Using Mathematical Models to Describe the Dynamics of the Spread of Traditional and Cryptocurrency Payment Systems

Victor Dostov[1], Pavel Shoust[2(✉)], and Elizaveta Popova[3]

[1] Federal State Budgetary Educational Institution of Higher Education "Saint-Petersburg State University", 7-9 Universitetskaya Emb, St Petersburg 199034, Russia
dostov@npaed.ru

[2] Russian Electronic Money and Remittance Association, 5/2 Orlikov Per, Moscow 107078, Russia
shoust@npaed.ru

[3] Saint-Petersburg National Research University of Information Technologies, Mechanics and Optics 49 Kronverkskiy Prospect, St Petersburg 197101, Russia
popova.elizaveta@list.ru

Abstract. As new payment systems emerge, it is important to predict their dynamics and separate payments from speculative transactions. Based on the classification of payment systems as 'one-sided' and 'two-sided' we use mathematical methods to predict their behavior over time. By introducing the fatigue factor and involvement factor, we draw the differential equation that could be used for the analysis of the payment systems' behavior. The analysis shows that any changes in the initial state of the system fade over time; one-time circumstantial changes (such as sudden regulatory change or promotional campaign) have an only temporary effect. Our equations can also be used to identify the prevailing type of transactions (P2P or C2B) in 'mixed' systems. Our model is verified by empirical data from payment card statistics, WebMoney registration rate and can be used to analyze Bitcoin usage as well. Further, research on using the model to explain and predict the competitive effects is also proposed. This is the first attempt at using differential equations for payment system analysis with a model verified by empirical evidence.

Keywords: P2P · B2C · Cryptocurrencies · Payment systems · The dynamics of the spread · Commodization · Payment system · Model · Bass equation · Bitcoin

1 Introduction

According to Summers (1994), a payment system is a set of rules, institutions and technical mechanisms for transferring funds between a sender and a recipient (in case of retail payment systems - from an individual to a legal or physical entity [1, 6]).

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Retail payments are an integral part of the functioning of modern society. Their role—in terms of the value, volume and variety of such payments — has increased particularly in recent decades. According to the World Payments Report [20], the volume of non-cash payments in 2016 grew by 10% per year [18]. The trend is expected to continue in the future, primarily fueled by the growth in the developing economies. At the same time, for almost the entire twentieth century, the market was very technologically uniform—almost exclusively dominated by payment cards (VISA, MasterCard and others) and money transfer systems, both domestic (like postal transfers) and international (Western Union, MoneyGram). Thomas Hugo (2014) in his «Measuring progress toward a cashless society» shows that the last 20 years saw a rapid growth of retail payment systems, both quantitative and (extensively) qualitative [16].

New payment systems continue to emerge. For example, the popularity of distributed ledgers-based payment systems, especially Bitcoin, made researchers wonder about the long-term prospects of such projects [9]. As new virtual currencies might be used for speculation as well as facilitating payments, it is also important to estimate the correlation between these two functions.

This study seeks to answer these questions. The idea is to develop mathematical models that could be used to study both “breakthrough” systems (new payment systems) and traditional solutions in the field of electronic payments.

However, only few research papers so far are dedicated to creating a model that would explain and predict the development of payment systems. The main attention of the researchers is focused on the study of legal [6], technological [4, 8], macroeconomic [16] aspects of the functioning of payment systems. Yet, the analysis and forecasting payment system’s development seems to be not only of theoretical but also practical value. The scientific tools that allow to analyze the evolution of such systems can create an empirical basis for decision-making not only in the private sector, but also in the development of national plans for the development of infrastructure and, ultimately, the transition to a cashless society.

Some payment systems are developing incrementally, some are growing rapidly, others leave the market, unable to attract customers. In other words, the market is extremely dynamic—from giants such as International Payment Schemes (IPS) to small crypto-startups [20]. There is an obvious interest in mathematical modeling of their dynamics. However, there are very few papers on this topic. Firstly, such systems are relatively young; secondly, they are quite specific and require special knowledge to understand the principles of their work. Third, unlike the “classical” economy of production and services, their impact on macroeconomic indicators is small, which demotivate “general” economists. A certain incentive for such research was created by the phenomenon of Bitcoin, however, the work usually focuses on deep, although generalized, technical analysis, for example, in the work of Fantazzini et al. [8]. Several papers devoted to systems of gross payments among banks, for example the work of Kopytin [4]. Nevertheless, from the point of the normal economy operation and the interests of the consumer, the importance of retail payment systems is huge and constantly growing. Therefore, we see academic and practical benefits of building a mathematical model of such systems.

