

Modeling an Optimal Control Problem for the Navigation of Mobile Robots in an Ocduded Environment Application to Unmanned Aerial Vehicles

Kahina Louadj[1(✉)], Philippe Marthon[2], and Abdelkrim Nemra[3]

[1] Laboratoire de Conception et Conduites Systèmes de Production (L2CSP), Tizi-Ouzou, Algérie
louadj_kahina@yahoo.fr
[2] IRIT-ENSEEIHT, Toulouse, France
Philippe.marthon@enseeiht.fr
[3] Laboratoire Robotique et Productique, Ecole Militaire Polytechnique, Bordj El Bahri, Algiers, Algérie
karim_nemra@yahoo.fr

Abstract. An optimal control problem of two Unmanned autonomous Vehicles to follow the trajectory and avoid the collision between them. The aim is to minimize energy, the distance between state and desired, and maximize the distance between the two drones. For this study, we used midpoint such discretization method, the simulation results ar given by Bocop software.

Keywords: Navigation · UAV · Trajectory planning

1 Introduction

Today, the emergence of robotics [20,21] led to make robots more autonomous and increasingly easy to drive. A robotic system is defined as an artificial system, equipped with sensors and actuators, designed to act on the outside world. In the beginning, these robots were installed in industries to perform us tasks that are painful, repetitive or impossible for humans. nowadays, we can find robots in different fields: industrial, domestic, medical, military, etc.

UAVs are autonomous or semi-autonomous flying machines capable of flying missions without human presence on board [6,7]. This earned them the name "Unmanned Arial Vehicle: UAV". Today we are witnessing a rise of drones in civil and military aeronautics. The potential application of these flying machines is very high, but there are still a number of difficulties that slow down their expansion. Researchers and designers have been working intensively over the past two decades to try to iron out these difficulties. This concerns the field of mechanical and aerodynamic design as well as the fields of embedded systems and automation. As for classic aircraft cases the design of a drone requires

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mastery of previous scientific fields. However, despite some similarities with aircraft, the design of drones raises completely new specific problems. Experiments and publications concerning the resolution of these problems are increasing exponentially.

In some problems, the workspace can be dynamic. the location of obstacles may change over time, making time an important parameter in evaluating the validity of drone movement, as opposed to path planning. Thus, the trajectory planning must respect the dynamic constraints that arise from the environment (mobile obstacles) and from the robotic system considered (its dynamics). We distinguish, essentially, two categories of methods of trajectory planning: methods based on a reference trajectory and those not using them. In fact, for the first category, the planner has a calculated reference trajectory in a model of the environment using a global method.

Optimal control theory has been formulated as an extension of the calculus of variations. Based on the theoretical foundation laid by several generations of mathematicians, optimal control has developed into a well established research area and nds its applications in many scienticelds, ranging from mathematics and engineering to biomedical and management sciences [9–11].

Trajectory planning consists of generating an optimal trajectory between two points in the configuration space [1–3,5]. This trajectory minimizes a particular criterion such as time, energy or distance traveled. The feasibility of the trajectory generated depends on the technique used, the chosen cost function and the different constraints.

Our goal is to find a control of two drones that can follow the trajectory and avoid the collision between the two drones. for this, we use discretization method such as the midpoint method and the simulations are made using the Bocop software [22]. The criterion is to minimize distance between state and state desired, energy and maximize the distance between the two drones [14–19].

The article is organized as follows; In Sect. 1 defines the general problem of navigation and as application is two quadrotor drones. In Sect. 2, the mathematical model of the optimal control problem of the two drones. The results are given by Bocop in Sect. 3 and finished by conclusion.

2 Definition of General Problem of Navigation

In Autonomous Vehicle's, the development of trajectory planners capable of guiding choices towards maximum efficiency is of crucial importance. One of the major problems in robotics lies in the establishment of autonomous robots. By autonomy, we mean that by means of a high level specification of the task to be executed by the robot, it will be able to realize it without any external intervention.

