

Investigation of the Hydraulic Unit Operation Features Based on Vibration System Data Mining

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Abstract. The software and hardware level used for on-line monitoring of the hydropower equipment functioning parameters as well as the large amounts of stored data create the necessary conditions for application of innovative methods and technologies of data analysis in the tasks of analytical assessment of equipment condition. This paper presents the results of detecting the operational patterns of the hydraulic unit in various modes and functioning conditions based on the data mining techniques – principal component analysis and cluster analysis – applied to the monitoring data of the vibration control system. In multidimensional data space, two principal components have been selected and interpreted taking into account the contribution of the data attributes to the principal components. On the plane of the first two principal components, a five-cluster structure has been constructed to define the moments of time when the system demonstrates a characteristic behaviour. In addition, the monitored parameters have been analysed in terms of time series at characteristic moments of time. As a result, the comprehensive multidimensional analysis of monitoring data has allowed us to discover the hydraulic unit operation patterns and dependencies, determine the character of the influence induced by its constructive elements and work out the ratio between the ranges of key parameters in various modes of equipment operation.

Keywords: Multi-dimensional data analysis · Principal component analysis · Cluster analysis · Vibration control system · Hydropower equipment · Estimation of technical condition

1 Introduction

Modern requirements for cost-effective maintenance of equipment dictate the need to shift from scheduled preventive maintenance to maintenance and repair of equipment based on its technical condition [1–4]. At present, the guidelines in the field of hydroelectric power plants establish monitoring requirements for the state of hydropower equipment during operation and determine the rules and procedure for forming an integral assessment of the technical condition of the main equipment [5, 6]. The level of software and hardware used for on-line monitoring of the functioning parameters of

© Springer Nature Switzerland AG 2019
S. Misra et al. (Eds.): ICCSA 2019, LNCS 11619, pp. 434–446, 2019.
https://doi.org/10.1007/978-3-030-24289-3_32

hydropower equipment, as well as the large amounts of accumulated data, create the necessary conditions for application of innovative methods and technologies of data analysis in the tasks of the analytical assessment of equipment condition without interruptions in its operation [7]. This work is aimed at detecting and studying the operational patterns of the hydraulic unit in various modes and conditions by considering the mutual influence of functional elements based on the application of principal component analysis and cluster analysis to the monitoring data of the vibration control system. Principal component analysis (PCA) is one of the most common techniques used to describe variation patterns in multidimensional data and one of the simplest and most robust ways of doing dimensionality reduction. PCA is a mathematical procedure that uses an orthogonal transformation to convert a set of observations of possibly correlated variables into a set of values of linearly uncorrelated variables called principal components [8]. Cluster analysis is a tool for discovering and identifying associations and structure within the data. Cluster analysis provides insight into the data by dividing the dataset of objects into groups (i.e. clusters) of objects, so that objects in a cluster are more similar to each other than to objects in other clusters [9]. Data mining techniques provide an effective tool for discovering previously unknown, nontrivial, practically useful and interpreted knowledge needed to make decisions [10].

This paper presents a comprehensive multi-dimensional analysis of monitoring data of the vibration control system for one of the hydraulic units of the Hydroelectric Power Station for the period from 25 June 2015 to 15 September 2015. In multidimensional data space, two principal components were selected and interpreted taking into account the contribution of the data attributes to the principal components. On the plane of the first two principal components, a five-cluster structure was constructed to define the moments of time when the system demonstrates a characteristic behaviour. In addition, the monitored parameters were analysed in terms of time series at characteristic moments of time. As a result, the comprehensive analysis of monitoring data allowed us to discover patterns and dependencies in the hydraulic unit operation, determine the character of the influence induced by its constructive elements and work out the ratio between the ranges of key parameters in various modes of equipment operation. Within this research, the analysis and visualisation of multidimensional data were conducted using the ViDaExpert tools [11].