In this paper, we assume that in the long term, the behavior of big number of customers that are defined by a wide set of parameters is equivalent to the behavior of a

single user, described with a small number of statistically measurable parameters. This hypothesis is confirmed by studies that signal the gradual convergence and simplification of the functionality of payment systems, reducing them to the basic services of money transfer from point "a" to point "b" - called "commodization" [2].

This paper is structured as follows. In the first section, we present the methodology, introduce the concept of the payment systems and their classification, which is relevant for subsequent analysis. We also consider the Bass Models and assess their applicability to the research at hand.

In the second section, we present a model for analyzing the development of payment systems, considering the fatigue and involvement coefficients. In the third section, we test the proposed approach to the available empirical material. In the final part, several conclusions are made regarding the theoretical and practical implications of the use of mathematical apparatus for predicting the growth of financial services. Several proposals for further research are formulated.

The results of this work can be used to predict the behavior of payment systems, payment instruments, cryptocurrencies, etc.

2 Methodology

In this study, "payment system" will be understood as a set of instruments, banking procedures and, typically, interbank funds transfer systems that ensure the circulation of money [23]. We use a holistic approach and consider the payment system as a single actor This means that the internal functioning of the system: internal legal, technological and economic parameters are beyond the interest of the study. In a simplified sense, we consider the payment system as a "black box" - its interaction with the environment can be described by a limited set of parameters. The function of this "black box" is the transfer of funds from the sender to the recipient.

Generally, the research is focused on the qualitative and quantitative influence of external parameters on payment systems.

For further purposes, we use the existing classification of financial services (and retail payment systems as their subset) on C2B and P2P [13]. In C2B (customer to business - hereinafter "one-sided") [21] systems, a limited number of customers make payments to the resource (stores) with infinite capacity. Interaction between customers and stores is carried out through a non-cooperative game (where senders and recipients can not directly determine the actions of each other). A simple example is a credit card payment to a store for some product. P2P payments (person to person - hereinafter "two-sided system") [21] are made between individuals within a closed audience (in this case, the parties can influence each other). Simple examples are international transfers such as Western Union or transfers from card to card, implemented by Sberbank in Russia. The last example shows that the same payment system can serve as P2P and C2B in function. As we will see later, depending on the predominance of one of these areas in the company's behavior of the model vary significantly.

Currently, several approaches to analyzing distribution dynamics of products and services in the market are used. From the methodological point, there are computer-based modeling and differential equation-based approaches. We do not deny the

usefulness of computer modeling for cases that cannot be described analytically, but here we suggest further implementation of differential equations. There are two major approaches that lead to formally similar results. The first is the Bass innovations diffusion theory [10], its basic equation takes the following form (1)

$$f(t) = p + [q - p]F(t) - q[F(t)]^2 \tag{1}$$

The Bass Model parameter representing the potential market, which is the ultimate number of purchasers of the product, is constant. Time intervals are numbered sequentially with the first full-time interval (usually a year) of sales at t = 1 in the Srinivasan-Mason form of the Bass Model equations. A time interval is denoted "t". The Bass model coefficient (parameter) of innovation is "P". The Bass model coefficient (parameter) of imitation is "q". The portion (fraction) of the potential market that adopts at time t is f(t). The portion (fraction) of the potential market that has adopted up to and including time t is F(t).

f(t) is the time derivation of F(t), which is expressed

$$f(t) = \frac{d[F(t)]}{dt} \tag{2}$$

The second approach to the market dynamics equations is based on the Furstholz equation [22] borrowed from the simulation of biological systems with limited resource growth; it is often also generalized in the form proposed by Lotka and Volterra [3, 14] which is mathematically similar to Bass model with different modifications.

