

Weighted Coefficients to Measure Agreement Among Several Sets of Ranks Emphasizing Top and Bottom Ranks at the Same Time

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Abstract. In this paper, two new weighted coefficients of agreement to measure the concordance among several (more than two) sets of ranks, putting more weight in the lower and upper ranks simultaneously, are presented. These new coefficients, the signed Klotz and the signed Mood, generalize the correspondent rank-order coefficients to measure the agreement between two sets of ranks previously proposed by the authors [1]. Under the null hypothesis of no agreement or no association among the rankings, the asymptotic distribution of these new coefficients was derived. To illustrate the worth of these measures, an example is presented to compare them with the Kendall's coefficient and the van der Waerden correlation coefficient.

Keywords: Rank-order correlation · Weighted coefficients · Concordance measures · Signed Klotz coefficient · Signed Mood coefficient · van der Waerden coefficient

1 Introduction

Concordance coefficients are widely used in several applied fields. The choice for a specific parametric or nonparametric measure depends on the type of data and the purpose of agreement evaluation. The intraclass correlation coefficients [2,3] and Kendall's coefficient of concordance W [4] are examples of these two types of measures. The nonparametric measures of agreement allow to evaluate the degree of agreement or concordance among several sets of ranks. Despite in most of these measures all ranks have the same weight when evaluating agreement, several studies focus on the evaluation of agreement among the upper (respectively, lower) ranks, neglecting the disagreement in the lower (respectively, upper) ranks. In these cases, weighted rank concordance coefficients are

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the appropriate choice [5]. Iman and Conover [6] proposed the top-down concordance, which weighted the lower and upper ranks differently. However, in some situations, may be important that coefficients give more weight to the lower and upper ranks simultaneously. In the case of two sets of ranks, several measures have been proposed, namely the van der Waerden correlation coefficient [7] and the new weighted rank correlations coefficients sensitive to agreement on top and bottom rankings [8].

There are several situations where n objects are ranked by more than two independent observers and in which the interest is focused on agreement on the top rankings. Kendall's coefficient of concordance [4] assigns equal weights to all rankings and thus can not be used for this purpose. In 2014, Coolen-Maturi introduced a new coefficient of concordance which was more sensitive to agreement on the top rankings [9].

In this paper, other weighted coefficients of agreement to measure the concordance among $m > 2$ sets of ranks, putting more weight in the lower and upper ranks simultaneously, are presented. These new coefficients generalize the rank-order coefficients to measure the agreement between two sets of ranks ($m = 2$) putting more weight in the extreme ranks, previously proposed by the authors [1].

The remainder of paper is organized in five sections. Weighted coefficients to measure the concordance among several sets of ranks putting emphasis on the extremes ranks are shown in Sect. 2. In Sect. 3, distribution of the generalized weighted agreement coefficients is derived. In Sect. 4, an illustrative example is presented and in Sect. 5 some future work is mentioned.

2 Weighted Coefficients to Measure the Concordance Among Several Sets of Ranks Putting Emphasis on the Extremes Ranks

Suppose that there are n subjects to be ranked by $m > 2$ independent observers or judges or in m moments, producing m sets of ranks.

Let R_{ij} represents the rank assigned to the jth subject by the ith observer, for $i = 1, 2, \ldots, m$ and $j = 1, 2, \ldots, n$. After obtaining the m sets of ranks for the n subjects, the corresponding scores were calculated.

Consider that S_{ij} denotes a generic score, associated to the rank that was awarded by the ith observer to the jth subject, for $i = 1, 2, \ldots, m$ and $j = 1, 2, \ldots, n$. The score S_{ij} can be the van der Waerden score $W_{ij} = \Phi^{-1}\left(\frac{R_{ij}}{n+1}\right)$ [10], the signed Klotz score $SK_{ij} = sign\left(R_{ij} - \frac{n+1}{2}\right)\left(\Phi^{-1}\left(\frac{R_{ij}}{n+1}\right)\right)^2$, or the signed Mood score $SM_{ij} = sign\left(R_{ij} - \frac{n+1}{2}\right)\left(R_{ij} - \frac{n+1}{2}\right)^2$ [1]. In Table 1, it can be observed the structure of the scores for the m observers.