The outline of this paper is as follows: Sect. 1 contains the introduction. Section 2 describes the original monitoring data. Section 3 presents the results of the multidimensional analysis of vibration monitoring data. Subsection 3.1 presents the results of the principal component analysis: identification and interpretation of principal components. Subsection 3.2 presents the results of cluster analysis: construction of the five-cluster structure and detailed analysis of data distribution on the clusters considering the values of key parameters. Section 4 contains the conclusions..

2 Monitoring Data Description

According to the theory of multi-dimensional data analysis, the original data are represented as a set of objects and a set of attributes. The set of objects contains the moments of time in the hydraulic unit operation throughout three months (i.e. from

07:00 am 25 June 2015 to 11:00 pm 15 September 2015) aggregated by hour. The set of attributes contains the parameters registered by Automated Control System of Technological Processes and Vibration Control System. Controlled parameters are measured by sensors located on the equipment on the left bank and on the downstream side. Data attributes are listed in Table 1.

Table 1. List of the data attributes.

No	Attribute	Description
1	A_POWER	Active power, MW
2	R_POWER	Reactive power, MW
3	RV_GB_DS	Relative vibration of the generator bearing (downstream, radially), μm
4	RV_GB_LB	Relative vibration of the generator bearing (left bank, radially), μm
5	RV_HB_DS	Relative vibration of the heel of the thrust bearing (downstream, vertically), μm
6	RV_HB_LB	Relative vibration of the heel of the thrust bearing (left bank, vertically), μm
7	RV_TB_DS	Relative vibration of the turbine bearing (downstream, radially), μm
8	RV_TB_LB	Relative vibration of the turbine bearing (left bank, radially), μm
9	AV_GB_DS	Absolute vibration of the generator bearing (downstream, radially), μm
10	AV_GB_LB	Absolute vibration of the generator bearing (left bank, radially), μm
11	AV_FH_DS	Absolute vibration of the heel of the thrust bearing (downstream, vertically), μm
12	AV_FH_LB	Absolute vibration of the heel of the thrust bearing (left bank, vertically), μm
13	AV_TB_DS	Absolute vibration of the turbine bearing (downstream, radially), μm
14	AV_TB_LB	Absolute vibration of the turbine bearing (left bank, radially), μm

Original dataset was analysed to identify preliminary correlation. The correlation coefficients between attributes are shown in Table 2. The results of correlation analysis demonstrated a strong linear relationship between the paired parameters (i.e. parameters measured by sensors on the same functional element on different sides). Since it is impractical to consider both parameters in the pair, the analysis is performed for parameters registered by sensors on the left bank. Within the analysed attributes, active power is a regulated parameter that is determined by the power generation plan.

The results of correlation analysis allow us to detect the following dependencies. Active power significantly affects the relative vibration of the heel of the thrust bearing (A_POWER ~ RV_HB_LB) and relative vibration of the turbine bearing (A_POWER ~ RV_TB_LB). At the same time, the values of relative vibration of the heel of the thrust bearing and the turbine bearing are strongly related to each other (RV_HB_LB ~ RV_TB_LB). An inverse relationship is observed between active power and both relative and absolute vibration of the generator bearing (A_POWER ~ -RV_GB_LB, A_POWER ~ -AV_GB_LB). The values of relative and absolute vibration of the generator bearing are also strongly interconnected

Table 2. Correlation coefficients between attributes.

No	2	3	4	5	6	7	8	9	10	11	12	13	14
1	**–0.72**	**–0.72**	**–0.79**	**0.80**	**0.76**	**0.80**	**0.79**	**–0.92**	**–0.88**	–0.01	–0.03	–0.50	0.15
2		0.71	**0.78**	–0.55	–0.46	–0.56	–0.57	0.71	0.65	0.10	0.14	0.51	0.26
3			**0.97**	–0.27	–0.22	–0.35	–0.34	**0.86**	**0.86**	0.26	0.32	0.33	0.06
4				–0.40	0.33	–0.47	–0.47	**0.89**	**0.87**	0.21	0.26	0.46	0.11
5					**0.99**	**0.95**	**0.95**	–0.54	–0.48	0.09	0.11	–0.48	0.11
6						**0.93**	**0.93**	–0.49	–0.44	0,05	0.08	–0.42	0.19
7							**0.99**	–0.55	–0.48	–0.08	–0.04	–0.59	–0.10
8								–0.54	–0.47	–0.07	–0.03	–0.62	–0.12
9									**0.99**	0.15	0.20	0.23	0.35
10										0.17	0.23	0.18	0.31
11											**0.99**	0.39	–0.18
12												0.32	–0.23
13													0.47