Since the market of payment systems is quite specific, we will draft equations from scratch and then discuss their relation to the above-mentioned models.

3 C2B Payment Systems

Let's imagine that company X launches a mass retail payment system from scratch for some new audience. An example would be the launch of Diners club credit cards in Russia in 1995 or PayPal e-wallets in the US in 1998. Suppose also that:

1. the behavior of a potential user is random, that is, the user chooses the moment to start using this payment system randomly (while the decision itself, of course, is influenced by user's needs and market supply);
2. the decision is made independently of current users and potential users. In our experience, this behavior is typical for commoditized systems. Thus, for C2B systems, we neglect, unlike the Bass model, the mutual influence of users (or, as Bass defines it, the effect of imitation). As will be shown below, this assumption is confirmed by the empirical data. We will also correlate our model and the Bass model below.

We also introduce the main indicators:

1. the current number of users x;
2. the maximum number of users, for example, the entire audience of a given country N. Therefore, the number of potential users not currently participating in the system is N-x;
3. audience capture rate, which reflects the probability that a given user will start using the service: a > 0 (the reverse time of the decision) within a given period.

Suppose a customer joins the payment system once and continues using it forever. Then the dynamics of the system, expressed in the number of users, is described by the simplest differential equation.

$$\frac{dx}{dt} = a(N - x) \tag{3}$$

with initial condition x (0) = 0. The solution is

$$x = N(1 - e^{-at}) \tag{4}$$

When t is small, the number of users grows linearly, when t is large, audience that has not joined the system decreases exponentially. Two simple but useful modifications can be introduced into Eq. (4) for further consideration.

Firstly, we introduce the fatigue factor b > 0, which describes the probability that an existing user will stop using the system (with the possibility to return). Then Eq. (4) becomes

$$\frac{dx}{dt} = a(N - x) - bx \tag{5}$$

with solution

$$x = \frac{a}{a+b} N\left(1 - e^{-(a+b)t}\right) \tag{6}$$

and audience growth rate

$$\frac{dx}{dt} = ae^{-(a+b)t} \tag{7}$$

Thus, we see that for small t

$$x = aNt \tag{8}$$

and user fatigue obviously does not affect the growth rate. With large t the system adopts a stationary value

$$x = \frac{a}{a+b}N \tag{9}$$

This value is achieved faster, but the volume of system's audience is smaller than if the fatigue factor is not accounted for.

The empirical nature of these equations should be emphasized. The exact definition of 'usage termination' is difficult to determine. It can be assumed that the average user uses a payment system with a frequency c $\gg$ a, and the termination is the non-usage of the system for a time, much more than a.

Let's look at the model where the client first joins the system and loses interest in using it after some time (However he/she may start to use it again in future, we do not suppose long-lasting negative experience). Again, in any case, such models remain empirical, although, in our opinion, extremely useful and meaningful for the analysis of the behavior of payment systems.

Secondly, suppose that at time t = 0, there is already some number of clients (x_0). Then (7) takes the form

$$x = \frac{a}{a+b}N\left(1 - \left(1 - \frac{(a+b)\frac{x_0}{N}}{a}\right)e^{-(a+b)t}\right) \tag{10}$$

Thus, the increase in the number of users with a non-zero initial condition is

$$\Delta x = x_0 e^{-(a+b)t} \tag{11}$$

Equation (10) can be rewritten in a simple form

$$x = x_\infty\left(1 - \left(1 - \frac{x_0}{x_\infty}\right)e^{-\frac{t}{T}}\right) \tag{12}$$

$$x_\infty = \left(\frac{a}{a+b}\right)N \tag{13}$$

$$T = \frac{1}{a+b}; \tag{14}$$

or

$$z = \left(1 - (1 - z_0)e^{-\frac{t}{T}}\right) \tag{15}$$

Where $z = \frac{x}{x_\infty}$. Note also that within the framework of this model we can consider the values of

$$\left(\frac{a}{a+b}\right)N < x_0 < N \tag{16}$$

then the value of x (t) will decrease, tending back-exponentially to the stationary one.