Note that, for any score among those three, we have $\sum_{j=1}^{n} S_{1j} = \sum_{j=1}^{n} S_{2j} = \cdots = \sum_{j=1}^{n} S_{mj} = 0$. In fact, in the case of van der Waerden and signed Klotz scores, one has: (i) if n odd then $\Phi^{-1}\left(\frac{k}{n+1}\right) = -\Phi^{-1}\left(\frac{n+1-k}{n+1}\right)$,

Table 1. Scores corresponding to the m sets of ranks

Observers/judges	Subjects						Total
	1	2	...	j	...	n	
1	S_{11}	S_{12}	...	S_{1j}	...	S_{1n}	$S_{1\bullet} = \sum_{j=1}^{n} S_{1j} = 0$
2	S_{21}	S_{22}	...	S_{2j}	...	S_{2n}	$S_{2\bullet} = \sum_{j=1}^{n} S_{2j} = 0$
$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$	$\ddots$	$\vdots$	$\vdots$
i	S_{i1}	S_{i2}	...	S_{ij}	...	S_{in}	$S_{i\bullet} = \sum_{j=1}^{n} S_{ij} = 0$
$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$	$\ddots$	$\vdots$	$\vdots$
m	S_{m1}	S_{m2}	...	S_{mj}	...	S_{mn}	$S_{m\bullet} = \sum_{j=1}^{n} S_{mj} = 0$
Total	$S_{\bullet 1}$	$S_{\bullet 2}$	...	$S_{\bullet j}$	...	$S_{\bullet n}$	$S_{\bullet\bullet} = 0$

for $k = 1, 2, \ldots, \lfloor \frac{n}{2} \rfloor$, and $\Phi^{-1}\left(\frac{(n+1)/2}{n+1}\right) = \Phi^{-1}\left(\frac{1}{2}\right) = 0$; (ii) if n even then $\Phi^{-1}\left(\frac{k}{n+1}\right) = -\Phi^{-1}\left(\frac{n+1-k}{n+1}\right)$, for $k = 1, 2, \ldots, \frac{n}{2}$. For the signed Mood scores, notice that: (i) if n odd then $\left(k - \frac{n+1}{2}\right)^2 = \left(n + 1 - k - \frac{n+1}{2}\right)^2$, for $k = 1, 2, \ldots, \lfloor \frac{n}{2} \rfloor$; (ii) if n even then $\left(k - \frac{n+1}{2}\right)^2 = \left(n + 1 - k - \frac{n+1}{2}\right)^2$, for $k = 1, 2, \ldots, \frac{n}{2}$.

Moreover, $\sum_{j=1}^{n} S_{1j}^2 = \sum_{j=1}^{n} S_{2j}^2 = \cdots = \sum_{j=1}^{n} S_{mj}^2$ is a non null constant, denoted by C_S, that depends on the sample size n. Therefore, in [1] the authors had defined the rank-order correlation coefficient between the two sets of rankings assigned to the n subjects by the observers/judges i and k as follows

$$R_S^{ik} = \frac{\sum_{j=1}^{n} S_{ij} S_{kj}}{\sum_{j=1}^{n} S_{ij}^2} = \frac{1}{C_S} \sum_{j=1}^{n} S_{ij} S_{kj},$$

which take values in the interval $[-1, 1]$.

In this work, the idea to build two new weighted coefficients of agreement is similar to the one used on the construction of Kendall's coefficient of concordance W [4,11]. This coefficient is based on the ratio between the variance of the sums of scores awarded to subjects, denoted by S_{SS}^2, and the maximum value that this variance can attain considering the values of m and n. So, the weighted

coefficients of agreement to measure the concordance among m sets of ranks, giving more weight to the extremes ones, can be defined by

$$A_S = \frac{S_{SS}^2}{\max S_{SS}^2}$$

with

$$S_{SS}^2 = \frac{1}{n-1}\sum_{j=1}^{n}\left(S_{\bullet j} - \overline{S}_{\bullet\bullet}\right)^2 = \frac{1}{n-1}\sum_{j=1}^{n} S_{\bullet j}^2,$$

attending that $\overline{S}_{\bullet\bullet} = \sum_{j=1}^{n} S_{\bullet j}/n = S_{\bullet\bullet}/n = 0$.