(RV_GB_LB ∼ AV_GB_LB). Moreover, a strong inverse relationship is observed between active and reactive power (A_POWER ∼ -R_POWER). Correlation analysis of the initial data has allowed us to establish the presence of relations between key parameters, determine the strength of their influence on each other and figure out the general characteristic patterns of the system.

3 Multi-dimensional Analysis of Vibration Monitoring Data

3.1 Principal Component Analysis

In order to apply the multi-dimensional data analysis, we formed a dataset which contains 1229 objects and 8 attributes. The principal component analysis was implemented to reduce the dimension of multi-dimensional space and identify patterns in the structure of multi-dimensional data. In general, the method allows identifying k components based on k initial attributes. Table 3 shows the results of calculating the eigenvectors of the covariance matrix arranged in order of descending eigenvalues.

The combination of Kaiser's rule and the Broken-stick model [12] helped to identify four principal components (PC1, PC2, PC3 and PC4). Figure 1 illustrates the eigenvalues of components. As Fig. 1 shows, Kaiser's rule determines one principal component – eigenvalue of the first component is significantly greater than the average value; the Broken-stick model gives four principal components – the line of Broken-stick model cuts off the eigenvalues of first four components.

The nature of the principal components is described by the weights of the attributes (i.e. eigenvalues) presented in Table 3. We can see that the first principal component (PC1) is characterised by a significant positive contribution of active power, relative vibration of the heel of the thrust bearing and the turbine bearing and a significant negative contribution of absolute and relative vibration of the generator bearing and reactive power. Obviously, the first component demonstrates all strong relations identified at the stage of correlation analysis. The second principal component (PC2) is

Table 3. Results of principal component calculation.

Components		**1**	**2**	**3**	**4**	5	6	7	8
Eigenvalues		0.543	0.192	0.137	0.091	0.027	0.008	0.002	0.001
Accumulated dispersion		0.543	0.734	0.871	0.962	0.989	0.997	0.999	1.000
1	A_POWER	**0.466**	0.128	0.072	0.007	0.219	0.294	0.755	0.238
2	R_POWER	**–0.394**	0.225	0.061	–0.388	0.788	–0.058	–0.041	0.121
3	RV_GB_LB	**–0.406**	0.254	–0.287	–0.237	–0.296	0.727	0.073	–0.103
4	RV_HB_LB	**0.362**	**0.359**	**–0.373**	**–0.307**	–0.160	–0.118	–0.341	0.592
5	RV_TB_LB	**0.393**	0.148	**–0.482**	–0.194	0.197	–0.105	–0.014	–0.710
6	AV_GB_LB	**–0.413**	0.038	**–0.453**	0.015	–0.202	–0.540	0.530	0.096
7	AV_FH_LB	**–0.413**	**0.585**	–0.123	**0.768**	0.180	0.041	–0.117	0.001
8	AV_TB_LB	–0.074	**0.612**	**0.563**	–0.269	–0.325	–0.253	0.109	–0.234

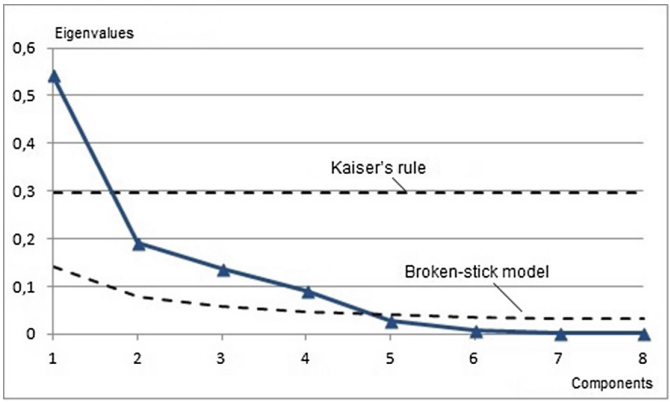


Fig. 1. Eigenvalues of components.