The development of such plans is a complex task that requires a fine and accurate modeling of the work environment and a precise definition of the objectives to be achieved. In general, trajectory planning approaches are classified into two major families: the so-called reactive or non-reference pathways consist of iteratively modifying, over time, a movement initially calculated to best

guarantee convergence towards the goal. The trajectory deformation approach allows a mobile robot to navigate safely in a dynamic environment so as to move away from obstacles and satisfy the kinematic constraints of the system [24–26].

The motion planning for a drone evolving in an environment is to determine a sequence of actions allowing it to move between two given configurations while respecting a certain number of constraints and criteria. This sequence of actions is defined by a succession of crossing points belonging to the trajectory of the considered mobile which must respect a certain number of constraints and criteria which generally depend on the nature of the task to be carried out, the nature of the environment and characteristics of the drone itself. This subject has sparked many studies. Anyway, the difficulty of such a problem depends largely on the nature of the environment which can be static ie not changing over time (such as walls, furniture ...), ie dynamic that there are entities (obstacles or other drones), besides the drone that move there. It should be noted, of course, that most real environments are dynamic.

Although the path deformation is to distance every moment the path followed by the robot of the current position of the obstacles, the trajectory deformation aims to anticipate their movements by removing this trajectory of the forecast model from the future movements of moving objects. In fact, the trajectory followed decomposes itself into a path followed by the drone and a law of speed (kinematics). Thus a trajectory tells us, at every moment, the position of the drone on its way. Mathematically, a trajectory is described by a parametric curve which represents the path to be covered in time. It is important to note here that when it comes to a dynamic environment, a motion planner is required to predict trajectory deformations to avoid possible collisions. To this end, the planner can distort the path he intends to follow, as well as modify his speed to avoid an obstacle: to avoid an animal crossing a road, a driver may brake or change his path [24–26].

In our case, Given n moving vehicles in an environment cluttered with fixed and moving obstacles, find the controls and trajectories of these vehicles to avoid collision and to complete their route in minimal time (safety and efficiency).

An admissible movement that the drone must perform between the initial position and a given end position. Depending on the nature of the environment, including the obstacles, we distinguish:

- **Path Planning:** Representing the simplest motion planning case in which the drone moves in an environment populated by static obstacles. Thus, we limit ourselves to the geometrical aspect of the problem which amounts to calculating a continuous sequence of configurations without collision between two given configurations.
- **Trajectory Planning:** In some problems, the workspace can be dynamic. The location of obstacles may change over time, making the temporal dimension an important parameter in evaluating the validity of the movement of the drone. Thus, the trajectory planning becomes unavoidable as soon as the problem requires taking into account the dynamic constraints of the robotic

system that are those concerning the environment or those that describe the dynamics of the drone.

We are interested in the decision of the navigation of the drone namely the planning of movement for a drone for which the taking into account of cinematic constraints this primordial, coupled with the need to consider its own geometry and without forgetting the good mastery of the environment in which it evolves.

3 Application to the Case of Two Quad-Rotor Drones

A quad-rotor is an aerial vehicle which rotates by producing differentials in thrust between it's for motors. The following section presents the mathematical model of the quad-rotor [6,7].

The motion of the quad-rotor can be divided into two subsystems, rotational subsystem (roll, pitch and yaw) and translation subsystem (altitude and x and y position). The mathematical model of the two quad-rotor can be written in a state space representation as follows

$$\begin{cases} \dot{x}_{1i} = \dot{\varphi} = x_{2i}, \\ \dot{x}_{2i} = \ddot{\varphi} = a_1 x_{4i} x_{6i} + a_2 x_{4i} \Omega + b_1 u_{1i}, \\ \dot{x}_{3i} = \dot{\theta} = x_{4i}, \\ \dot{x}_{4i} = \ddot{\theta} = a_3 x_{2i} x_{6i} + a_4 x_{2i} \Omega + b_2 u_{2i}, \\ \dot{x}_{5i} = \dot{\psi} = x_{6i}, \\ \dot{x}_{6i} = \ddot{\psi} = a_5 x_{2i} x_{4i} + b_3 u_{3i}, \\ \dot{x}_{7i} = \dot{z} = x_8, \\ \dot{x}_{8i} = \ddot{z} = \dfrac{u_{4i}}{m}(cosx_{1i} sinx_{3i} cosx_{5i} + sinx_{1i} sinx_{5i}), \\ \dot{x}_{9i} = \dot{x} = x_{10i}, \\ \dot{x}_{10i} = \ddot{x} = \dfrac{u_{4i}}{m}(cosx_{1i} sinx_{3i} sinx_{5i} - sinx_{1i} cosx_{5i}), \\ \dot{x}_{11i} = \dot{y} = x_{12i}, \\ \dot{x}_{12i} = \ddot{y} = \dfrac{cosx_{1i} cosx_{3i}}{m} u_{4i} - g, \\ -\pi/2 \leq x_{1i} \leq \pi/2, -\pi/2 \leq x_{3i} \leq \pi/2, \\ -\pi \leq x_{5i} \leq \pi, x_k i(0) = 0, k = \overline{1,12}, i = \overline{1,2} \\ -20 \leq u_{ji} \leq 20, \ j = \overline{1,4}, \ 0 \leq u_{4i} \leq 20. \end{cases} \tag{1}$$