Although counterintuitive for non-mathematics, two phenomena are typical for Poisson's processes: the rate of audience growth is defined by the sum of the involvement coefficient and the fatigue coefficient; and changes in initial conditions quickly fade with time. It is also important that the long-term behavior of the system, (9) cannot be inducted from its initial stage (8), does not allow estimation of b, which reduces auditorium at long terms. This explains, partly, the existing hyper-optimism in the assessment of payment startups and their small success rate [7].

There is also another interpretation of the results. The time t = 0 does not have to coincide with the start of the system. Differential equations of this type have no memory and their behavior at $t \gg T$ is determined only by coefficients a, b and not by the initial value. Thus, if at the time t0 we have drastically changed the number of users of the system and/or the coefficients, then the future behavior of the system will also be described by the formula (10). Therefore, we can say that changes in the coefficients significantly affect the long-term behavior of the C2B system, while one-time promotions that attract a lot of new customers without affecting the coefficients – do not. Similar effects will be shown by external events, such as advertising or regulatory changes (for example, requiring a one-time identification): they may also have a rapid effect on the audience. If these changes have not significantly affected a and b (for example, this identification is very simple and generally available), the effect of this impact will be quickly mitigated at time T.

It is also useful to mark that these equations are quite general. They can be applied not only just for payments, but also for B2C services, for example, in terms of issuing loans and microcredits; in general—to most financial services, where customers are repetitively using a service with an infinite capacity of one of the payment participants group. This may not be true for the C2B systems at the initial stage, when, for example, the number of stores that accept the newly emerged payment instrument is very limited. Analysis of such cases requires further fine-tuning of the model that will be attempted at further stages of our work.

4 P2P Payment Systems

Along with payment systems for goods and services, where the behavior is determined by each user independently, there are payment systems with binary behavior, where the decisions of some users are affected by the decision of others. A simple example is money transfer system or card-to-card system, you need your counterparty to use it as well (of course, it is possible that both sides join the system at the same time, but this is less probable). Then the simplest dynamics of the P2P system will be described by the equation:

$$\frac{dx}{dt} = a(N - x)x \tag{17}$$

with the solution

$$x = \frac{x_0 N e^{aNt}}{x_0(e^{aNt} - 1) + N} \tag{18}$$

At small t the audience grows exponentially

$$x = x_0 e^{aNt} \tag{19}$$

at large t, the number of people who has not joined the system, decreases as

$$x = \frac{N^2}{x_0} e^{-aNt} \tag{20}$$

It is obvious that there is no non-trivial solution at $x_0 = 0$ (at least one client is needed in our assumptions) and, unlike the first model, the dynamics should be more sensitive to the size of the initial audience.

You can also add a fatigue factor b into this equation. In this case, the result is similar to the Ferhulst equation.

$$\frac{dx}{dt} = a(N - x)x - bx \tag{21}$$

which is well studied in relation to, for example, biological models and allows for an analytical, albeit more cumbersome, solution, which, for the sake of clarity, can be described as:

$$x = \frac{K x_0 e^{rt}}{K - x_0 + x_0 e^{rt}} \tag{22}$$

or

$$\frac{x}{K} = \frac{e^{rt}}{\frac{K}{x_0} - 1 + e^{rt}} \tag{23}$$

where

$$K = \frac{(aN - b)}{a} \tag{24}$$

asymptotic stationary solution (21), and

$$r = aN - b \tag{25}$$

Subsequently, we will also need a number of new customers (registrations) R per unit of time

$$R = a(N - x)x = a\left(N - \frac{Kx_0e^{rt}}{K - x_0 + x_0e^{rt}}\right)\frac{Kx_0e^{rt}}{K - x_0 + x_0e^{rt}} \tag{26}$$

We see that the behavior of such a system is more complex than for the first one. On small time intervals, where x(t) $\ll$ K the system shows fast growth

$$x = x_0e^{rt} \tag{27}$$

At point $x(t) = \frac{K}{2}$, the function x (t) has an inflection (see Fig. 1) and the growth rate starts to decrease. At large values of t

$$x = K \tag{28}$$

That means there is a limit of users independent of the initial conditions. Thus, the system has either a stage of rapid growth which eventually leads to the period of a stationary value or only a stage where the system moves to a stationary value. This equation also has decreasing solutions at $x_0 > K$. Figure 1 shows the graph of the solution for the 1st type equation (Eq. (6)).