Therefore

$$A_S = \frac{\frac{1}{n-1}\sum_{j=1}^{n} S_{\bullet j}^2}{\max\left(\frac{1}{n-1}\sum_{j=1}^{n} S_{\bullet j}^2\right)} = \frac{\sum_{j=1}^{n} S_{\bullet j}^2}{\max\sum_{j=1}^{n} S_{\bullet j}^2}$$
$$= \frac{\sum_{j=1}^{n}\left(\sum_{i=1}^{m} S_{ij}\right)^2}{\max\sum_{j=1}^{n} S_{\bullet j}^2}.$$

When all the observers agree on the ranks assigned to the n subjects, the rank R_{ij} is equal to j, for $i = 1, 2, \ldots, m$ and, consequently, all the m scores S_{ij} are equal. So, in this case, the sum of the scores assigned to the jth subject, for $j = 1, 2, \ldots, n$, is given by mS_{ij}. So, one has

$$\max S_{SS}^2 = \max\sum_{j=1}^{n} S_{\bullet j}^2 = \sum_{j=1}^{n}\left(mS_{ij}\right)^2$$
$$= m^2\sum_{j=1}^{n} S_{ij}^2 = m^2\ C_S.$$

Thus, the weighted coefficients to measure the concordance among m sets of ranks, putting emphasis on the extremes ones, are given by

$$A_S = \frac{\sum_{j=1}^{n}\left(\sum_{i=1}^{m} S_{ij}\right)^2}{m^2\ C_S}. \tag{1}$$

These coefficients take values in the interval $[0, 1]$.

It is possible to verify that there is a relationship between the weighted coefficients A_S and the average $\overline{R}_S$ of the weighted rank-order correlation coefficients between the $\binom{m}{2}$ possible pairs of ranks (i,k), for $i,k = 1,2,\ldots,m$. In fact, one has

$$\begin{aligned}
\overline{R}_S &= \frac{1}{\binom{m}{2}} \sum_{i<k} R_S^{ik} \\
&= \frac{2}{m(m-1)} \sum_{i<k} \left[\frac{1}{C_S} \sum_{j=1}^{n} S_{ij} S_{kj} \right] \\
&= \frac{1}{m(m-1)C_S} \sum_{j=1}^{n} \left[2 \sum_{i<k} S_{ij} S_{kj} \right] \\
&= \frac{1}{m(m-1)C_S} \sum_{j=1}^{n} \left[\left(\sum_{i=1}^{m} S_{ij} \right)^2 - \sum_{i=1}^{m} S_{ij}^2 \right] \\
&= \frac{1}{m(m-1)C_S} \left[\sum_{j=1}^{n} \left(\sum_{i=1}^{m} S_{ij} \right)^2 - \sum_{i=1}^{m} \sum_{j=1}^{n} S_{ij}^2 \right] \\
&= \frac{1}{m(m-1)C_S} \left[\sum_{j=1}^{n} \left(\sum_{i=1}^{m} S_{ij} \right)^2 - m\ C_S \right].
\end{aligned}$$

Attending to (1) it follows that

$$\sum_{j=1}^{n} \left(\sum_{i=1}^{m} S_{ij} \right)^2 = m^2\ C_S\ A_S, \tag{2}$$

so $\overline{R}_S$ can be written as follows

$$\overline{R}_S = \frac{1}{m(m-1)C_S} \left(m^2\ C_S\ A_S - m\ C_S \right) = \frac{m\ A_S - 1}{m-1}.$$

This allow us to conclude that each one of the weighted agreement coefficients A_S can be obtained from the average $\overline{R}_S$ of the weighted rank-order correlation coefficients between the $\binom{m}{2}$ possible pairs of ranks (i,k), for $i,k = 1,2,\ldots,m$, that is

$$A_S = \frac{(m-1)\overline{R}_S + 1}{m}.$$

3 Distribution of the Generalized Weighted Agreement Coefficients

For small values of n and m, it is possible to obtain the exact quantiles for the generalized weighted agreement coefficients A_S, considering all possible permutations $(n!)^m$ of rankings. This is very demanding computationally and it becomes

impossible for larger values of n and m. However, to slightly larger values n and m, it is still possible to obtain approximate quantiles using Monte Carlo simulation. For larger values of n and m, it can be presented the asymptotic distribution of A_S, under the null hypotheses of no agreement or no association between the rankings, i.e., under the hypotheses of random assignment of ranks by all observers/judges.

Theorem 1: Under the null hypotheses of no agreement or no association among the rankings, the statistic $m(n-1)A_S$ has an asymptotic Chi-Square distribution with $(n-1)$ degrees of freedom, as $m \to \infty$.

