed in instances with shorter setup times and $(S, T, M, N) = (20, 15, 50, 100)$, the same set where the maximum computational time was also reached, 754,3 s. The larger gaps and computational times were observed in instances with largest M, 50 potential facility sites. Within these sets of instances, gap increases as the number of scenarios increases for shorter setup times. On the other hand, for longer setup times, gap decreases in some problems as the number of scenarios increases. The heuristic's performance was better in large sized problems with longer setup times than with shorter ones.

In terms of solution quality, the average minimum gap achieved by CPLEX is better than the average minimum gap obtained by the heuristic, although for many instance sets the minimum gap achieved is the same. However, less instances were solved by the solver, in particular large sized problems. The execution times required by the solver are clearly higher than those required by the heuristic. In the majority of problems, except instances with $(M, N) = (50, 100)$,

Table 6. Computational results (longer setup times and 20 scenarios).

T	M	N	Gap (%)						Time (sec)					
			Heur			Cplex			Heur			Cplex		
			Min	Aver	Max	Min	Aver	Max	Min	Aver	Max	Min	Aver	Max
10	10	100	0,00	0,00	0,01	0,00	0,00	0,00	0,11	0,13	0,16	5,04	6,18	7,60
	10	200	0,00	0,00	0,00	0,00	0,00	0,00	0,25	0,28	0,31	16,19	18,59	20,20
	20	100	0,00	0,28	0,92	0,00	0,22	1,12	0,14	0,17	0,21	12,62	14,24	17,16
	20	200	0,01	0,04	0,06	0,00	0,00	0,00	0,29	0,36	0,40	27,22	32,54	35,12
	50	100	0,14	0,72	1,33	0,00	0,63	1,72	0,25	100,9	418,8	43,3	70,5	103,4
	50	200	0,17	0,55	1,01	0,00	9,54	24,22	0,56	0,63	0,72	139,9	209,1	343,3
Aver			**0,05**	**0,27**	**0,55**	**0,00**	**1,73**	**4,51**	**0,27**	**17,1**	**70,1**	**40,7**	**58,5**	**87,8**
15	10	100	0,00	0,01	0,05	0,00	0,00	0,00	0,23	0,28	0,36	8,91	10,59	12,98
	10	200	0,00	0,00	0,00	0,00	0,00	0,00	0,50	0,60	0,77	26,69	29,08	32,12
	20	100	0,00	0,13	0,37	0,00	0,00	0,00	0,34	0,38	0,44	18,41	24,08	28,30
	20	200	0,00	0,02	0,08	0,00	0,00	0,00	0,81	0,86	0,93	50,61	54,73	64,01
	50	100	0,03	0,43	0,85	0,00	0,18	0,59	0,50	31,82	156,9	52,1	110,7	187,9
	50	200	0,49	0,88	1,08	–	–	–	1,09	1,34	1,64	–	–	–
Aver			**0,09**	**0,25**	**0,41**	**0,00**	**0,04**	**0,12**	**0,58**	**5,9**	**26,8**	**31,4**	**45,8**	**65,1**
			0,07	**0,26**	**0,48**	**0,00**	**0,96**	**2,51**	**0,42**	**11,5**	**48,5**	**36,5**	**52,8**	**77,5**

Solver was unable to solve any of the instances with $(T, M, N) = (15, 50, 200)$.

the maximum computational time of the heuristic is smaller than the minimum time of CPLEX. On average, the heuristic presents better mean and maximum times. On average, the gap is also better for the heuristic than for CPLEX in the majority of problems, except for longer setup times and larger horizons. CPLEX presents a better behavior for instances with longer setup times than with shorter, and is also able of solving more problems.

It is possible to observe that, as expected, the objective function value is always higher when longer setup times are considered. Table 7 reports the average relative increase (in percentage) of the (Best) objective function values for longer setup times, considering the average over the five instances. In the majority of instances, this increase in total cost corresponds to an increase of assignment costs and a decrease of fixed facility costs. However, there are also instances where both fixed facility and assignment costs increase. Table 7 also reports the minimum and maximum number of operational facilities for each combination of (S, T, M, N). Instances with longer setup times have less operational facilities. It should be noted that not only the number of operational facilities can change, but also the period of time when facilities become operational. This may have to do with the fact that with shorter setup times it is possible to take better advantage of the compromises between total fixed and assignment costs, since facilities can be operational during a longer period of time within the planning horizon.

Table 7. Relative increase of objective function values (RIof, in (%)) and number of operational facilities with shorter (♯ Sh) versus longer (♯ Lo) setup times.