characterised by a significant positive contribution of absolute vibration of the heel of the thrust bearing and the turbine bearing as well as a positive contribution of relative vibration of the heel of the thrust bearing. The third principal component (PC3) is characterised by a significant positive contribution of absolute vibration of the turbine bearing, a significant negative contribution of relative vibration of the turbine bearing, absolute vibration of the generator bearing and negative contribution of relative vibration of the heel of the thrust bearing. The fourth principal component is characterised by a significant positive contribution of absolute vibration of the heel of the thrust bearing and negative contribution of reactive power. Taking into account the high information content of the first and second main components in the data structure (73% of accumulated dispersion), the further multi-dimensional analysis and interpretation of its results are performed in the context of the first two principal components.

3.2 Cluster Analysis

In order to identify the structure and discover patterns within the data, the cluster analysis was performed using the common centroid-based clustering method *k*-means. The *k*-means clustering is done by minimizing the sum of squares of distances between data and the corresponding cluster centroid. For *k*-means method, the most important and difficult question is the identification of the number of clusters that should be considered. In this study, the number of clusters was determined through the PCA technique: the number of clusters that depend on the number of principal components. Thus, referring back to the previous section, four principal components form the five-cluster structure (k = 5). The results of the clustering of the studied multi-dimensional dataset on the PCA plot are presented in Fig. 2.

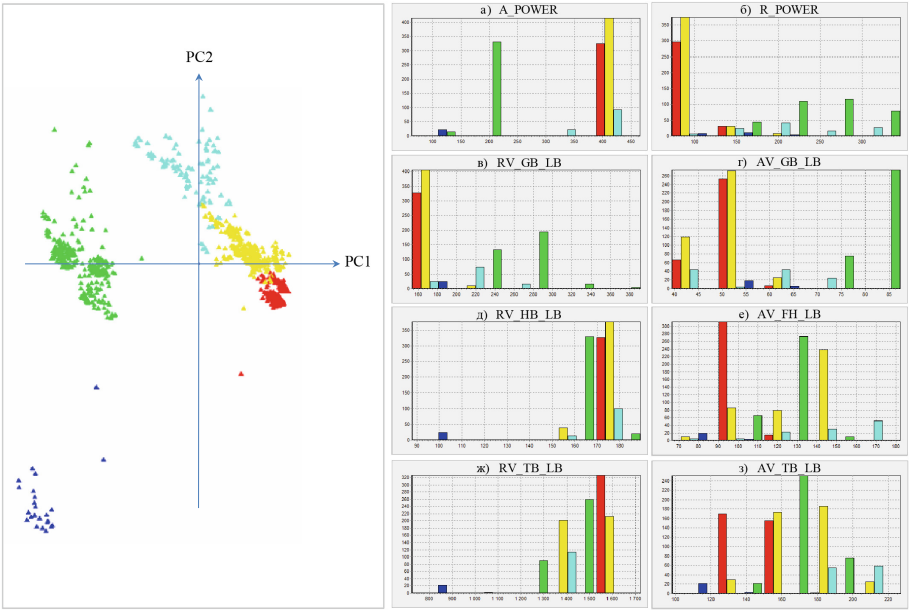


Fig. 2. Five-cluster data structure. (Color figure online)

The analysed objects are divided into three large groups that are characterised by low, medium and high values of active power. In the largest group, where high values of active power are observed, additional three clusters stand out by the values of vibrations. Thus, the five-cluster data structure includes: Cluster 1 (blue) – 24 objects, Cluster 2 (green) – 349 objects, Cluster 3 (light blue) – 114 objects, Cluster 4 (yellow) – 415 objects and Cluster 5 (red) – 327 objects.