Proof 1: In p. 173 of [7] is postulated the following Theorem: *c*onsidering the number of observations in each block fixed and denoted by n, and the scores $a(1), \ldots, a(n)$ also fixed and arbitrary but not constants, and the number of blocks m tending to ∞, the statistic

$$Q_m = \frac{n-1}{m}\left(\sum_{j=1}^{n}\left(a(j)-\overline{a}\right)^2\right)^{-1}\sum_{j=1}^{n}\left(\sum_{i=1}^{m}a\left(R_{ij}\right)-m\,\overline{a}\right)^2$$

has asymptotically a Chi-Square distribution with $(n-1)$ degrees of freedom.

For the three scores considered in this work, denoted by S_{ij}, we get: $\overline{a} = \overline{S}_{\bullet\bullet} = 0$; $a(j) = S_j$ which is the score correspondent to the rank $R_{ij} = j$ (recall that when all the observers agree on the ranks assigned to the n subjects, the m scores S_{ij} are equal and denoted by S_j); $a\left(R_{ij}\right) = S_{ij}$. Therefore,

$$\begin{aligned} Q_m &= \frac{n-1}{m}\left(\sum_{j=1}^{n}a(j)^2\right)^{-1}\sum_{j=1}^{n}\left(\sum_{i=1}^{m}a\left(R_{ij}\right)\right)^2 \\ &= \frac{n-1}{m}\left(\sum_{j=1}^{n}S_j^2\right)^{-1}\sum_{j=1}^{n}\left(\sum_{i=1}^{m}S_{ij}\right)^2. \end{aligned}$$

So, considering $C_S = \sum_{j=1}^{n} S_j^2 = \sum_{j=1}^{n} S_{ij}^2$ and (1), it comes

$$Q_m = \frac{n-1}{m}C_S^{-1}\left(m^2 C_S A_S\right) = m(n-1)A_S,$$

leading to what is intended to demonstrate. □

In the case of many ties among the assigned ranks within judges, these tied observations are replaced by the average of the corresponding ranks that the observations would have if they were not tied. In this case, the usual F statistic for the two-way analysis of variance over the set of ranks can be used, as this statistics corrects for ties. This statistic has an approximate F distribution with $(n-1)$ and $(m-1)(n-1)$ degrees of freedom, as was shown by Iman and

Davenport for the case of Friedman statistics [12]. Similarly to what was done by these authors, we can state the following

Proposition 1: Considering the weighted coefficients A_S, the statistic F is given by $(m-1)A_S/(1-A_S)$, which has an asymptotic F distribution with $(n-1)$ and $(m-1)(n-1)$ degrees of freedom.

Proof 2: To deduce the statistic F, the usual one for the two-way analysis of variance over the set of ranks or, more precisely, over the set of scores obtained from ranks, the total variability in the data (SS_T) is partitioned into three components: the between subjects sum of squares (SS_{BS}), the between judges sum of squares (SS_{BJ}) and the error sum of squares (SS_E). In this case, the F statistic is the quotient between two ratios, the between subjects mean square ($MS_{BS} = SS_{BS}/(n-1)$) and the error mean square ($MS_E = SS_E/((m-1)(n-1))$, being $(n-1)$ and $(m-1)(n-1)$ the degrees of freedom of SS_{BJ} and SS_E, respectively.

In our case, the total sum of squares is given by

$$SS_T = \sum_{i=1}^{m}\sum_{j=1}^{n}\left(S_{ij} - \overline{S}_{\bullet\bullet}\right)^2 = \sum_{i=1}^{m}\sum_{j=1}^{n} S_{ij}^2;$$

the between subjects sum of squares is

$$\begin{aligned} SS_{BS} &= \sum_{i=1}^{m}\sum_{j=1}^{n}\left(\overline{S}_{\bullet j} - \overline{S}_{\bullet\bullet}\right)^2 = \sum_{i=1}^{m}\sum_{j=1}^{n}\overline{S}_{\bullet j}^2 \\ &= \sum_{i=1}^{m}\sum_{j=1}^{n}\left(\frac{S_{\bullet j}}{m}\right)^2 = \frac{1}{m^2}\sum_{i=1}^{m}\sum_{j=1}^{n} S_{\bullet j}^2 \\ &= \frac{m}{m^2}\sum_{j=1}^{n} S_{\bullet j}^2 = \frac{1}{m}\sum_{j=1}^{n}\left(\sum_{i=1}^{m} S_{ij}\right)^2 \\ &= \frac{1}{m}m^2 C_S A_S \quad \text{attending to (2)} \\ &= mC_S A_S; \end{aligned} \tag{3}$$