T	M	N	RIof (%)		$S = 10$		$S = 20$	
			$S = 10$	$S = 20$	♯ Sh	♯ Lo	♯ Sh	♯ Lo
10	10	100	11,2	10,4	7–9	5–7	8–9	5–8
	10	200	9,9	10,1	9–10	6–8	9–10	7–9
	20	100	9,0	10,7	10–13	8–12	11–13	8–11
	20	200	11,5	10,2	15–19	11–14	15–17	10–15
	50	100	12,1	11,1	15–21	14–19	17–20	13–15
	50	200	10,1	10,5	25–28	20–25	26–29	15–24
Aver			**10,7**	**10,5**				
15	10	100	9,1	12,4	8–9	7–8	8–10	7–8
	10	200	9,6	9,0	10–10	9–10	10–10	8–9
	20	100	13,3	12,2	14–17	11–14	14–15	11–12
	20	200	12,7	12,8	18–19	14–14	15–18	12–15
	50	100	11,5	12,7	21–26	16–21	19–24	14–19
	50	200	14,4	13,7	28–35	23–28	27–33	22–25
Aver			**11,8**	**12,1**				

5 Conclusions

A dynamic facility location problem incorporating setup time uncertainty has been introduced. As far as the authors know, it is the first time this problem is addressed, and it represents a situation that often happens in practice: projects due get delayed, and opening a facility implies, most of the times, a set of different and interrelated tasks that have to be executed. A primal–dual heuristic has been developed, and computational results reported. The heuristic is capable of calculating optimal or near-optimal solutions in much shorter computational times than the general solver. Future work will explicitly consider compromises between investment cost and setup time uncertainty.

References

1. Albareda-Sambola, M., Alonso-Ayuso, A., Escudero, L.F., Fernández, E., Pizarro, C.: Fix-and-relax-coordination for a multi-period location-allocation problem under uncertainty. Comput. Oper. Res. **40**, 2878–2892 (2013)
2. Al-Momani, A.H.: Construction delay: a quantitative analysis. Int. J. Proj. Manag. **18**(1), 51–59 (2000)
3. Arantes, A., da Silva, P.F., Ferreira, L.M. D.: Delays in construction projects-causes and impacts. In: 2015 International Conference on Industrial Engineering and Systems Management (IESM), pp. 1105–1110. IEEE (2015)
4. Assaf, S.A., Al-Hejji, S.: Causes of delay in large construction projects. Int. J. Proj. Manag. **24**(4), 349–357 (2006)

5. Baron, O., Milner, J., Naseraldin, H.: Facility location: a robust optimization approach. Prod. Oper. Manag. **20**(5), 772–785 (2011)
6. Bilde, O., Krarup, J.: Sharp lower bounds and efficient algorithms for the simple plant location problem. Ann. Discret. Math. **1**, 79–97 (1977)
7. De Rosa, V., Hartmann, E., Gebhard, M., Wollenweber, J.: Robust capacitated facility location model for acquisitions under uncertainty. Comput. Ind. Eng. **72**, 206–216 (2014)
8. Dias, J., Captivo, M.E., Clímaco, J.: Efficient primal-dual heuristic for a dynamic location problem. Comput. Oper. Res. **34**(6), 1800–1823 (2007)
9. Dias, J.: Can we really ignore time in Simple Plant Location Problems? (No. 7/2015), Inesc-Coimbra (2015)
10. Dias, J.M., Marques, M.d.C.: A multiobjective approach for a dynamic simple plant location problem under uncertainty. In: Murgante, B., Misra, S., Rocha, A.M.A.C., Torre, C., Rocha, J.G., Falcão, M.I., Taniar, D., Apduhan, B.O., Gervasi, O. (eds.) ICCSA 2014. LNCS, vol. 8580, pp. 60–75. Springer, Cham (2014). https://doi.org/10.1007/978-3-319-09129-7_5
11. Dunke, F., Heckmann, I., Nickel, S., Saldanha-da-Gama, F.: Time traps in supply chains: is optimal still good enough? Eur. J. Oper. Res. **264**(3), 813–829 (2018)
12. Erlenkotter, D.: A dual-based procedure for uncapacitated facility location. Oper. Res. **26**(6), 992–1009 (1978)
13. Golenko-Ginzburg, D.: On the distribution of activity time in PERT. J. Oper. Res. Soc. **39**(8), 767–771 (1988)
14. Hernández, P., et al.: A branch-and-cluster coordination scheme for selecting prison facility sites under uncertainty. Comput. Oper. Res. **39**, 2232–2241 (2012)
15. Louveaux, F.V.: Discrete stochastic location models. Ann. Oper. Res. **6**(2), 21–34 (1986)
16. Louveaux, F.V., Peeters, D.: A dual-based procedure for stochastic facility location. Oper. Res. **40**(3), 564–573 (1992)
17. Marques, M.d.C., Dias, J.: Simple dynamic location problem with uncertainty: a primal-dual heuristic approach. Optimization **62**(10), 1379–1397 (2013)
18. Marques, M.d.C, Dias, J.: Dynamic location problem under uncertainty with a regret-based measure of robustness. Int. Trans. Oper. Res. (2015). https://doi.org/10.1111/itor.12183
19. Snyder, L.V.: Facility location under uncertainty: a review. IIE Trans. **38**, 537–554 (2006)
20. Trietsch, D., Baker, K.R.: PERT 21: fitting PERT/CPM for use in the 21st century. Int. J. Proj. Manag. **30**(4), 490–502 (2012)
21. Van Roy, T.J., Erlenkotter, D.: A dual-based procedure for dynamic facility location. Manag. Sci. **28**(10), 1091–1105 (1982)