The data distribution as per the clusters in terms of parameters is presented in Fig. 2 as a set of histograms. Cluster 1 (blue) (2% of data) differs from other clusters, it is characterised by low values of active power (appr. 120 MW, Fig. 2a) and all other parameters that correspond to the moments when the hydraulic unit is stopped or

started. Cluster 2 (green) (28% of data) is characterised by average values of power activity (appr. 215 MW, Fig. 2a) and rather high values of other parameters including maximum values of absolute and relative vibration of the generator bearing (Fig. 2c and d). Cluster 3 (light blue) (9% of data) is characterised by maximum values of active power (appr. 430 MW, Fig. 2a) and maximum values of absolute vibration of the heel of the thrust bearing support and the turbine bearing (Fig. 2e and h). Cluster 4 (yellow) (34% of data) is characterised by close to maximum values of active power (appr. 415 MW, Fig. 2a), maximum values of relative vibration of the turbine bearing (Fig. 2g) and close to minimum values of absolute and relative vibration of the generator bearing (Fig. 2c and d). Cluster 5 (red) (27% of data) is characterized by high values of active power (appr. 400 MW, Fig. 2a), minimum values of reactive power (Fig. 2b), minimum values of absolute and relative vibration of the generator bearing (Fig. 2c and d) and low values of the absolute vibration of the heel of the thrust bearing (Fig. 2e) and the turbine bearing (Fig. 2h). Highlighted clusters combine the moments of time with typical behaviour of the system that corresponds to different modes and conditions in which the power generating equipment operates.

In order to study the dependencies between the parameters in certain modes and conditions of operation of the hydraulic unit, the obtained clusters were analysed in a more detailed way in this research. As an example, consider Cluster 2 (green) of the five-cluster structure. The moments of time combined by this cluster correspond to the average level of active power, but, at the same time, the values of vibration of the generator bearing reach their maximum. To analyse the nature of the system behaviour at high values of particular parameters the data were clustered over a range of values of these parameters. Figure 3 shows the results of clustering over the range of values of relative vibration of the generator bearing in the plane of the first two principal components. The colour of points in the figure corresponds to a certain range of values: blue colour indicates the lowest values of the parameter, red colour indicates the highest values of the parameter. On the data map, there is an area that includes five objects from Cluster 2 with maximum values of relative vibration of the generator bearing.

To analyse the dependencies between the monitoring parameters, the considered moments of time were detailed to minutes and a comparative diagram for the time series of monitored parameters for one of the characteristic periods of time was constructed (Fig. 4). Figure 4 covers the period of time from 19:00 h to 22:00 h on 21st August 2015. The area marked with vertical lines depicts one of the characteristic periods of time from 20:00 h to 21:00 h when the values of relative vibration of the generator bearing (RV_GB_LB) are at their maximum. As Fig. 4 shows, a change in active power at the upper limit of average values and an increase in reactive power lead to an increase in relative vibration of the generator bearing (RV_GB_LB) (appr. 60%) and a significant increase in absolute vibration of the heel of the thrust bearing (AV_FH_LB) (appr. 45%) and absolute vibration of the turbine bearing (AV_TB_LB) (appr. 17%), however the values of absolute vibration of the generator bearing (AV_GB_LB) remain unchanged. Apart from this, in the considered period, there is an increase in the relative vibration of the heel of the thrust bearing (RV_HB_LB) and the turbine bearing (RV_TB_LB). This behaviour was observed mainly in the evening from 19:00 h to 22:00 h with the duration of not more than one hour per day in the second half of August.