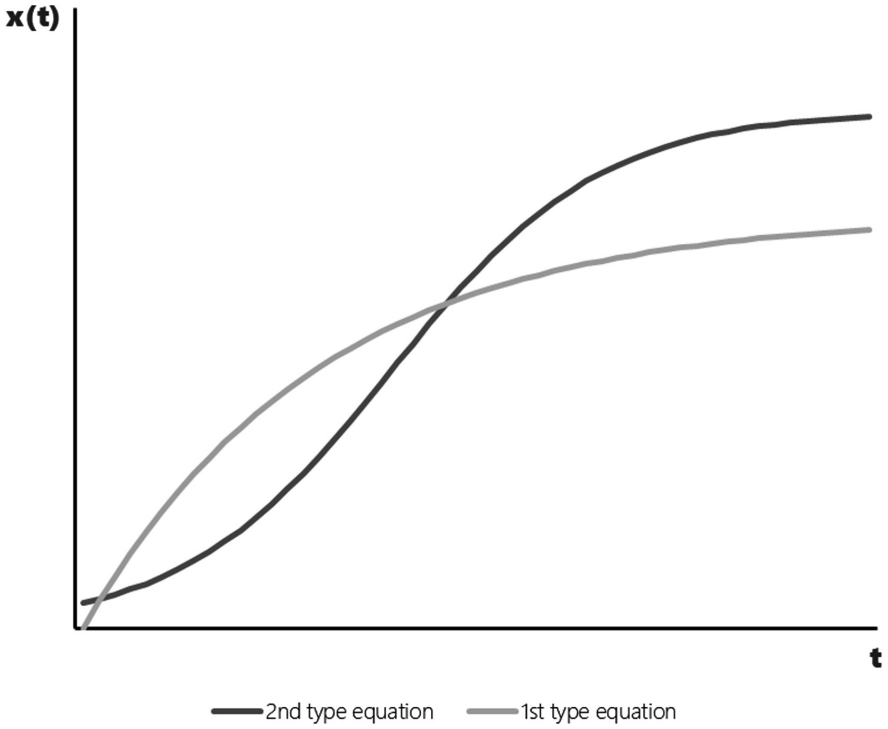

Fig. 1. Graph the solution of the logistic equation.

Similarly, the number of registrations can also behave non-monotonously. At small t it is small

$$R = \left(\frac{r}{K}\right)(N - x_0)x_0, \tag{29}$$

At large t

$$R = a(N - x)x = r(N - K), \tag{30}$$

If 2 K > N, then R reaches an extreme between these values. For small x_0, the extremum is reached at the point

$$t = \frac{1}{r} ln \frac{KN}{(2K - N)x_0} \tag{31}$$

The lack of the system's memory is relevant here as well. Its behavior is determined by the set of a and b. However, we see that such system is more sensitive to the value of x_0 and advertising or regulatory effects might have more persisting effects.

Practical conclusions can already be drawn from these simple discussions. For example, we have data about system dynamics behavior over a period. After analyzing the second derivative, we can make assumptions about the dominant type of transactions in the system—whether these are P2P or C2B transactions.

We can also analyze the results of the advertising campaign and see whether it has persisting consequences. Finally, if we know about the prevalence of payments (C2B) or transfers (P2P)—we can confidently predict the effects of advertising or the regulatory actions, or changes in the coefficients, and to understand whether the system is at the stage of growth or saturation. It is also possible to emphasize the information from the reverse analysis. In particular, the fatigue coefficient b can indicate the potential of the system to retain clients. This provides additional information for further fine-tuning of marketing strategies and directions of product development.

Analysis of publications on market dynamics inevitably leads to the analogy of the Ferhulst Eq. (21) and subsequent solutions to the Bass equation [10]. While Ferhulst equations and Bass equation are mathematically very similar, there some differences in the interpretation of the parameters.

Bass interprets the appearance of a quadratic multiplier as a consequence of the simulation effect. In P2P payment systems, the nature of this multiplier is more technical – you can't make a transfer if your recipient is not a user of the system. Thus, we can expect that for P2P systems the role of this parameter will be greater than in the diffusion theory of consumer goods and the results will be closer to the empirical data.