the between judges sum of squares is

$$SS_{BJ} = \sum_{i=1}^{m}\sum_{j=1}^{n}\left(\overline{S}_{i\bullet} - \overline{S}_{\bullet\bullet}\right)^2 = 0;$$

and finally, the error sum of squares is

$$\begin{aligned}
SS_E &= \sum_{i=1}^{m}\sum_{j=1}^{n}\left(S_{ij}-\overline{S}_{\bullet j}\right)^2 \\
&= \sum_{i=1}^{m}\sum_{j=1}^{n} S_{ij}^2 - 2\sum_{i=1}^{m}\sum_{j=1}^{n} S_{ij}\overline{S}_{\bullet j} + \sum_{i=1}^{m}\sum_{j=1}^{n}\overline{S}_{\bullet j}^2 \\
&= \sum_{i=1}^{m}\sum_{j=1}^{n} S_{ij}^2 - 2\sum_{j=1}^{n} S_{\bullet j}\frac{S_{\bullet j}}{m} + \sum_{i=1}^{m}\sum_{j=1}^{n}\left(\frac{S_{\bullet j}}{m}\right)^2 \\
&= \sum_{i=1}^{m}\sum_{j=1}^{n} S_{ij}^2 - \frac{2}{m}\sum_{j=1}^{n} S_{\bullet j}^2 + \frac{m}{m^2}\sum_{j=1}^{n} S_{\bullet j}^2 \\
&= \sum_{i=1}^{m}\sum_{j=1}^{n} S_{ij}^2 - \frac{1}{m}\sum_{j=1}^{n} S_{\bullet j}^2 \\
&= \sum_{i=1}^{m} C_S - \frac{1}{m}\sum_{j=1}^{n}\left(\sum_{i=1}^{m} S_{ij}\right)^2 \\
&= mC_S - \frac{1}{m}m^2 C_S A_S \quad \text{attending to (2)} \\
&= mC_S\left(1-A_S\right). \qquad (4)
\end{aligned}$$

Considering (3) and (4), it is proved that

$$\begin{aligned}
F = \frac{MS_{BS}}{MS_E} &= \frac{\dfrac{SS_{BS}}{n-1}}{\dfrac{SS_E}{(m-1)(n-1)}} = \frac{(m-1)mC_S A_S}{mC_S(1-A_S)} \\
&= \frac{(m-1)A_S}{1-A_S} \;\overset{\circ}{\sim}\; F(n-1,(m-1)(n-1)).
\end{aligned}$$

□

4 Example

In this example, we are going to consider three judges and twenty subjects to show the usefulness of the new weighted coefficients. The four top and bottom ranks are in total agreement, while the remaining ranks were obtained through a permutation of the numbers 5 to 16.

The three sets of ranks $\{(R_{1j}, R_{2j}, R_{3j}), j = 1, \ldots, 20\}$ are listed in Table 2 and displayed in Fig. 1. In Table 2, the correspondents van der Waerden, signed Klotz and signed Mood scores are also shown.

The Kendall's coefficient, the van der Waerden correlation coefficient, the signed Klotz and the signed Mood coefficients were calculated for these data. The 95% bootstrap confidence intervals were estimated by the percentile method, based on 100, 000 samples [13]. The results, obtained using the package *bootstrap* [14] of R software, are displayed in Table 3.

Table 2. Sets of ranks (R_{1j}, R_{2j}, R_{3j}), $j = 1, \ldots, 20$, and the corresponding van der Waerden (W_{1j}, W_{2j}, W_{3j}), signed Klotz $(SK_{1j}, SK_{2j}, SK_{3j})$ and signed Mood scores $(SM_{1j}, SM_{2j}, SM_{3j})$.