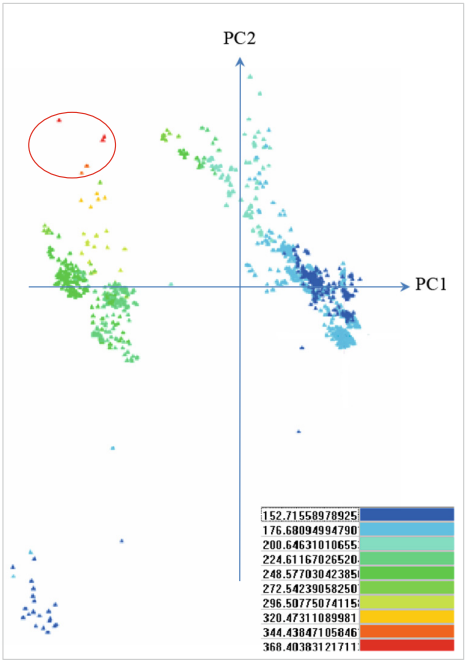


Fig. 3. Results of clustering over the range of values of relative vibration of the generator bearing (RV_GB_LB) on the PCA plot. (Color figure online)

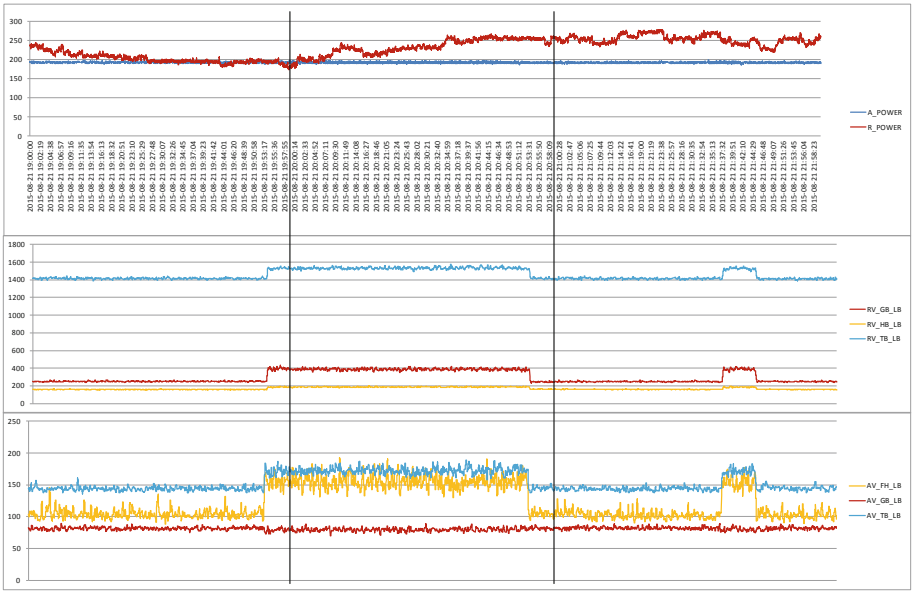


Fig. 4. Diagram of comparing the time series of monitored parameters at the moments with high values of RV_GB_LB.

Figure 5 shows the candlestick charts where the upper and lower shadows set the minimum and maximum value of the parameter in the original dataset. This graph demonstrates the relationship between the values of the main parameters and their ranges in the analyzed time interval.

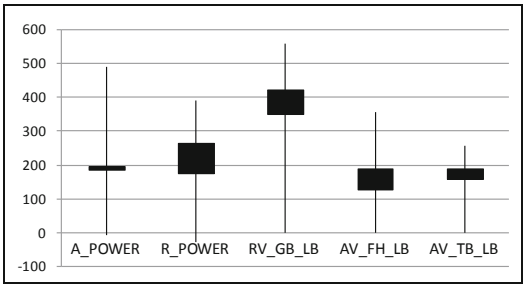


Fig. 5. Diagram of parameter ratios in the moments of time with maximum values of RV_GB_LB.

As another example, consider Cluster 5 (red) of the five-cluster structure. The moments of time combined by this cluster correspond to the high level of active power, but, at the same time, the values of vibration of the generator bearing reach their minimum, whereas the values of relative vibration of the heel of the thrust bearing and the turbine bearing are quite high. Figure 6 shows the results of clustering over the range of values of relative vibration of the turbine bearing in the plane of the first two principal components. On the data map, there is an area that includes the 215 objects from Cluster 5 with high values of relative vibration of the turbine bearing.