We also take into account the fatigue effect, due to which the user can stop using the system at any time, which is absent in the original Bass model.

5 Application to Real-Life Cases

Unfortunately, detailed data on payment systems are usually qualified by companies as confidential and are not publicly available. However, we obtained data sets for some empirical cases.

Let's consider the statistics of the European Central Bank on non-cash payments 2015 [12]. Here we make a raw hypothesis that the number of payments is approximately proportional to the number of active cards. The increase in the number of such operations is well described by the solution of the 1st type: it is a good fit with characteristic T = 15 years (Fig. 2).

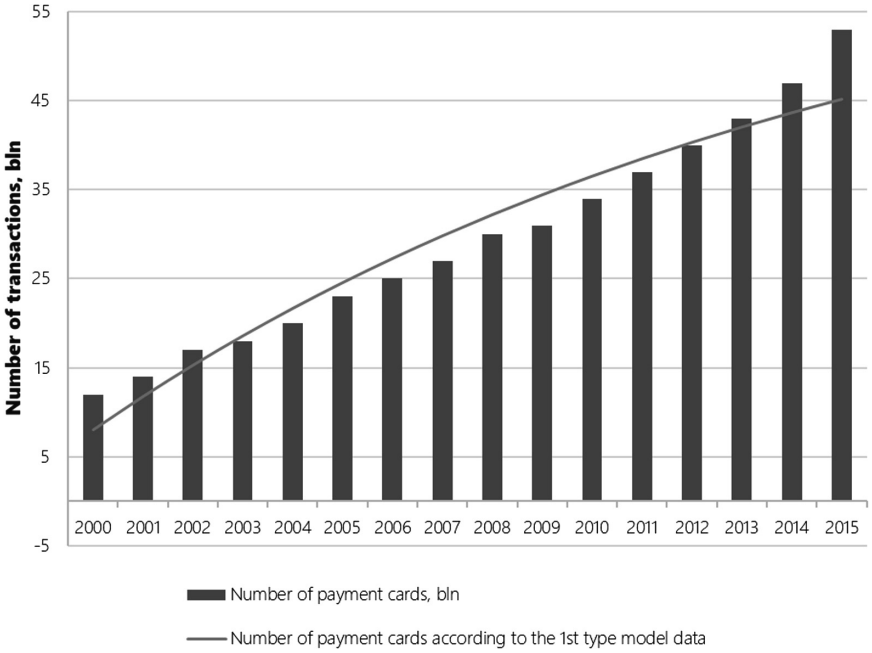

Fig. 2. Card payments in the EU and the 1st model data

According to the Bank of Korea report for 2013, in 2012, for the first time since 2004, when they began to collect statistics, the trend reflecting the number of issued cards per year has turned downward [19]. Here we make a raw hypothesis that the number of issued cards is approximately proportional to the number of active cards. We also see a good fit with characteristics of T = 10 years; n = 180 mln (Fig. 3).

Let us also consider the data of the Central Bank of the Russian Federation on payment cards in use [5] by half-year periods. Model fits well with T = 6 years, N = 390 mln. Application of the model to these cases also supports evident facts that younger markets are growing faster and bigger country has the bigger market capacity (Fig. 4).