Where $g(m/s^2)$: gravity acceleration;$I_x, I_y, I_z(kg/m^2)$: roll, pitch and yaw inertia moments respectively, $J_r(kg/m^2)$: the rotor inertia; $m(kg)$: mass; $x, y, z(m)$: longitudinal, lateral and vertical motions respectively; $\phi, \theta, \psi(rad)$: roll, pitch and yaw angles, respectively; $w_k(rad/s)$: rotor angular velocity, where, k equal to 1, 2, 3 and 4; $d(m)$: the distance between the quad-rotor center of mass and the propeller rotation axis; $u_1, u_2, u_3(N.m)$: aerodynamical roll, pitch and yaw moments respectively; $u_4(N)$: lift force.

With $\Omega = w_1 - w_2 + w_3 - w_4$, $a_1 = \dfrac{I_y - I_z}{I_x}$, $a_2 = \dfrac{-J_r}{I_x}$, $a_3 = \dfrac{I_z - I_x}{I_y}$, $a_4 = \dfrac{J_r}{I_y}$, $a_5 = \dfrac{I_x - I_y}{I_z}$, $b_1 = \dfrac{d}{I_x}$, $b_2 = \dfrac{d}{I_y}$, $b_3 = \dfrac{d}{I_z}$.

One of the main objectives of this paper is to find control which ensures the convergence of positions $\{x, y, z\}$, to their desired trajectories respectively $\{x_d, y_d, z_d\}$, while maintaining stability of roll and pitch angle $\{\psi, \phi, \theta\}$ in final time.

Then, the criterion is formulated as follows:

$$J = \int_0^{t_f} \rho_1[\sum_{i=1}^{2}(x_{7i} - x_{di})^2 + (x_{9i} - y_{di})^2 + (x_{11i} - z_{di})^2] - \rho_2[(x_{71} - x_{72})^2 + (x_{91} - x_{92})^2 + (x_{111} - x_{112})^2] + (1 - (\rho_1 + \rho_2))(\sum_{i=1}^{2}(u_{1i}^2 + u_{2i}^2 + u_{3i}^2 + u_{4i}^2)dt \rightarrow min \quad (2)$$

where $t_f(second)$: final time; $\{x_d, y_d, z_d\}$: desired trajectory of $\{x, y, z\}$.

4 Results Simulation

$\omega = 340; d = 0.23;\ m = 0.6;\ I_x = I_y = 7.5e-3,\ I_z = 1.3e-2;\ J_r = 6e-5;$ $g = 9.8$.

The red line is the delimiter of $x_1,\ x_3,\ x_5$ respectively. And the blue line is the trajectories.

The results given by Bocop software are presented in Figs. 1, 2, 3, 4, 5, 6, 7 and 8. And Figs. 9, 10, is the criterion for $\rho = 1e-9$. Else, the optimal solution is ensured in 100 iterations with 3.69 s. And $J = 7.2963$. The results are compatible with desired state, and satisfies the objective of this problem's.