In a similar vein, the considered moments of time were detailed to minutes and another comparative diagram for the time series of monitored parameters for one of the characteristic periods of time was constructed (Fig. 7). Figure 7 covers the period of time from 02:00 h to 09:00 h on 1st July 2015. The area marked with vertical lines depicts one of the characteristic periods of time from 05:00 h to 06:00 h when the values of relative vibration of the turbine bearing (RV_TB_LB) are high. As Fig. 7 suggests, a slight decrease in active power in the range of high values (appr. 450 MW) leads to a slight decrease in relative vibration of the turbine bearing (RV_TB_LB) and to a significant increase in absolute vibration of the heel of the thrust bearing (AV_FH_LB) (appr. 50%) and in absolute vibration of the generator bearing (AV_GB_LB) (appr. 30%). This behavior of the system was mainly observed in the daytime continuously up to 16 h at the end of June or at the beginning of July.

Figure 8 shows a candlestick chart that illustrates comprehensively the relationship between values and ranges of key parameters in the analyzed time interval. It should be noted that the observed high values of relative vibration of the turbine bearing (appr. 1600 μm) are a distinctive feature and property of the researched hydraulic unit.

The comprehensive multi-directional analysis of monitoring data has allowed us to determine the periods when the hydraulic unit manifests its typical behaviour and the operating conditions characteristic for them, identify dependencies between parameters and their influence on each other and on the behavior of the system as a whole

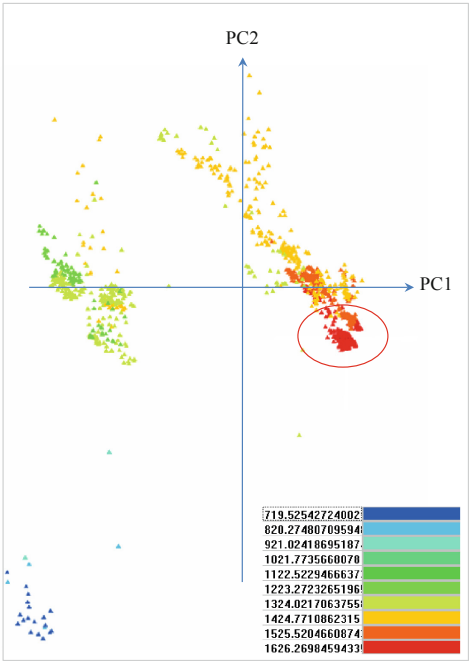


Fig. 6. Results of clustering over the range of values of relative vibration of the turbine bearing (RV_TB_LB) on the PCA plot.

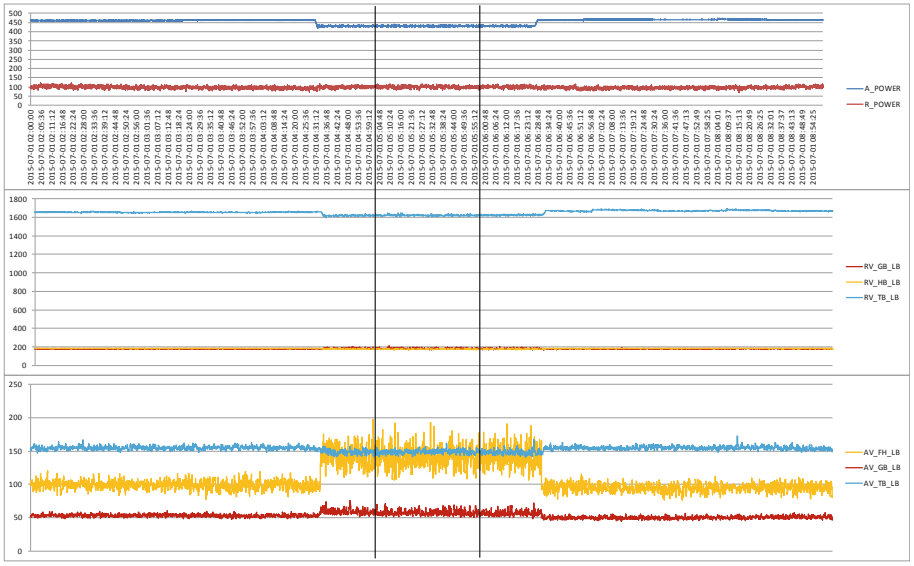


Fig. 7. Diagram of comparing the time series of monitored parameters at the moments with high values of RV_TB_LB.