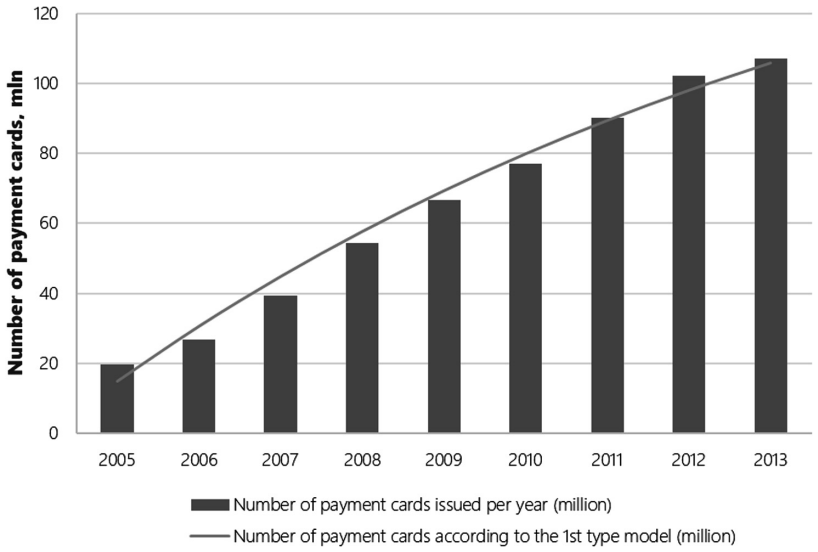

Fig. 3. Number of debit card in South Korea and the 1st model data

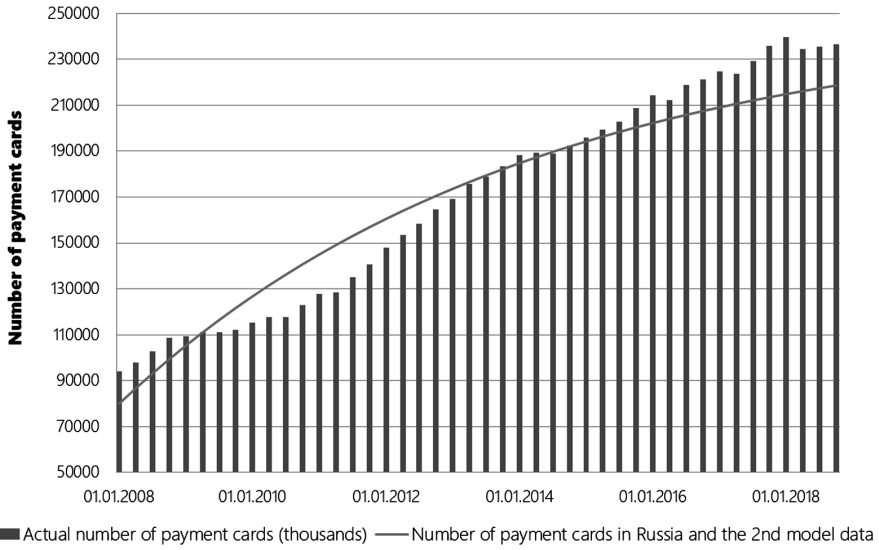

Fig. 4. The number of payment cards in Russia and the 2nd model data

5.1 WebMoney

An example of data from open sources is WebMoney statistics [17]. These statistics provide cumulative data up to 2014. After 2014 the company has only been publishing the data on a particular date. Therefore, we calculated this data for annual values.

The presence of inflection on the chart clearly indicates the predominance of P2P services, although C2B segment is also catered to by the system. As can be seen from the graph, at r = 0.48; K = 21000 (thousands) and N = 25500 (thousands); x_0 = 80. The graph of the formula (24) perfectly describes the empirical data.

If we try to predict the number of registrations for 2019 using the type two equation based on the available data, we will get 2.3 million registrations per year. This equates to 6 301 registrations per day, which is in line with the real-life data (Fig. 5).