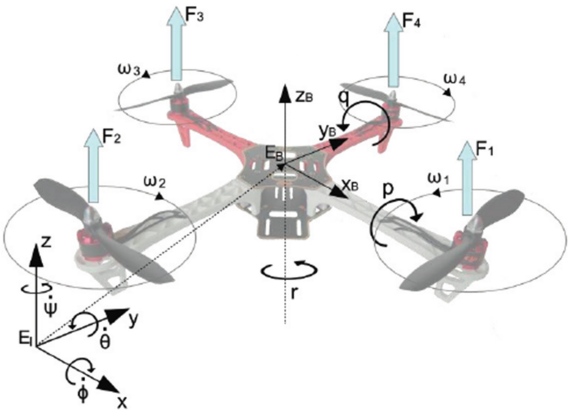

Fig. 1. Quad-rotor

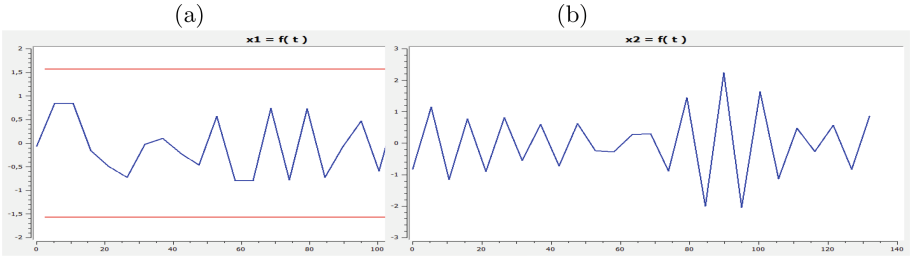

Fig. 2. Trajectory state of x_1 and x_2 respectively.

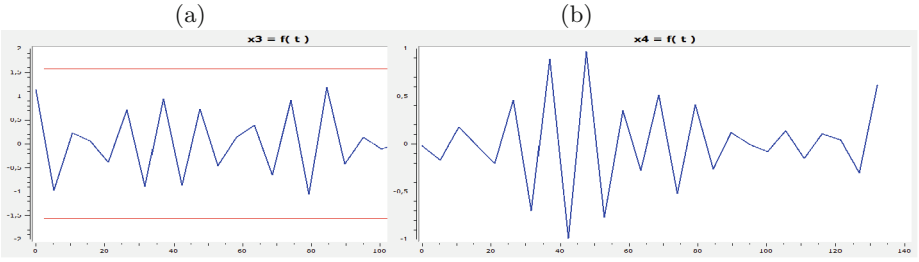

Fig. 3. Trajectory state of x_3 and x_4 respectively.

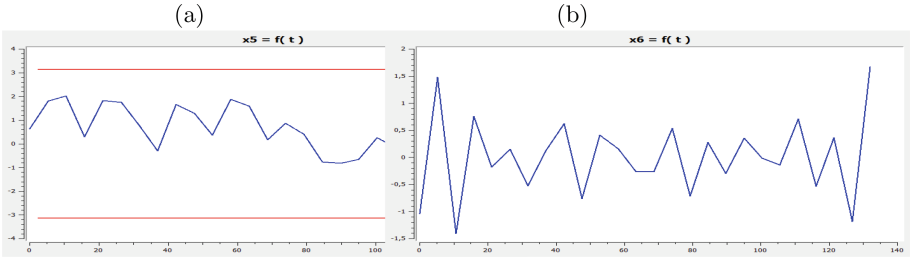

Fig. 4. Trajectory state of x_5 and x_6 respectively.

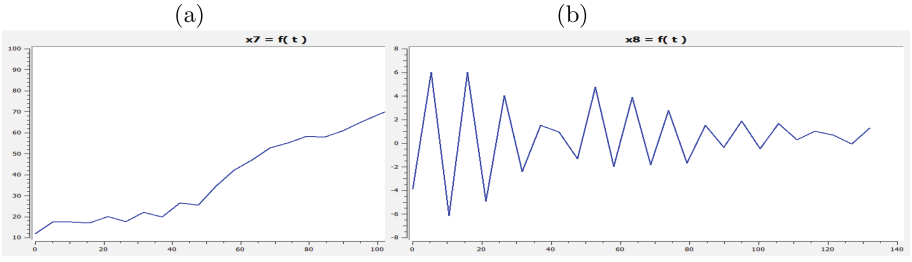

Fig. 5. Trajectory state of x_7 and x_8 respectively.