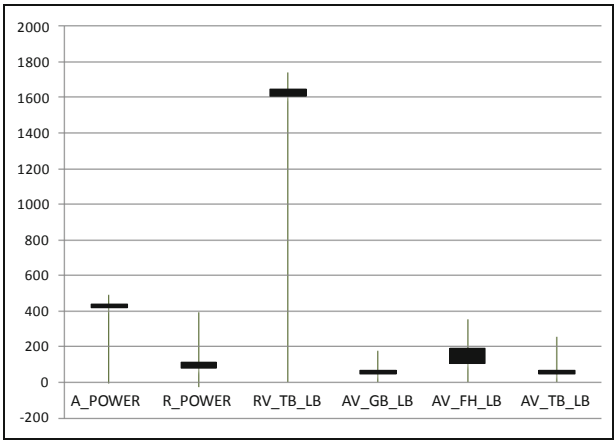


Fig. 8. Diagram of parameter ratios in the moments of time with high values of RV_TB_LB.

considering the accepted values. The modes of operation that lead to high values of vibration can be considered as adversely increasing the equipment wear, which must be taken into account when assessing the technical condition and managing the hydraulic unit life.

4 Conclusion

This paper presents the study of features of the hydraulic unit operation carried out on the basis of comprehensive analysis of monitoring data of the vibration control system. Application of the multi-dimensional data analysis techniques – principal component analysis and cluster analysis – identified the moments of time with the typical behaviour of the system that correspond to various modes and conditions in which the hydropower equipment operates.

It was determined that at an average level of active power the vibration of the generator bearing reaches its maximum values, whereas at a high level of active power it is minimal, but the relative vibration of the heel of the thrust bearing and the turbine bearing has quite high values. Moreover, the low level of reactive power corresponds to a high level of active power. Also, this study has allowed us to detect the evidence of mutual influence of functional elements of the hydraulic unit in various modes of its operation. The turbine bearing and the thrust bearing proved to have a similar behaviour pattern, their relative vibration increases with a raise in the active power level whereas absolute vibration, on the contrary, decreases. At the same time, the behaviour of the generator bearing is different, its absolute and relative vibration decreases with a raise in the active power level.

The detailed analysis of the characteristic moments of time has allowed us to identify the dependencies of the accepted values for key parameters. For instance, a slight change in the amplitude of active power at the upper limit of average values (appr. 200 MW) leads to a significant increase in the relative vibration of the generator

bearing (appr. 60%), absolute vibration of the heel of the thrust bearing (appr. 45%) and absolute vibration of the turbine bearing (appr. 60%); a slight decrease in active power in the range of high values (appr. 450 MW) leads to a slight decrease in the relative vibration of the turbine bearing (in the range of its maximum values, appr. 1600 μm) and to a significant increase in the absolute vibration of the thrust bearing support (appr. 50%) and in the absolute vibration of the generator bearing (appr. 30%).

The revealed dependencies between parameters, ranges and ratios of their values in various operating modes present the unique characteristics and distinctive properties of the particular hydraulic unit and jointly form the so-called "analytical portrait" of the hydraulic unit. A change in the technical condition of the hydraulic unit over time may manifest itself in a change in the detected dependencies between parameters, and therefore by comparing the "analytical portraits" it will be possible to identify the dynamics of equipment wear. Formalisation and verification of statistical reliability of the identified dependencies and characteristic ratios of the parameter values will help form a knowledge base with the acceptable and unacceptable modes of the hydraulic unit operation and the features of its behaviour in various conditions. The technological basis that is being developed will provide a new solution for analytical assessment of the state of equipment without putting it out of operation and resource management of the energy system.

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