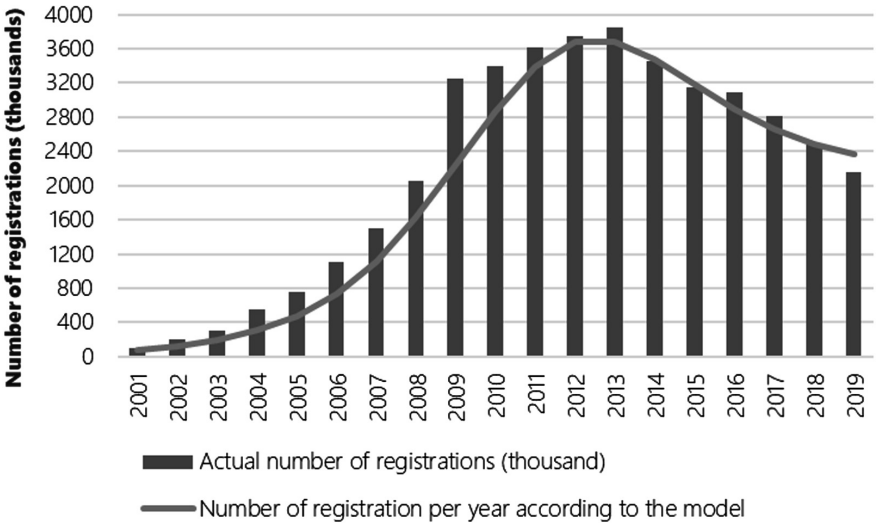

Fig. 5. WebMoney: number of wallets opened per year, thousands

An ambitious goal is to analyze such payment instrument as bitcoin and other cryptocurrencies. P2P transactions dominate in cryptocurrency schemes, so let's apply the equation to the graphs of the number of active Bitcoin [11] and Ethereum [15] wallets (Fig. 6).

As original graphs with factual data are based on the irregular temporal scale to accentuate fluctuations, analytical graphs look grainy as well: this is only a presentation problem. Obviously, the graphic representation of solutions to the (18) is smooth. We can see that the equation of the 2nd type (24) describes the behavior of both cryptocurrencies well for x_0 = 15 000, K = 550 000, r = 0,95 for bitcoin and x_0 = 1 500, K = 320 000 r = 0,5, excluding the around 2017 hype period, when the behavior of the systems was determined not by transactions, but by speculative investments. We can assume that both Bitcoin and Ethereum have reached a steady level and we will not see noticeable growth in their audience unless significant modifications to the business model or functionality are introduced.

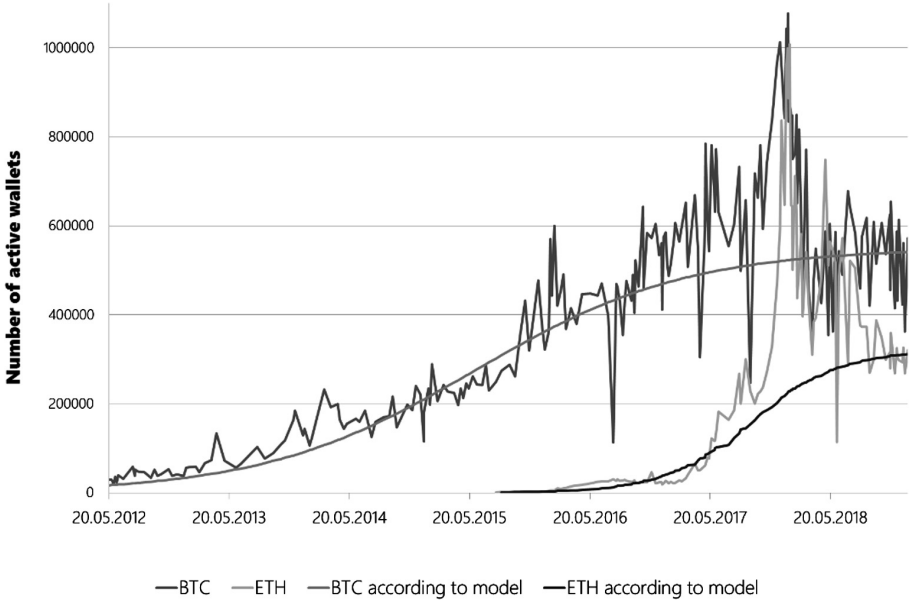

Fig. 6. Bitcoin, Ethereum. Active wallets statistic

6 Summary

This article has demonstrated the effectiveness of simple modeling methods for mass retail payment systems, including such disruptive systems as Bitcoin. The authors demonstrated that the behavior of such systems in the absence of competition can be described by one of two equations types, depending on the nature of the dominant transactions. These equations allow us to estimate the dynamics of the audience, the number of new registrations, to predict the results of advertising and regulatory actions. We have analyzed the behavior of the Bitcoin system and found that, very much like more traditional systems, it is likely to reach a floor limit in the long-run. Interestingly, surges in speculative transactions do not affect the payment functionality dynamics over time. Equations will be useful as a tool for assessing the economic behavior of such systems and as a basis for more detailed research of the financial market. The model describes some cases of early stages of payment systems development and does not include the competition concerns, which will be explored in further research.

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