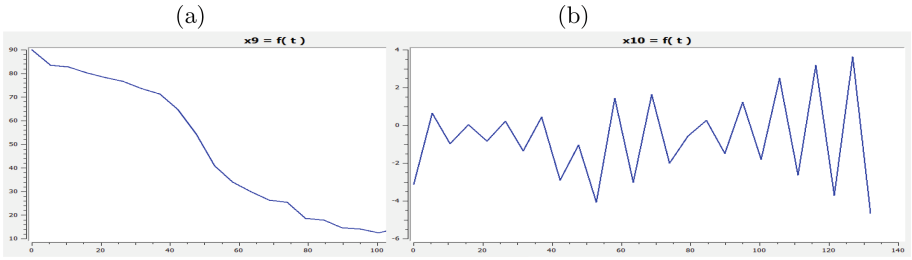

Fig. 6. Trajectory state of x_9 and x_{10} respectively.

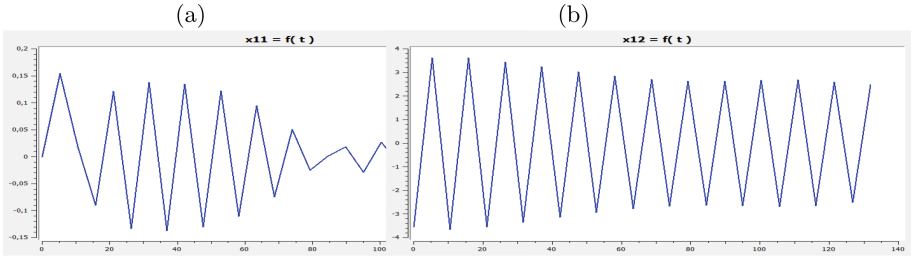

Fig. 7. Trajectory state of x_{11} and x_{12} respectively.

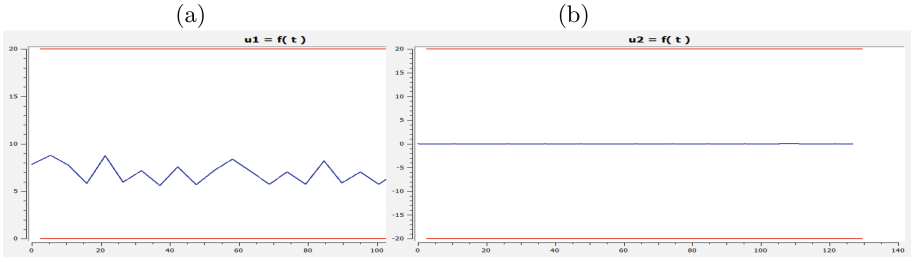

Fig. 8. Control u_1 and u_2 respectively.

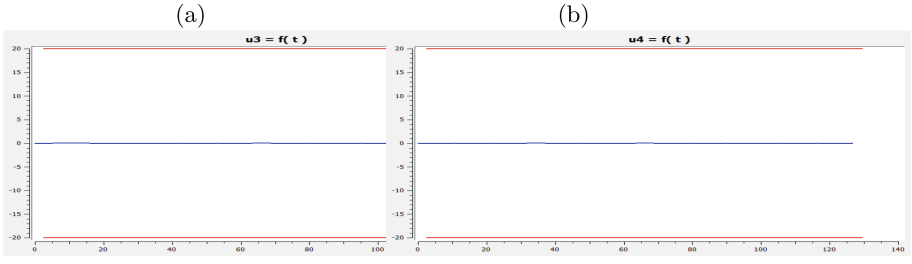

Fig. 9. Control of u_3 and u_4 respectively.

5 Conclusion

In this work, we are Modeling an optimal control problem for the navigation of mobile robots in an occluded environment Application to unmanned aerial vehicles. And we have solved this problem using discretization method with Bocop software. An optimal control problem of unmanned aerial vehicle to minimize the energy and distance between the state and desired trajectory, and maximize the distance between the two quadrotor in free final time. The results are adequate for our purpose in the computational time is 3.69 s in 100 iterations with Bocop software. The convergence is fast and the computational time is small.

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