R_{1j}	R_{2j}	R_{3j}	W_{1j}	W_{2j}	W_{3j}	SK_{1j}	SK_{2j}	SK_{3j}	SM_{1j}	SM_{2j}	SM_{3j}
1	1	1	−1.668	−1.668	−1.668	−2.784	−2.784	−2.784	−90.250	−90.250	−90.250
2	2	2	−1.309	−1.309	−1.309	−1.714	−1.714	−1.714	−72.250	−72.250	−72.250
3	3	3	−1.068	−1.068	−1.068	−1.140	−1.140	−1.140	−56.250	−56.250	−56.250
4	4	4	−0.876	−0.876	−0.876	−0.768	−0.768	−0.768	−42.250	−42.250	−42.250
5	16	9	−0.712	0.712	−0.180	−0.508	0.508	−0.032	−30.250	30.250	−2.250
6	15	15	−0.566	0.566	0.566	−0.320	0.320	0.320	−20.250	20.250	20.250
7	14	5	−0.431	0.431	−0.712	−0.186	0.186	−0.508	−12.250	12.250	−30.250
8	13	7	−0.303	0.303	−0.431	−0.092	0.092	−0.186	−6.250	6.250	−12.250
9	12	12	−0.180	0.180	0.180	−0.032	0.032	0.032	−2.250	2.250	2.250
10	11	16	−0.060	0.060	0.712	−0.004	0.004	0.508	−0.250	0.250	30.250
11	10	10	0.060	−0.060	−0.060	0.004	−0.004	−0.004	0.250	−0.250	−0.250
12	9	14	0.180	−0.180	0.431	0.032	−0.032	0.186	2.250	−2.250	12.250
13	8	6	0.303	−0.303	−0.566	0.092	−0.092	−0.320	6.250	−6.250	−20.250
14	7	11	0.431	−0.431	0.060	0.186	−0.186	0.004	12.250	−12.250	0.250
15	6	13	0.566	−0.566	0.303	0.320	−0.320	0.092	20.250	−20.250	6.250
16	5	8	0.712	−0.712	−0.303	0.508	−0.508	−0.092	30.250	−30.250	−6.250
17	17	17	0.876	0.876	0.876	0.768	0.768	0.768	42.250	42.250	42.250
18	18	18	1.068	1.068	1.068	1.140	1.140	1.140	56.250	56.250	56.250
19	19	19	1.309	1.309	1.309	1.714	1.714	1.714	72.250	72.250	72.250
20	20	20	1.668	1.668	1.668	2.784	2.784	2.784	90.250	90.250	90.250

Table 3. Rank-order correlation coefficients and 95% bootstrap confidence intervals.

Coefficient	Correlation	95% C.I.
Kendall's coefficient	0.809	$(0.594, 0.923)$
van der Waerden	0.866	$(0.672, 0.950)$
Signed Klotz	0.972	$(0.886, 0.992)$
Signed Mood	0.932	$(0.802, 0.979)$

The Kendall's coefficient, that assigns equal weights to all rankings, takes the value 0.809, whereas the other three coefficients that put more weight simultaneously on top and bottom ranks reach higher values, in particular the new coefficients proposed by the authors, the signed Klotz and the signed Mood coefficients. The greater value corresponds to the signed Klotz coefficient, which achieve the value 0.972.

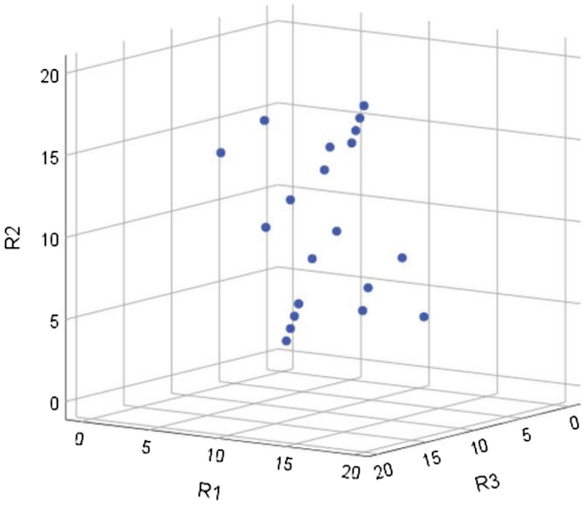

Fig. 1. 3D Scatter plot of the 20 pairs of three sets of ranks.

5 Future Work

In the continuity of this work, we intend to carry out a Monte Carlo simulation study to compare the performance of the new coefficients with other weighted coefficients that also emphasized the top and bottom ranks at the same time, but also with the Kendall's coefficient, that assigns equal weights to all rankings.

It would also be interesting to apply these new coefficients to real data in order to evaluate their performance against the usual coefficients.

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